Perspective on Turbulence Modeling using Reynolds Stress Models : Modification for pressure gradients

Tobias Knopp, Bernhard Eisfeld

DLR Institute of Aerodynamics and Flow Technology

Experimental data shown were obtained in cooperation with

Daniel Schanz, Matteo Novara, Erich Schülein, Andreas Schröder DLR AS

Nico Reuther, Nicolas Buchmann, Rainer Hain, Christian Cierpka, Christian Kähler

Univ Bundeswehr München, Institute for Fluid Mechanics and Aerodynamics

Motivation: Digital aerospace products

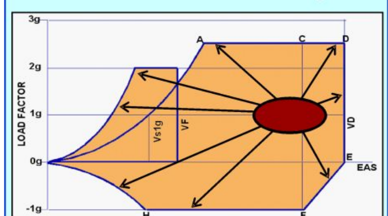
Numerical simulation based future aircraft design

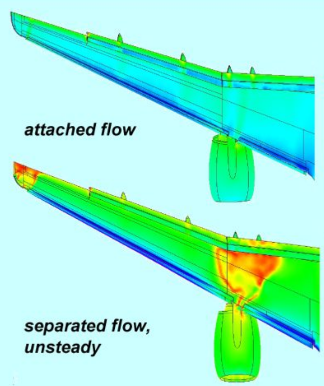
Challenges

- **A/δ_{99} extremely large: large Re and large A**
- **#simulations large for design and optimization**

=> RANS, not LES

Full flight envelope coverage: *CFD mostly done near cruise point*





configurations:

clean

airbrakes deployed

high lift

50	flight points
100	mass cases
10	a/c configurations
5	maneuvers
20	gusts (gradient lengths)
4	control laws

~ 20,000,000 simulations

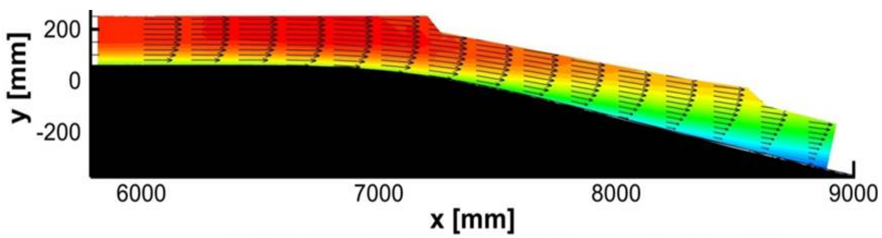
Engineering experience for **current** configurations and technologies

~ 100,000 simulations

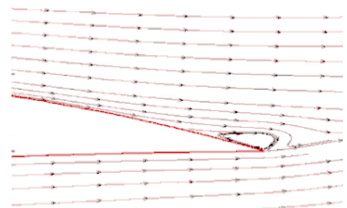
DLR strategy for RANS model improvement

- **Optimism to describe “first order” steady state aerodynamic flow phenomena within the RANS concept**

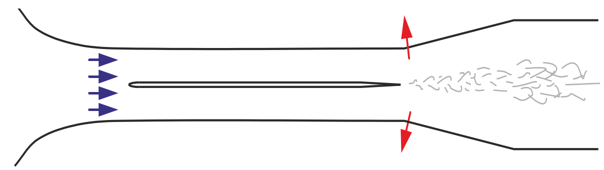
Turbulent boundary layers at APG



Separation and reattachment

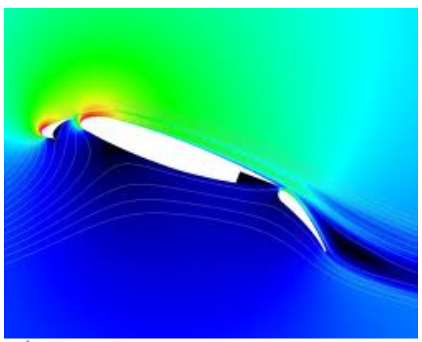


Wake flow at APG

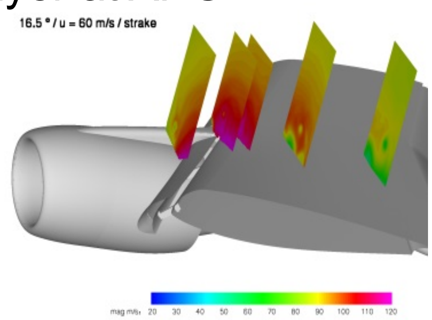


[28] Hoffenberg & Sullivan, 1998

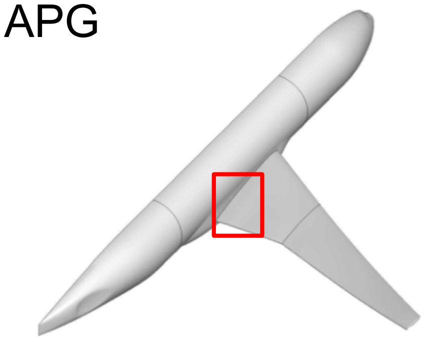
Interaction of wake flow and boundary layer at APG



