Perspectives on Reynolds Stress Modeling
Part I: General Approach

Umich/NASA Symposium on Advances in Turbulence Modeling

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Overview

- Introduction
- Reynolds Stress Modeling
- Flow Physics
- Modeling Perspectives

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Introduction

Why RANS?

- Aerodynamic design/optimization
  ➔ short response times required
- RANS (EVM) based CFD very successful

Virtual product

Why RSM?

- "Virtual product"
  - ➔ CFD at off-design conditions
  - ➔ Lack of accuracy
- Representation of more complex flow physics required
  ➔ RSM naturally provides opportunities

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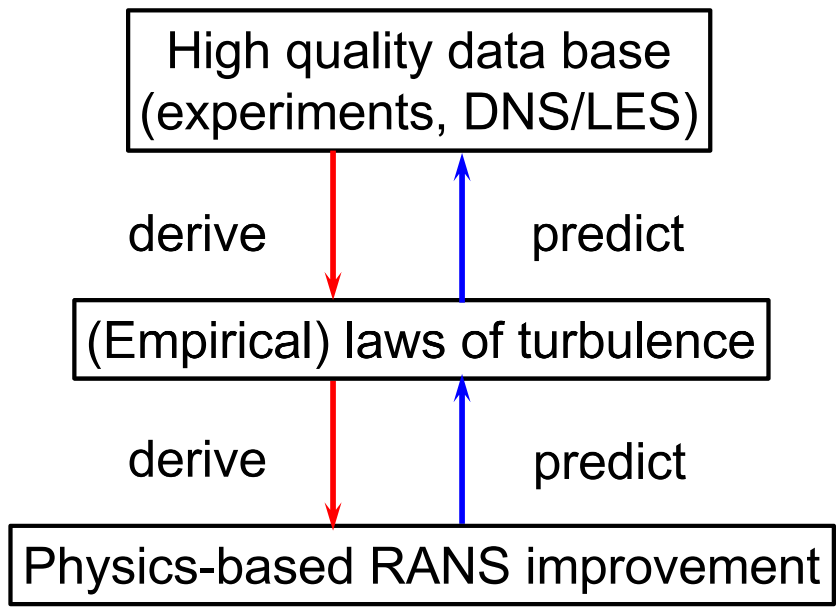
Introduction

Strategy

Physics-based modeling



- General approach for shear flows ➔ now
- APG boundary layers ➔ presentation by Tobias Knopp

Overview

- Introduction
- **Reynolds Stress Modeling**
- Flow Physics
- Modeling Perspectives

Reynolds Stress Modeling

Transport equation

- Incompressible formulation

$$\frac{\partial \overline{R}_{ij}}{\partial t} + \overline{U}_k \frac{\partial \overline{R}_{ij}}{\partial x_k} = P_{ij} + \Pi_{ij} - \varepsilon_{ij} + D_{ij}$$

- Production

$$P_{ij} = -\overline{R}_{ik} \frac{\partial \overline{U}_j}{\partial x_k} - \overline{R}_{jk} \frac{\partial \overline{U}_i}{\partial x_k}$$

Exact, no modeling involved

- Dissipation ε_{ij}
  - High Re ➔ isotropic
    - ε from length-scale equation
    - Anisotropy effects near walls ➔ e.g. Jakirlic

- Diffusion D_{ij}
  - Gradient driven modeling
  - Minor influence on overall performance

Reynolds Stress Modeling

Pressure-strain correlation Π_{ij}

- Traceless (incompressible)
  ➔ no contribution to k-budget
  ➔ re-distribution of Reynolds stresses

Rotta's analysis (1951)

$$\Pi_{ij} = \Pi_{ij}^{(s)} + \Pi_{ij}^{(r)} + \Pi_{ij}^{(b)}$$

- Slow term

  $\Pi_{ij}^{(s)}$ Return to isotropy (independent of mean flow)
  ➔ (non-linear) function of all Reynolds stress anisotropies b_{kl}

