On Reynolds-stress Modelling of Turbulence: Conventional vs. Eddy-resolving Closure

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Objective



➤ To formulate a (near-wall) RANS-based model on the secondmoment closure level capable of capturing the turbulence fluctuations → towards an Eddy-Resolving RSM (ER RSM)













Pressure-strain term model: slow (quadratic) + rapid (linear) $\Phi_{ij,1} = -\varepsilon \left[C_1 a_{ij} + C_1 \left(a_{ik} a_{kj} - \frac{1}{3} A_2 \delta_{ij} \right) \right] \quad \Phi_{ij,2} = -C_2 \left(P_{ij} - \frac{2}{3} \delta_{ij} P_k \right)$

> DNS-aided determination of the model coefficients (FDR'13)







Pressure-strain term model: slow (quadratic) + rapid (linear)







Anisotropic dissipation correlation model







> Wall boundary conditions based on the asymptotic behaviour of Taylor microscale (λ) and its exact relationship to ε^h , i.e. ε (JFM 656, FTaC'13)







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Background RSM: $\overline{u_i u_j} - \varepsilon^h$, performances 1



Channel flow: developing vs. fully-developed



*Heukelbach, K., Jakirlić S., Nakić, R. and Tropea, C. (2002): Influence of turbulence on the stability of liquid sheets. *Int. Conf. on Liquid Atomization and Spray Systems - ILASS*, Zaragoza, Spain, September 9-11



Background RSM: $\overline{u_i u_j} - \varepsilon^h$, performances 2 -boundary layers subjected to different PG







Background RSM: $\overline{u_i u_j} - \varepsilon^h$, performances 3 -boundary layers subjected to different PG





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Background RSM: $\overline{u_i u_j} - \varepsilon^h$, performances 4



- sink-flow b. l.: turbulence development/decay





Background RSM: $\overline{u_i u_j} - \varepsilon^h$, performances 5 - *sink-flow b. l.: turbulence development/decay*









Background RSM: $\overline{u_i u_j} - \varepsilon^h$, performances: - flow over a backward-facing step

> Tested in a series of BFS configurations over a range of Re-numbers (up to 10⁵)

Background RSM: $\overline{u_i u_j} - \varepsilon$, performances: - *transonic flow configurations*

Jakirlić, S., Eisfeld, B., Jester-Zürker, R. and Kroll, N. (2007): *Int. J. Heat and Fluid Flow*, Vol. **28(4):** 602-615

RSM, complex separated flows

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Introduction: eddy-resolving models

- **RANS** computational framework ⇒ **fully-modelled turbulence**
- LES framework \Rightarrow high portion of resolved turbulence ($\ge 80\%$)
- The basic of any Hybrid RANS/LES method is a RANS-based model formulation describing the modelled fraction of turbulence
- This model is appropriately "sensitized" to account for turbulence unsteadiness (fluctuating turbulence) by introducing either

 - the von Karman length scale $L_{\nu K} = \kappa S / |\nabla^2 U|$ nominally a grid-spacing-free model formulation (SAS-related models, Menter et al.)
- Accordingly, the model equations (formulated and validated within the Steady RANS framework, describing the fully-modeled turbulence), adopt automatically (by interplaying with the grid resolution) to the highly-unsteady (unresolved, <u>residual</u>) sub-scale turbulence

Computational illustrations, VLES/PANS

Vortex structure visualization: Q-criterion (6500 s-2)

illustration of the vortex structure refinement in terms of the systematically enhanced predictive capabilities of the applied models

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Drag, lift and side force coefficients, $F_i = \iint \tau_{ji} n_j dA$, with $\tau_{ij} = -p(x, y, z) \delta_{ij} + 2\mu S_{ij}$:

RANS (RSM): Total vs. homogeneous viscous dissipation rate - ε vs. ε_h

DNS, Moser et al. -0.05 Fully-developed channel flow: $\text{Re}_{\text{m}}=22.000 \text{ (Re}_{\tau}=590)$ 0.1 1 10 y⁺ 100

RANS (RSM): Scale-supplying equation: specific dissipation rate ω , i.e. ω_h

 $\succ \omega^{h}$ -equation has been derived directly from the previous ε^{h} -equation:

$$\frac{D}{Dt}\left(\omega^{h} = \frac{\varepsilon^{h}}{k}\right) \Rightarrow \frac{D\omega^{h}}{Dt} = \frac{1}{k}\frac{D\varepsilon^{h}}{Dt} - \frac{\varepsilon^{h}}{k^{2}}\frac{Dk}{Dt}$$

with k and ϵ^{h} equations (GGDH model for diffusion in $D\epsilon^{h}/Dt$ was replaced by the SGDH model – in terms of v_{t} - type):