Interaction of strake vortex and boundary layer at APG



Separation in the wing-body junction at APG



DLR strategy for RANS model improvement

High-quality data base (exp., DNS/LES)

DLR strategy for RANS model improvement

High-quality data base (exp., DNS/LES)

(Empirical) laws of turbulence

DLR strategy for RANS model improvement

High-quality data base (exp., DNS/LES)

(Empirical) laws of turbulence

Physics-based improvement of RANS

DLR strategy for RANS model improvement

High-quality data base (exp., DNS/LES)

(Empirical) laws of turbulence

Physics based improvement of RANS

Surface roughness

Experiments by Nikuradze

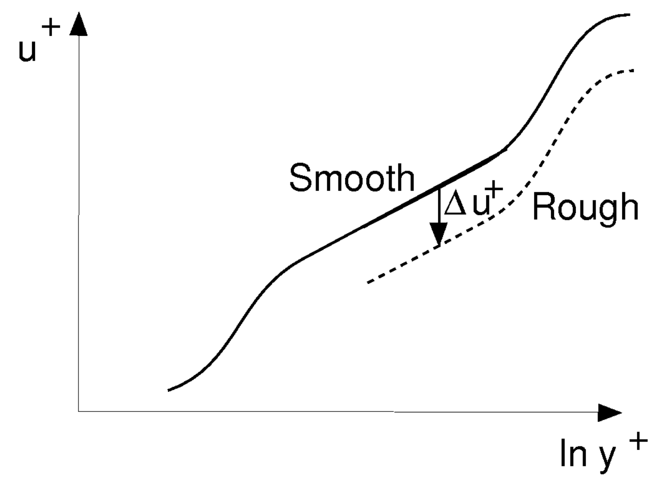
DLR strategy for RANS model improvement

High-quality data base (exp., DNS/LES)

(Empirical) laws of turbulence

Physics-based improvement of RANS

Surface roughness



$\Delta u^+ = f(k_r^+)$ $\quad$ $k_r^+ = k_r u_\tau / v$

DLR strategy for RANS model improvement

Surface roughness

High-quality data base (exp., DNS/LES)

(Empirical) laws of turbulence

Physics-based improvement of RANS

$$\nu_t = \kappa u_\tau (y + 0.03 k_r)$$

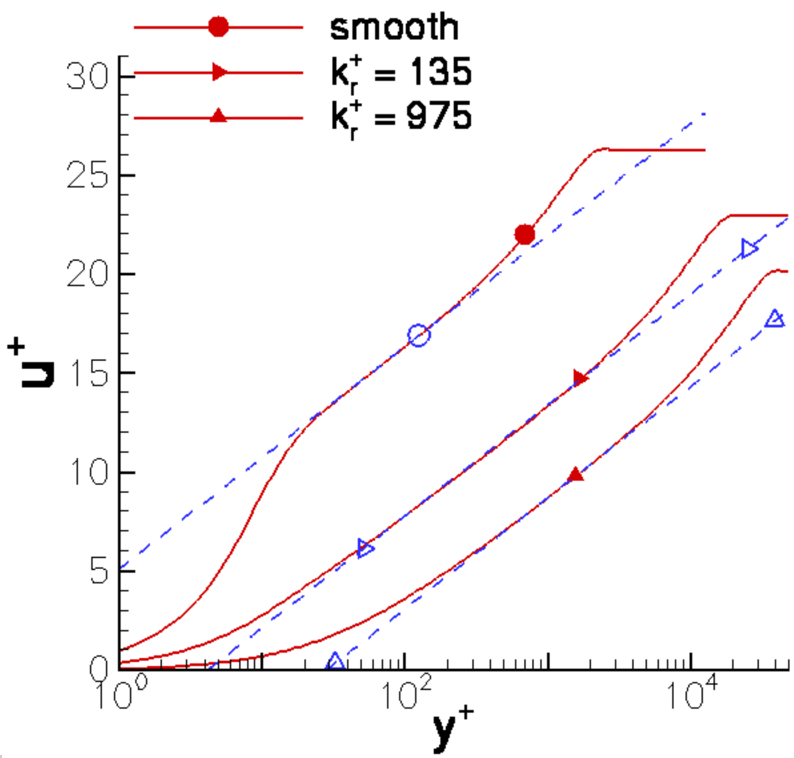
DLR strategy for RANS model improvement

Surface roughness

High-quality data base (exp., DNS/LES)