- Rapid term

  $$\Pi_{ij}^{(r)} = M_{ijkl} \frac{\partial \overline{U}_k}{\partial x_l}$$ Influence of mean flow

- Influence of boundaries

  $\Pi_{ij}^{(b)}$ Wall-reflexion terms (slow + rapid)

Reynolds Stress Modeling

Rapid-term modeling

- M_{ijkl} = function of all Reynolds stress anisotropies b_{mn}
- Constraints, e.g. symmetry (Rotta)
  ➔ reduction of terms/coefficients

Approaches

- Standard
    - M_{ijkl} linear in b_{mn} (e.g. LRR)
- Non-linear extension
    - M_{ijkl} = power series in b_{mn}
      ➔ More degrees of freedom
        - Opportunities
            - Additional physics (realizability, two-component limit)
        - Concerns
            - Rapid Distortion Theory
            - Numerics

Reynolds Stress Modeling

Calibration of RSM

- Boundary layer theory
  ➔ Turbulent equilibrium (Rotta/Hinze)

$P_{ij} - \varepsilon_{ij} + \Pi_{ij} = 0$ ➔ trace $P^{(k)} = \varepsilon$ (two-equation models)

- RSM: 3 equations for 3 coefficients = $f(b_{mn})$
  - Independent of velocity profile
  - Shear stress anisotropy by Bradshaw hypothesis
    ➔ in boundary layers $|b_{xy}| = 0.15$
  - Normal stress ratios by rule of thumb,
    e.g. Wilcox 4:2:3

Why is the Bradshaw hypothesis valid?

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Flow Physics

Theoretical considerations

- Turbulent equilibrium

$$P_{ij} - \varepsilon_{ij} + \Pi_{ij} = 0$$

- Self-similarity of U and R_{xy} + isotropic dissipation (high Re)

➔ $P_{ij}, \varepsilon_{ij}, \Pi_{ij}, P^{(k)}, \varepsilon$ Self-similar with identical profile function

- Scaling arguments for Π_{ij}
  - Slow term

$$\Pi_{ij}^{(s)} \propto \varepsilon F_{ij}^{(s)}(b_{mn}) \quad \Rightarrow \quad F_{ij}^{(s)}(b_{mn}) = const. \quad \Rightarrow \quad b_{mn} = const.$$

  - Rapid term ➔ consistent with b_{mn} = const.

➔ Flow physics principle
Boundary layer equations (turb. equilibrium) + self-similarity/self-preservation
➔ constant Reynolds stress anisotropy

- Applies to various shear flows
- Bradshaw hypothesis is special case

➔ Is the theory correct?

Reynolds Stress Anisotropy

Experimental confirmation: Plane jet

- Indicator function:

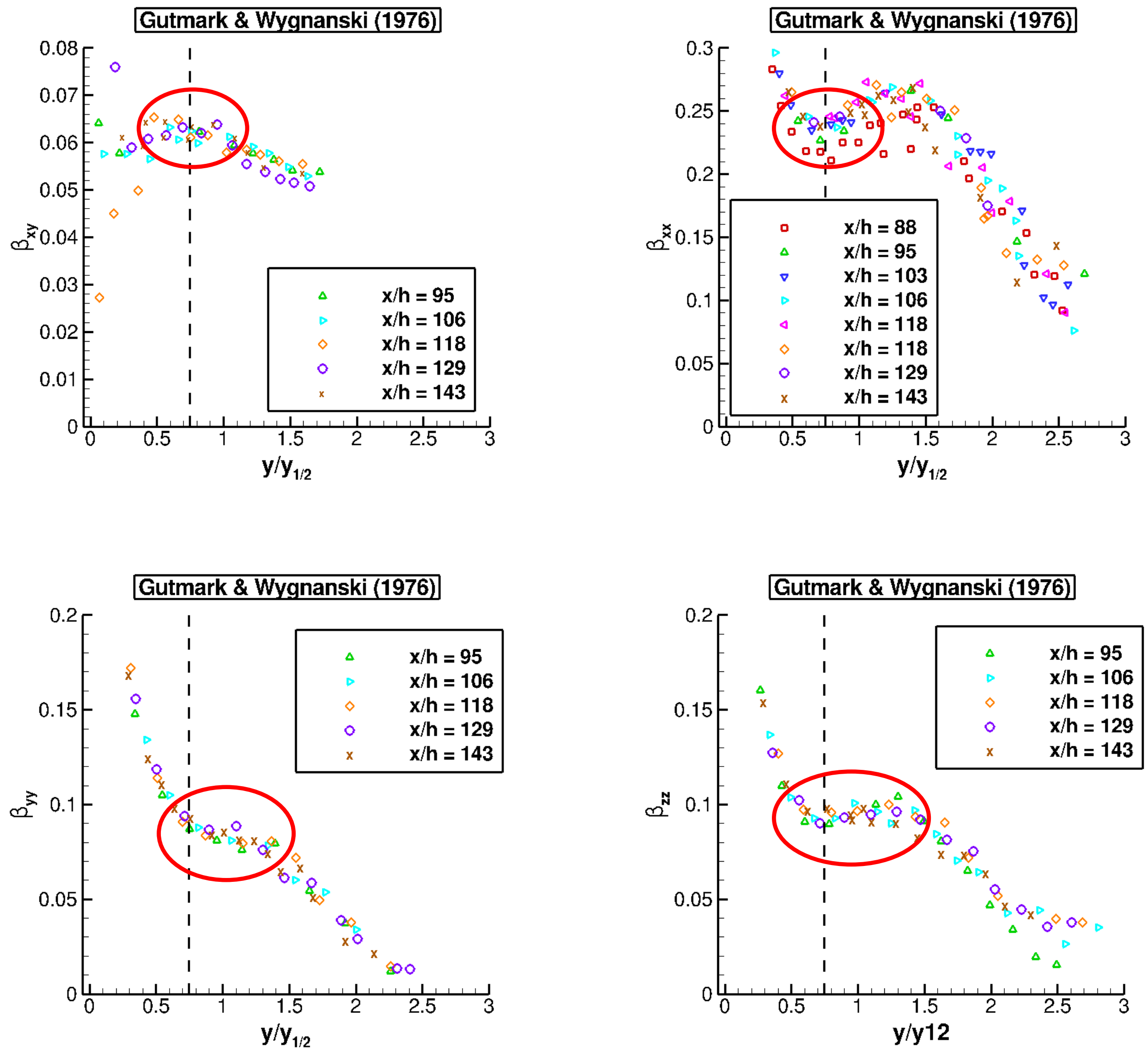
$$\beta_{ij} = \frac{\left\lfloor \overline{R}_{ij} / \left(\Delta \overline{U}_{\max} \right)^2 \right\rfloor_{\exp}}{f_{xy}(\xi)}$$