 $\frac{Dk}{Dt} = \frac{\partial}{\partial x_k} \left[\left(\frac{1}{2} v + \frac{v_t}{\sigma_k} \right) \frac{\partial k}{\partial x_k} \right] + P_k - \varepsilon^h \qquad \frac{D\varepsilon^h}{Dt} = \frac{\partial}{\partial x_k} \left[\left(\frac{1}{2} v + \frac{v_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon^h}{\partial x_k} \right] + C_{\varepsilon I} \frac{\varepsilon^h}{k} P_k - C_{\varepsilon 2} \frac{\varepsilon^h \varepsilon^h}{k} + P_{\varepsilon,3}$

Resulting equation for ω -homogeneous

$$\frac{D\omega^{h}}{Dt} = \frac{\partial}{\partial x_{k}} \left[\left(\frac{1}{2} v + \frac{v_{t}}{\sigma_{\omega}} \right) \frac{\partial \omega^{h}}{\partial x_{k}} \right] + \left(C_{\varepsilon I} - 1 \right) \frac{\omega^{h}}{k} P_{k} - \left(C_{\varepsilon 2} - 1 \right) \omega^{h} \omega^{h} + \frac{2}{k} \left(\frac{1}{2} v + \frac{v_{t}}{\sigma_{\omega}} \right) \frac{\partial \omega^{h}}{\partial x_{k}} \frac{\partial k}{\partial x_{k}} + \frac{1}{k} P_{\varepsilon,3}$$

SRANS (IS-RSM) Scale-supplying equation: SAS-featured ω_h -equation, 1

> Additional term, adopted from Menter & Egorov's k- ω SST SAS model, has been introduced into the ω_h -equation to make the model "instability sensitive" $\frac{D\omega_{h,SAS}}{Dt} = \frac{D\omega_h}{Dt} + \underbrace{C_{RSM,1} \max\left(P_{SAS}^*, 0\right)}_{P_{SAS}}$

$$P_{SAS}^{*} = 2.3713\kappa S^{2} \left(\frac{L}{L_{vk}}\right)^{1/2} - 3C_{RSM,2}k \max\left(\frac{(\nabla \omega_{h})^{2}}{\omega_{h}^{2}}, \frac{(\nabla k)^{2}}{k^{2}}\right) \quad \begin{array}{c} C_{RSM,1} = 0.004, \\ C_{RSM,2} = 8 \end{array}$$

with L being the turbulent length scale, S an invariant of the mean strain tensor S_{ij} and L_{vk} – the von Karman length scale

$$L = k^{1/2} / \omega^{h} \quad L_{\nu K} = \kappa S / |\nabla^{2} U| \quad \nabla^{2} U (\equiv U'') = \sqrt{\frac{\partial^{2} U_{i}}{\partial x_{i}^{2}}} \frac{\partial^{2} U_{i}}{\partial x_{i}^{2}} \quad S (\equiv U') = \sqrt{2S_{ij}S_{ij}}$$

The model doesn't comprise explicitly a grid-dependent parameter:
 towards a grid-spacing-free model formulation

Computational illustrations: IS-RSM

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Scale-supplying equation: SAS-featured ω_h -equation, 2

Selected results of 2D-hill simulation with the proposed DRSM-SAS (ω_h -eq.), Re_H=10600, 3

Shear stress component

Selected results of 2D-hill simulation with the proposed DRSM-SAS (ω_h -eq.), Re_H=10600, 4

Flow over a 2D fence, IS-RSM: P_{SAS}-term

Flow over a 2D fence, RSM vs. IS-RSM : Reynolds shear stress and t.k.e.

Flow over a 2D fence, RSM vs. IS-RSM: mean streamlines

(x_R/H)_{EXP}=11.7

Results of back-step simulation with the proposed RSM-RANS and SAS-RSM (ω_h -eq.), Re_H=5100

0.004 0.25 0.003 0.2 0.002 0.15 0.001 പ്പം 0.1 ΰ 0 -0.001 0.05 -0.002 SAS-RSM SAS-RSM **RANS-RSM** RANS-RSM DNS: Le et al., Re_H=5100 DNS: Le et al., Re_H=5100 -0.003 \odot Exp.: Jovic, Driver, Reu=5000 Exp.: Jovic, Driver, Reu=5000 -0.05 -0.004 10 х/Н 15 20 Ω 5 x/H 15 20 10 As a result of the unsteadiness capturing the SAS-RSM captures correctly high negative peak and shape of Cp and Cf developments at such a low Reynolds number 2 6 0 x/H4

100000

100000

Tandem Cylinder Configuration (Re_D=166000), description, 3

- The influence of difference in the distance between cylinders is reflected in strong structural changes.
- In the case of the longer distance the flow between the cylinders is bistable and switches between continuous shedding and intermittent shedding.
- In the short distance case the flow in the wake behind the downstream cylinder exhibits a continuous shedding behaviour. Here, the cylinders are close to each other resembling one long obstacle.