(Empirical) laws of turbulence

Physics-based improvement of RANS



DLR strategy for RANS model improvement

Turbulent boundary layer at APG

High-quality data base (exp., DNS/LES)

(Empirical) laws of turbulence

Physics-based improvement of RANS

DLR strategy for RANS model improvement

High-quality data base (exp., DNS/LES)

(Empirical) laws of turbulence

Physics-based improvement of RANS

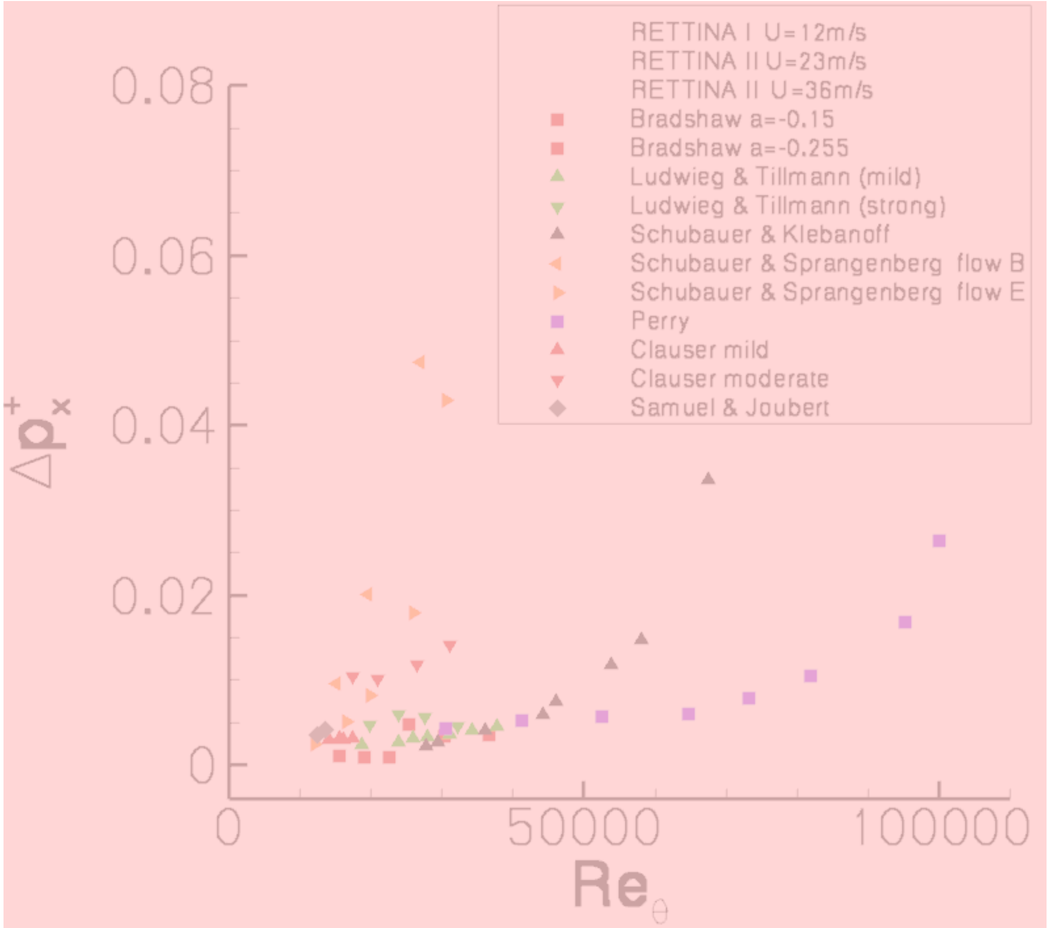
Turbulent boundary layer at APG

Relevant parameter space?