$f_{xy}(\xi)$ Theoretical profile of $R_{xy}/(\Delta U_{max})^2$

- All β_{ij} = const.
  ➔ identical profiles
  ➔ constant anisotropy

Exp. data confirm theory

B. Eisfeld:
Reynolds Stress Anisotropy in
Self-Preserving Turbulent Shear Flows,
DLR-IB-AS-BS-2017-106

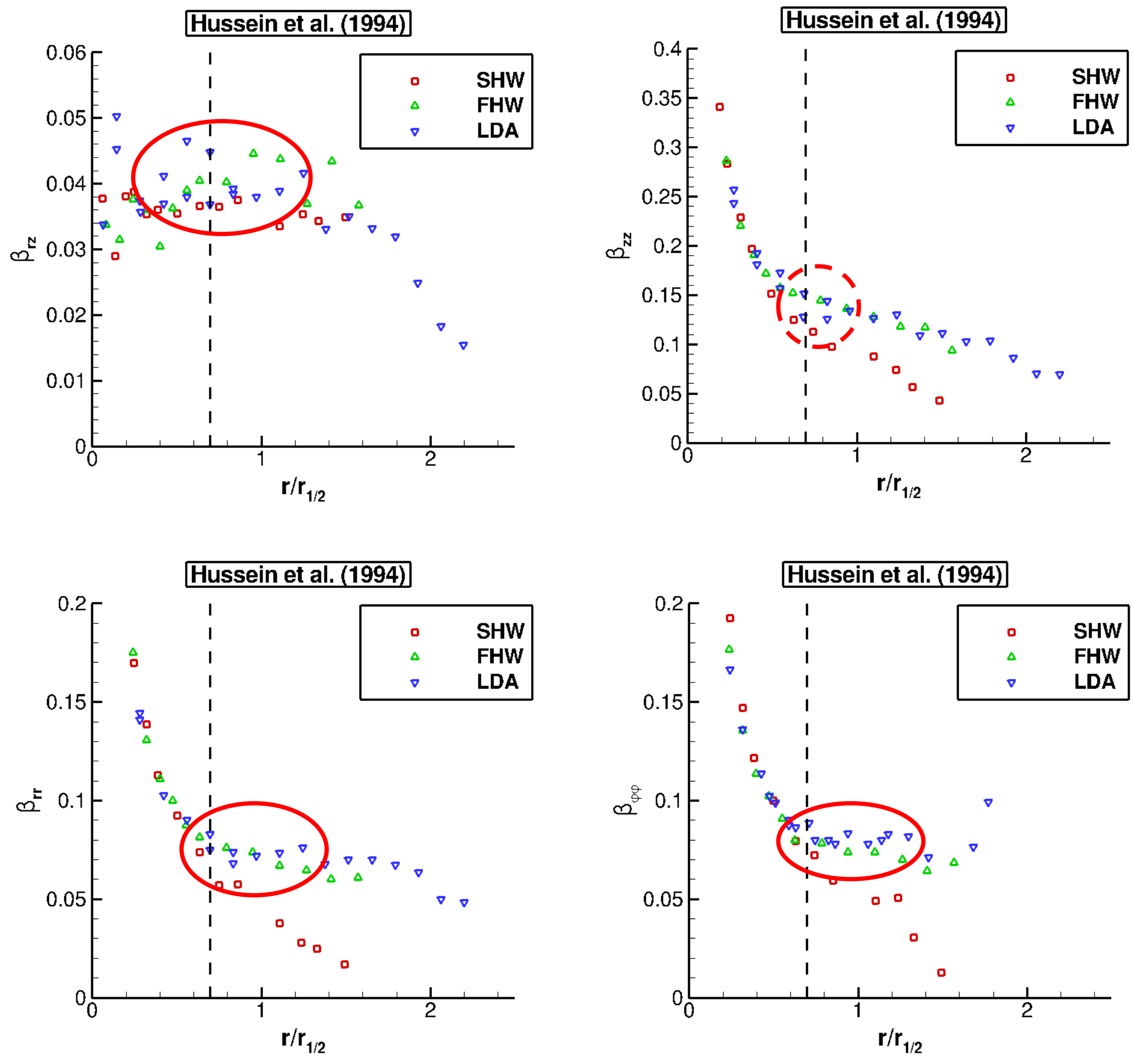


Reynolds Stress Anisotropy

Experimental confirmation: Axisymmetric jet

- Region of constant indicator function

➔ Exp. data confirm theory



B. Eisfeld:
Reynolds Stress Anisotropy in
Self-Preserving Turbulent Shear Flows,