Scale-supplying equation: SAS-featured ω_h -equation, P_{SAS} -Term

$$P_{SAS}\left(\overline{u_iu_j}-\omega_h\right),\,\omega_h=\varepsilon_h/k$$

Results of Tandem Cylinder L/D=3.7 (SAS-RSM, RANS-RSM) (ω_h-eq.), Re_D=166000, Mean velocity

Results of Tandem Cylinder L/D=3.7 (SAS-RSM, RANS-RSM) (ω_h -eq.), Re_D=166000, t.k.e.

Results of Tandem Cylinder L/D=3.7 (SAS-RSM, RANS-RSM) (ω_h -eq.), Re_D=166000, pressure distribution

Results of Tandem Cylinder L/D=3.7 (SAS-RSM, RANS-RSM) (ω_h -eq.), Re_D=166000, rms of fluctuating pressure

Baseline configuration: Instantaneous velocity field

Instantaneous velocity field:

- Highly unsteady turbulent flow field captured by the IS-RSM
 - Inflow: inflow duct simulated recycling method (Baba Ahmadi, Tabor; 2009) simultaneously with the diffuser without imposing any initial fluctuations

Baseline configuration: Time-averaged velocity field

- Good agreement between SAS-RSM and Exp. (flow behaviour with separation at upper wall has been properly captured); RANS-RSM shows large corner separation on upper-right corner
- Substantially better quantitative agreement between SAS-RSM and Exp. (large overestimation of velocity at z/B=0.5 obtained by RANS-RSM due to attached flow at upper wall)

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3D Diffuser: Discharge-Based Flow-Control:

- Asymetric 3D diffuser Re = 10000
 - Database 2 (present),
 Baseline + Actuated configs.:
 - Exp.: Grundmann *et al.* (2011): *Exp. Fluids* 50 (1):217-231
 - Time averaged velocity profiles (Pitot probe)
 - Pressure distribution (pressure taps)
- Streamwise-oriented arrangement of a Plasma-Actuator:
 - force directed towards the corners
 - increased momentum transport in the wall normal direction

Baseline vs. actuated configurations flow reversal – separation zone: <u>IS-RSM + PA</u>

Pulsed mode 40% DC Baseline Case Continuous mode 100% DC \Rightarrow Enhancement of the \Rightarrow Re-stress anisotropy \Rightarrow Strong continuous acceleration induced secondary (40% of the bulk flow intensity) momentum transfer motion (2 % of the and turbulence activity towards corners (25%), correlated with turbulence weakening bulk flow intensity) activity intensification $V^{2}+W^{2})^{1/2}/U$ 15 times enlarged W/0.5 \$ 0.5 0.5 25 2.5 2 2 z/h z/h z/h

Baseline vs. actuated configurations flow reversal – separation zone: <u>IS-RSM + PA</u>

Iso surfaces of axial velocity component:

- Pulsed mode 40% DC
- $\Rightarrow Size reduction of the recirculation bubble$
- Baseline Case
- ⇒ recirculation bubble occupies the entire upper wall
- Continuous mode 100% DC
- \Rightarrow recirculation bubble occupies the entire deflected side

Baseline vs. actuated configurations Surface (bottom wall) pressure distribution

- Pressure recovery coefficient along bottom wall:
- Actuated Configuration
 - Pulsed mode DC = 40%
 - \Rightarrow increase pressure recovery
 - Continuous mode DC = 100%
 - \Rightarrow decrease pressure recovery
- Both captured correctly by applying the IS-RSM Model

2D Hill, Re_H=10600, $(\overline{u^2})_{modelled}/(\overline{u^2})_{total}$:

uu_mod/uu_tot Coarse grid: • 0.2 0.4 80 x100 x 30 = 240.000 cv 0.5 n uu_mod/uu_tot 0.2 0.4 • Fine grid: 0 0.5 $160 \times 160 \times 60 = 1.536.000 \text{ cv}$

2D Hill, Re_H=10600, t.k.e. field:

- Coarse grid: 80 x100 x 30 = 240.000 cv
- Fine grid: 160 x160 x 60 = 1.536.000 cv

2D Hill, Re_H=10600, Velocity field:

SAS Term revisited, 1

Origin of the SAS-term (Menter & Egorov, 2003-2010) is the exact kLequation, with L representing the integral lengths scale:

$$\frac{\mathrm{D}(kL)}{\mathrm{D}t} \Rightarrow \int_{-\infty}^{\infty} \frac{\partial U(\vec{x} + r_{y})}{\partial y} R_{12} dr_{y} \rightarrow \\ \frac{\partial U(\vec{x})}{\partial y} \int_{-\infty}^{\infty} R_{12} dr_{y} + \left[\frac{\partial^{2} U(\vec{x})}{\partial y^{2}} \int_{-\infty}^{\infty} R_{12} r_{y} dr_{y} \right] + \frac{1}{2} \frac{\partial^{3} U(\vec{x})}{\partial y^{3}} \int_{-\infty}^{\infty} R_{12} r_{y}^{2} dr_{y} + \cdots$$

By adopting $R_{12} \propto v_t \partial U/\partial y$ and $v_t \propto Lk^{1/2}$ with $L \propto k^{1/2}/\omega$

$$\rightarrow \qquad \frac{\mathrm{D}(kL)}{\mathrm{D}t} \Rightarrow \frac{\partial^2 U(\bar{x})}{\partial y^2} \int_{-\infty}^{\infty} R_{12} r_y dr_y \rightarrow R_{12} \frac{\partial^2 U(\bar{x})}{\partial y^2} L^2 \propto \left[\left(k^{1/2} L^2 \right) \kappa S^2 \left(\frac{L}{L_{vK}} \right)^n \right]$$

SAS Term revisited, 2

$$\frac{D\omega_{h,SAS}}{Dt} = \frac{D\omega_{h}}{Dt} + \underbrace{C_{RSM,1} \max\left(P_{SAS}^{*},0\right)}_{P_{SAS}} \qquad \nabla^{2}U\left(\equiv U^{"}\right) = \sqrt{\frac{\partial^{2}U_{i}}{\partial x_{j}^{2}}} \frac{\partial^{2}U_{i}}{\partial x_{j}^{2}}$$

$$P_{SAS}^{*} = \sqrt{k} \left| \nabla^{2} U \right| - 3C_{RSM,2} k \max \left(\frac{\left(\nabla \omega_{h} \right)^{2}}{\omega_{h}^{2}}, \frac{\left(\nabla k \right)^{2}}{k^{2}} \right) \qquad C_{RSM,1} = 0.087, \\ C_{RSM,2} = 1.375$$

In addition, the model is made numerically more robust by blending the Reynolds-stress tensor entering the equation of motion:

$$\overline{u_{l}u_{j}} = 0.7 * \overline{u_{l}u_{j}}_{RSM} + 0.3 * \overline{u_{l}u_{j}}_{Boussinesq}$$
explicit treatment implicit treatment $\Rightarrow \frac{2}{3}k\delta_{ij} - v_{t}\left(\frac{\partial \overline{U}_{l}}{\partial \overline{x_{j}}} + \frac{\partial \overline{U}_{j}}{\partial \overline{x_{l}}}\right)$

SAS Term revisited, 2d-hill (70000 cv's)

Tandem Cylinder: PSD surface pressure

Tandem Cylinder: PSD sound pressure

Concluding remarks, IS-RSM, 1

- A "conventional", near-wall RANS-RSM model was extended in line with the SAS procedure resulting in an eddy-resolving turbulence model
- the enhancement of the ω_h production associated with the P_{SAS} term is, in line with the SAS proposal by Menter and Egorov (2010), particularly appropriate for highly unsteady separated shear layer regions.
- Herewith, an adequate suppression of the modelled turbulence intensity towards the corresponding sub-scale level is provided, enabling a turbulence activity intensification originating from the resolved motion
- It implies the model's self-adaptation by balancing between the resolved and modelled (unresolved) contributions to the turbulence kinetic energy.

Concluding remarks, IS-RSM, 2

- All present computations have been performed by OpenFOAM Code; CDS scheme for diffusive transport; blended CDS(98%)/UDS(2%) scheme for convection; three-times level scheme for temporal discretization)
- In all periodic flows (channel, 2D hill) and bluff-body configurations (tandem cylinder) no initial fluctuations were imposed; the results of the steady RANS computations were applied as initial conditions.
- In the "finite-size" flow configurations (2D fence, 3D diffuser) a precursor simulations of fully-developed channel/duct flows have been performed using the same method (also here the results of the steady RANS computations were applied as initial conditions)
- In the meantime the model has ben formulated in conjunction with the equation governing the ε_h quantity

Thank you for your attention! Questions?