DLR

Data base set-up: The parameter space

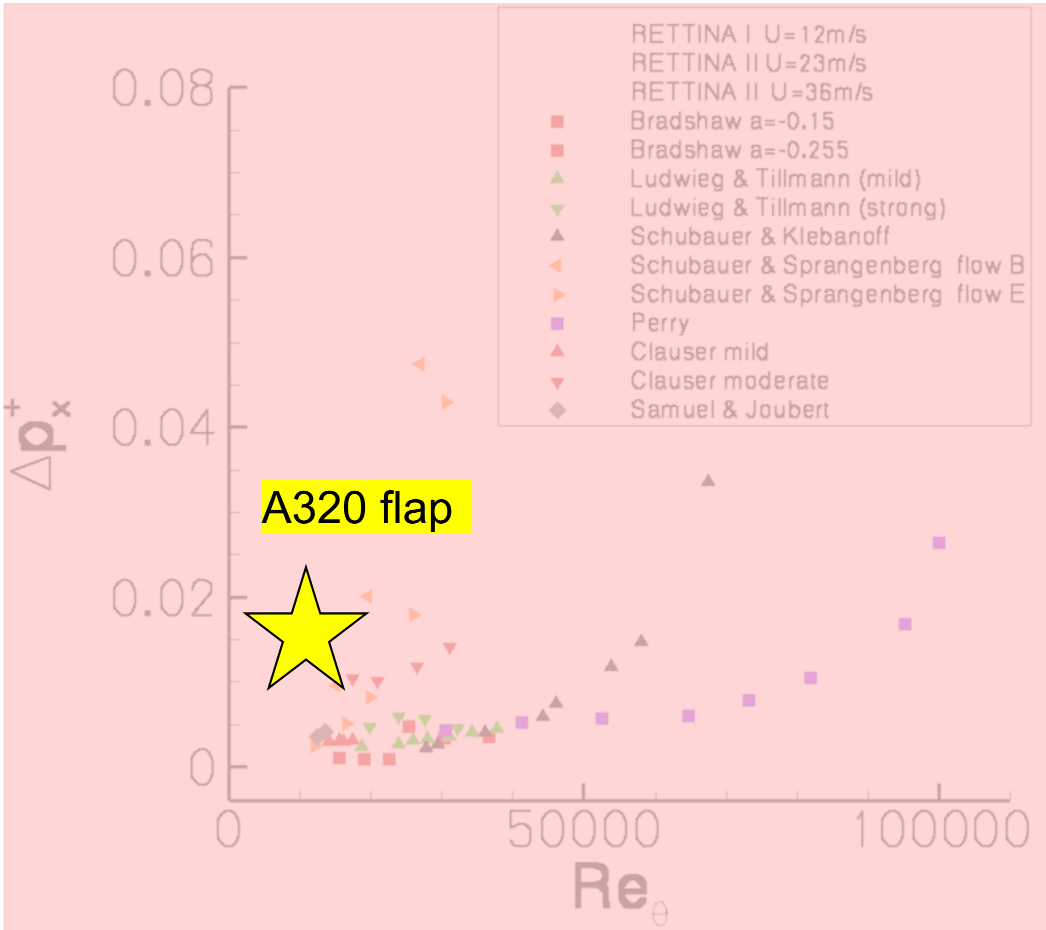
Pressure gradient parameter



Reynolds number

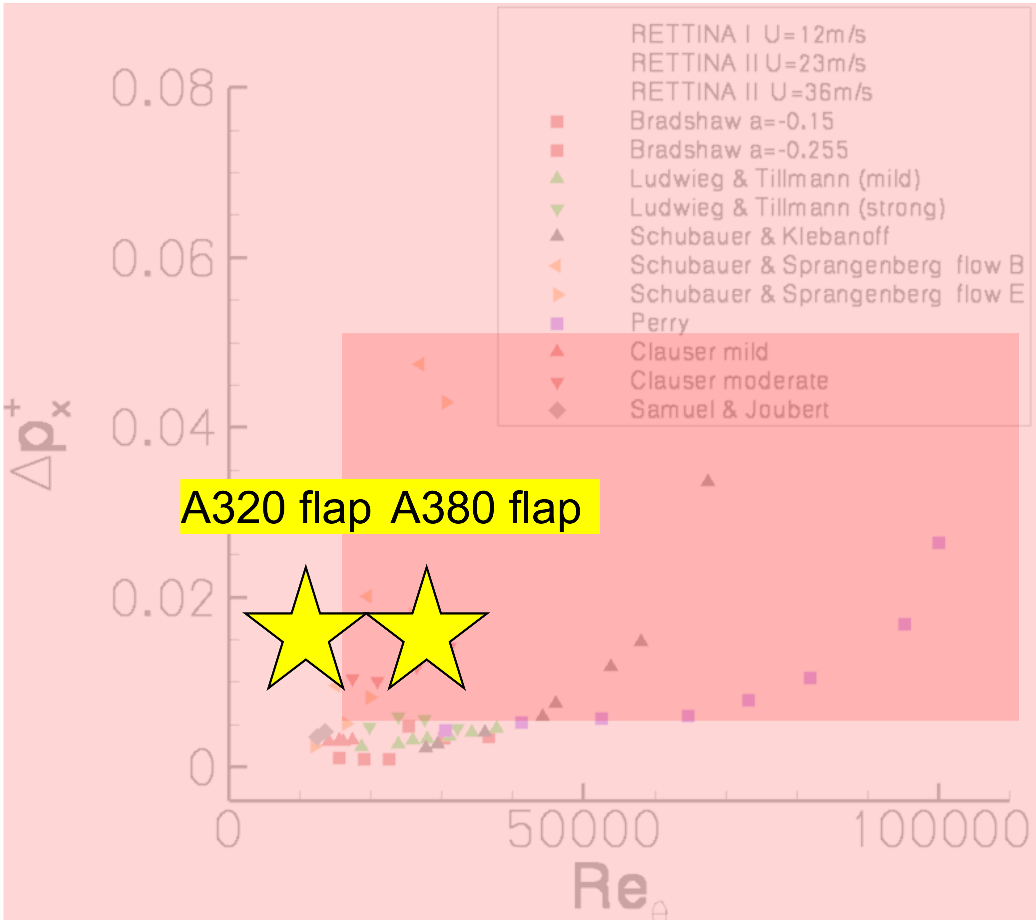