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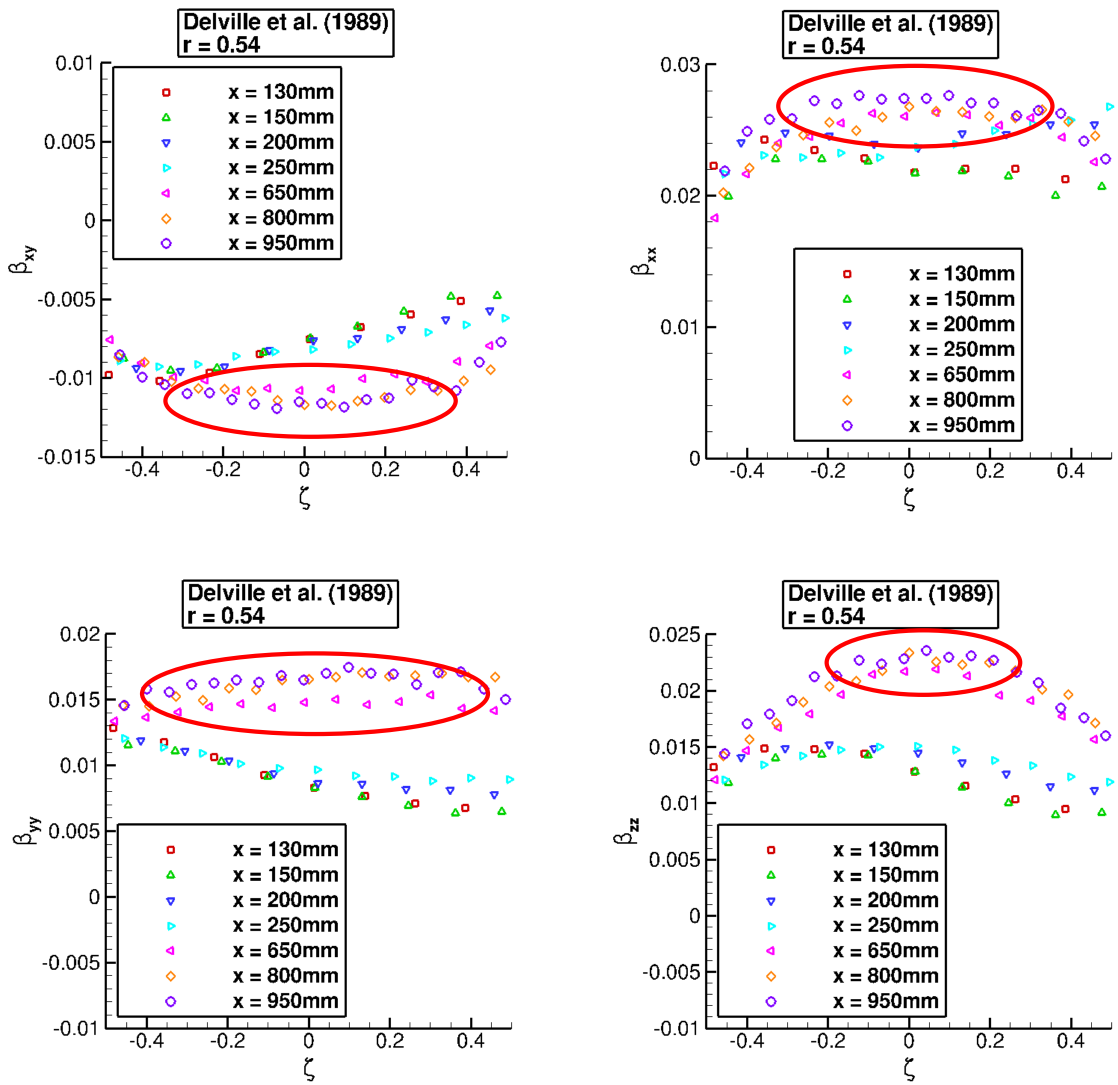
Reynolds Stress Anisotropy

Experimental confirmation: Plane mixing layer

- Region of constant indicator function

➔ Exp. data confirm theory



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DLR-IB-AS-BS-2017-106

Flow Physics

Reynolds stress anisotropy

- Provided by indicator function in constant region

$$b_{ij} = \frac{\beta_{ij}}{\beta_{kk}} - \frac{1}{3}\delta_{ij}$$

- Shear stress anisotropy (estimates)