Data base set-up: The parameter space

Pressure gradient parameter

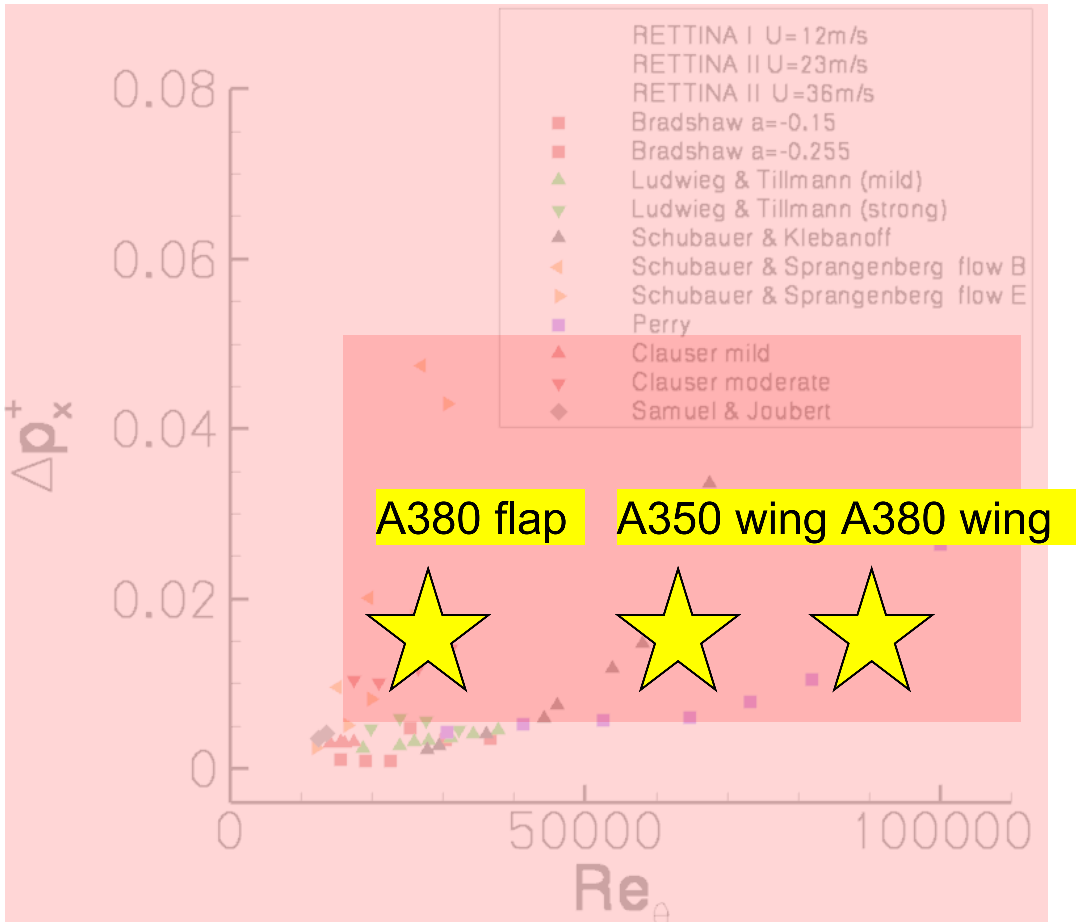


Reynolds number

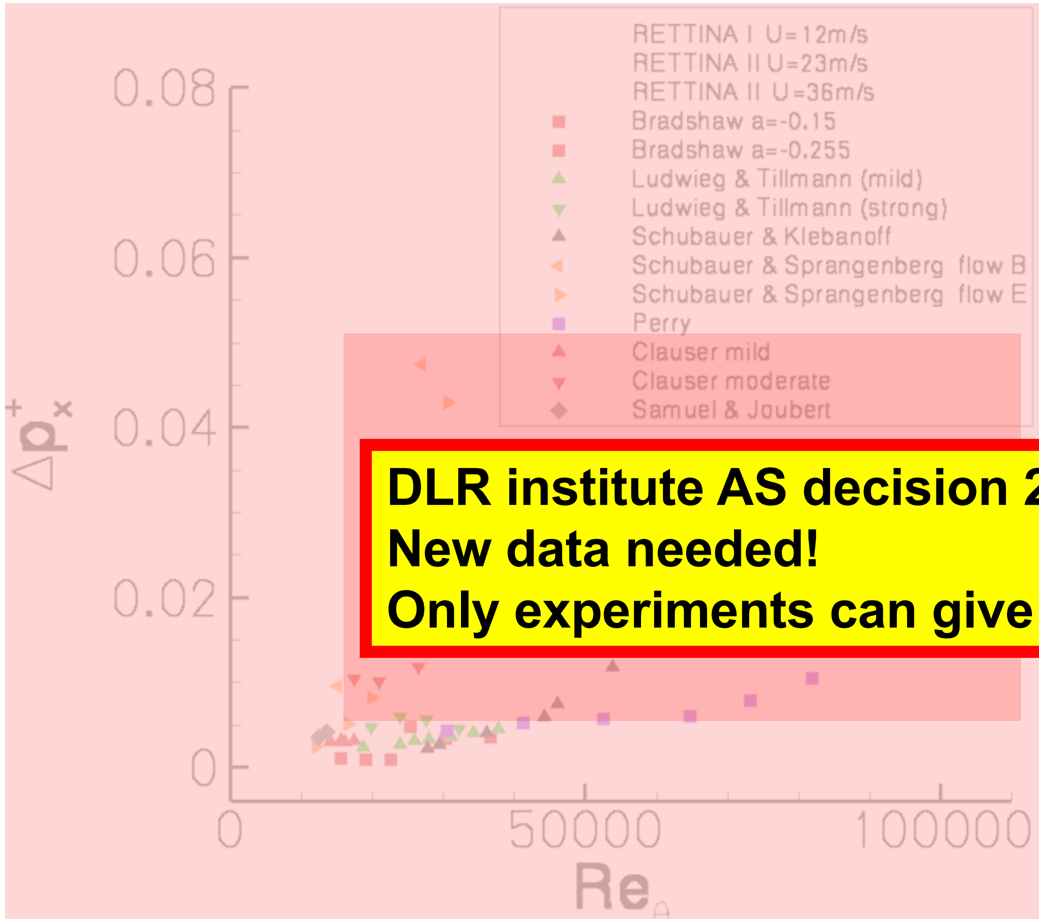
Data base set-up: The parameter space



Data base set-up: The parameter space



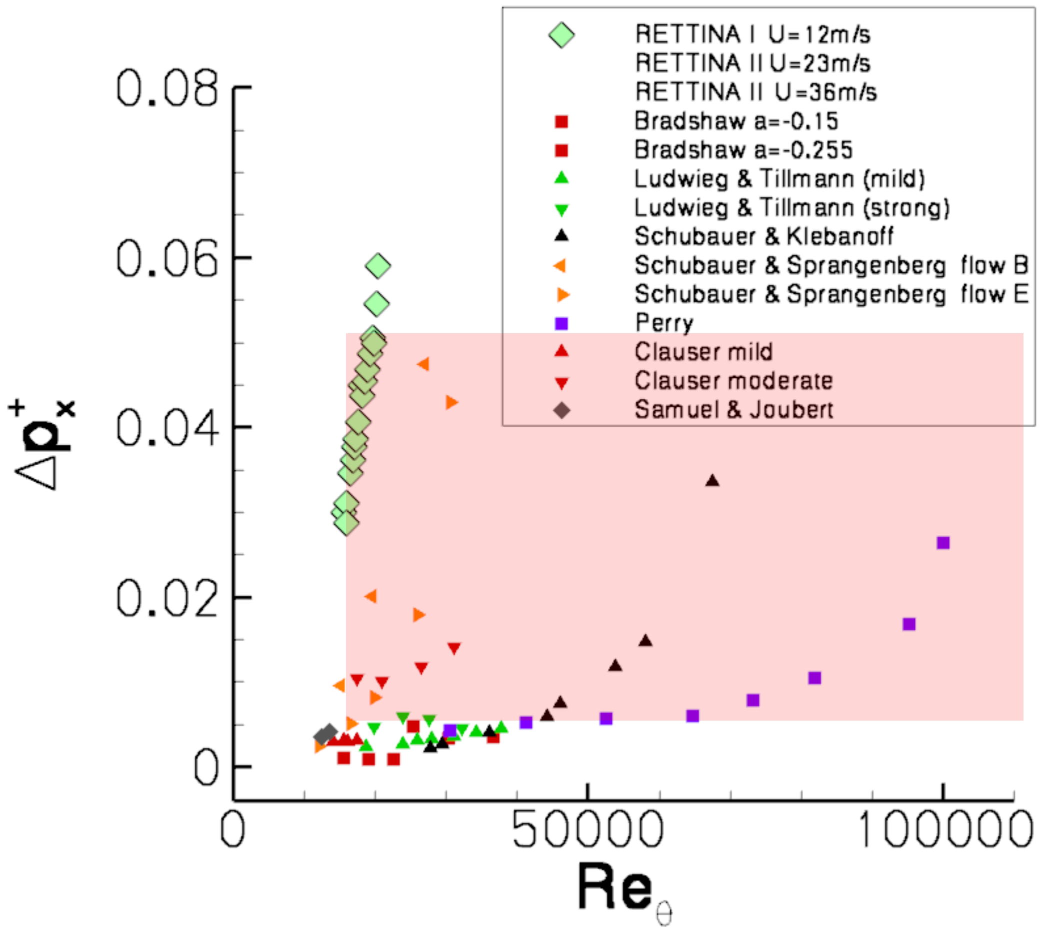