Flow	$\lvert b_{xy} \rvert$	
Boundary layer	0.150	Calibration value
Plane jet	0.147	Similar ➔ spreading o.k.
Axisym. jet	0.131	Smaller ➔ R_{xy} overestimated (spreading)
Mixing layer	0.164±0.012	Larger ➔ R_{xy} underestimated

- Plane jet ≠ axisymmetric jet ➔ “round jet/plane jet anomaly“
- Boundary layer ≠ mixing layer ➔ Re-attachment delayed

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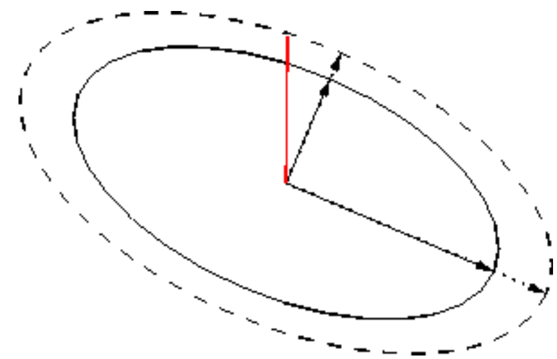
Modeling Perspectives

Mismatch of R_{xy}

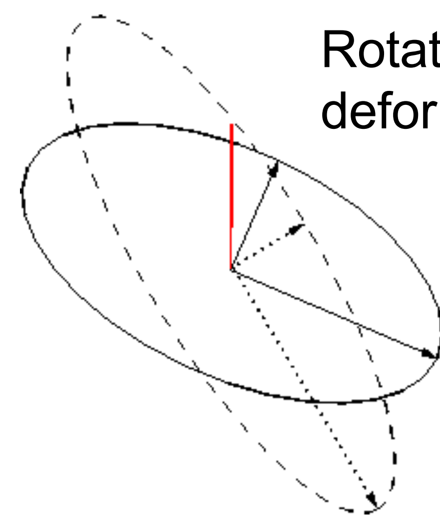
- Scaling approach
  - Modify k and keep anisotropy (boundary layer calibration)
    ➔ Modify length-scale determining equation

Scaling



- Alternative approach (experimental observation)
  - Modify anisotropy and keep k (length-scale) = change orientation of principle axes
    ➔ Consistent with RSM technology
    ➔ Adapt model calibration

Rotation+ deformation



- Combination required?

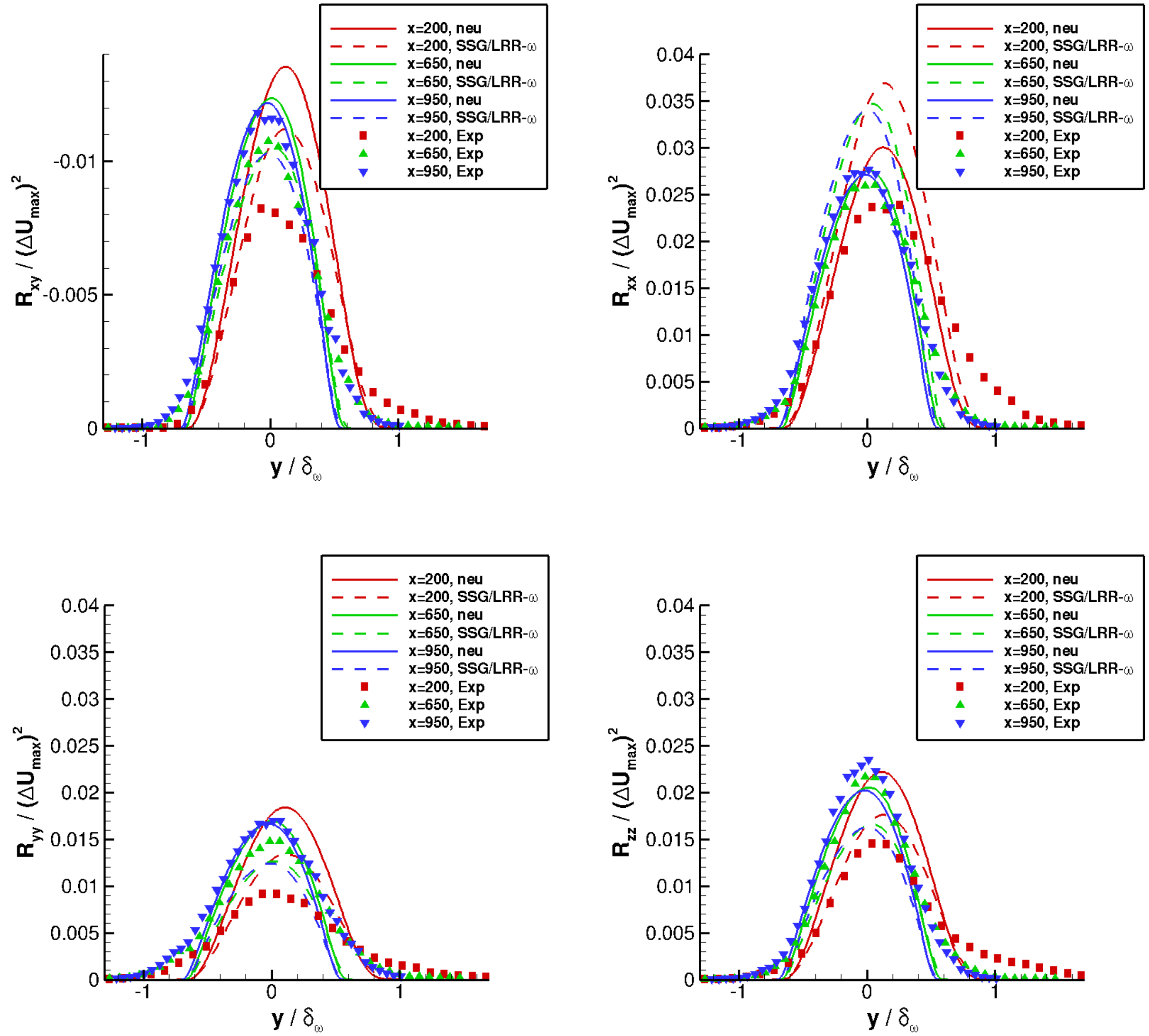
Note: Self-adaptation of model ➔ "zonal" approach

Modeling Perspectives

Example

- Baseline = SSG/LRR-ω
- Rough recalibration of SSG-part for mixing layer (Delville et al. data)