Data base set-up: The parameter space



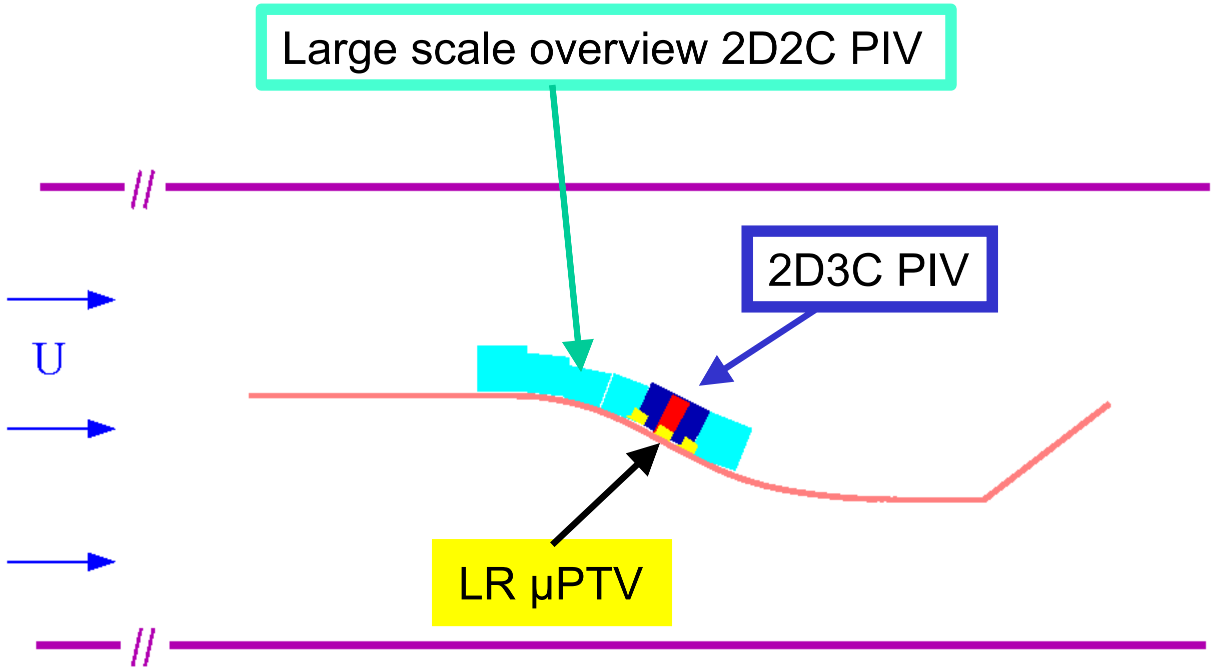
DLR institute AS decision 2009:
New data needed!
Only experiments can give large Re!