➔ get R_{ij} right at most downstream position



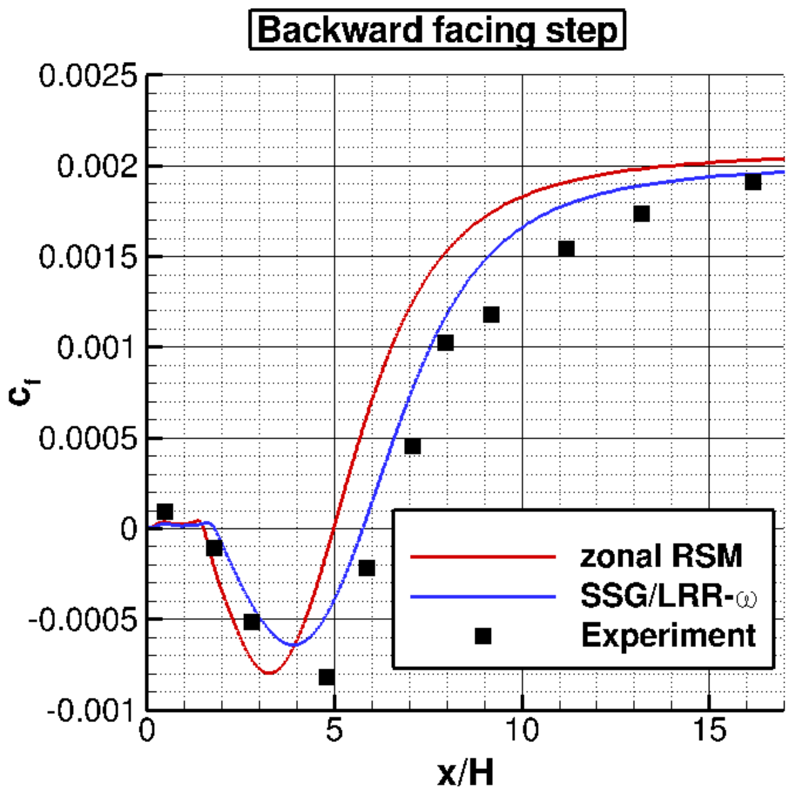
Note: for demonstration only

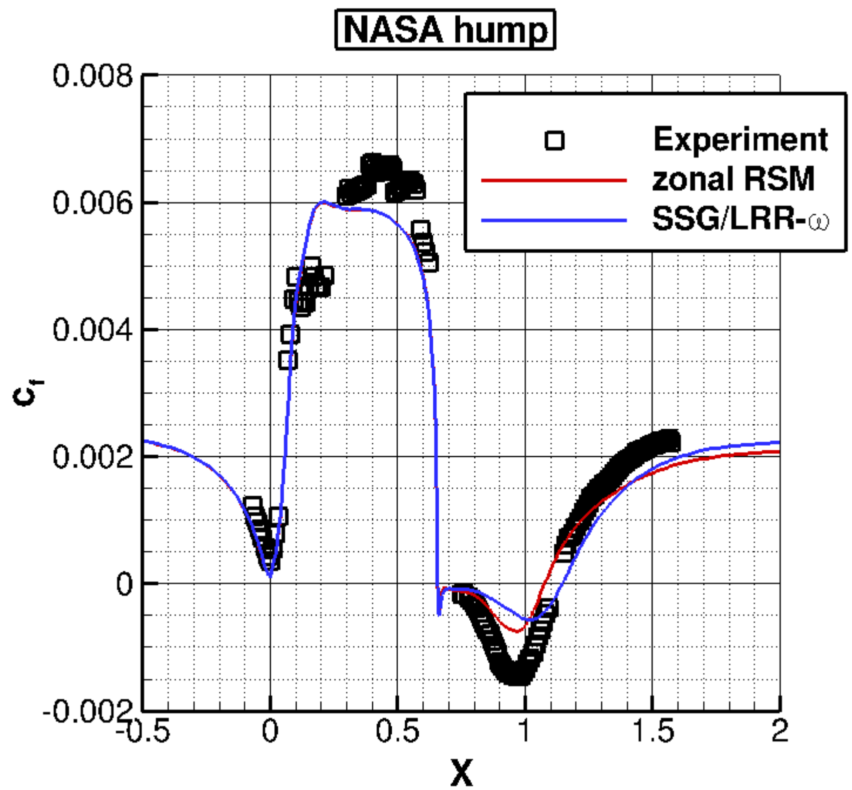
Modeling Perspectives

Example

- Application to separated flows

  ➔ Separation length reduced (for demonstration only!)





Half-jet mixing layer ≈ backward facing step ➔ model of reattachment

Modeling Perspectives

Requirements for future improvement

- Reliable anisotropy data for free shear flows and boundary layers
    - Highly accurate experiments
    - DNS
    - Requirement for self-preservation
        - High enough Re
        - Downstream development documented
- Sensors for self-adaptation
    - e.g. SAS-related?
    - Application of machine-learning methods
    - Requirements
        - Reliability
        - Suitability for RANS-based CFD
- Model analysis
    - Calibration
    - Interaction of Reynolds stress anisotropy and length-scale equation
- Improvement for APG boundary layers ➔ presentation by Tobias Knopp

Joint effort required