Data base set-up: The parameter space

Exp I.
Assess PIV at moderate Re



Experiment #1 (2011): RETTINA I exp. (DLR + UniBw Munich)

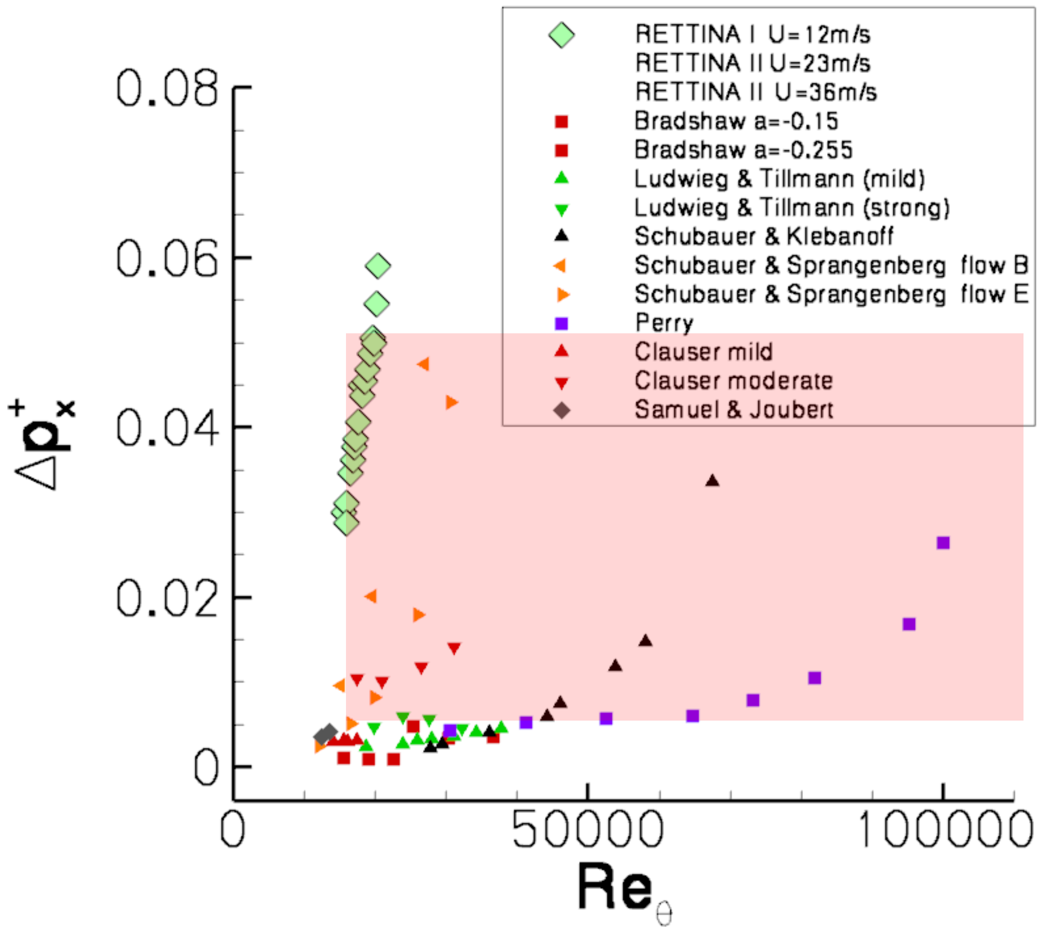


U_∞= 6 … 12m/s

Re_θ up to 10000 and δ_{99} =0.1m
Thick boundary layers enable PIV

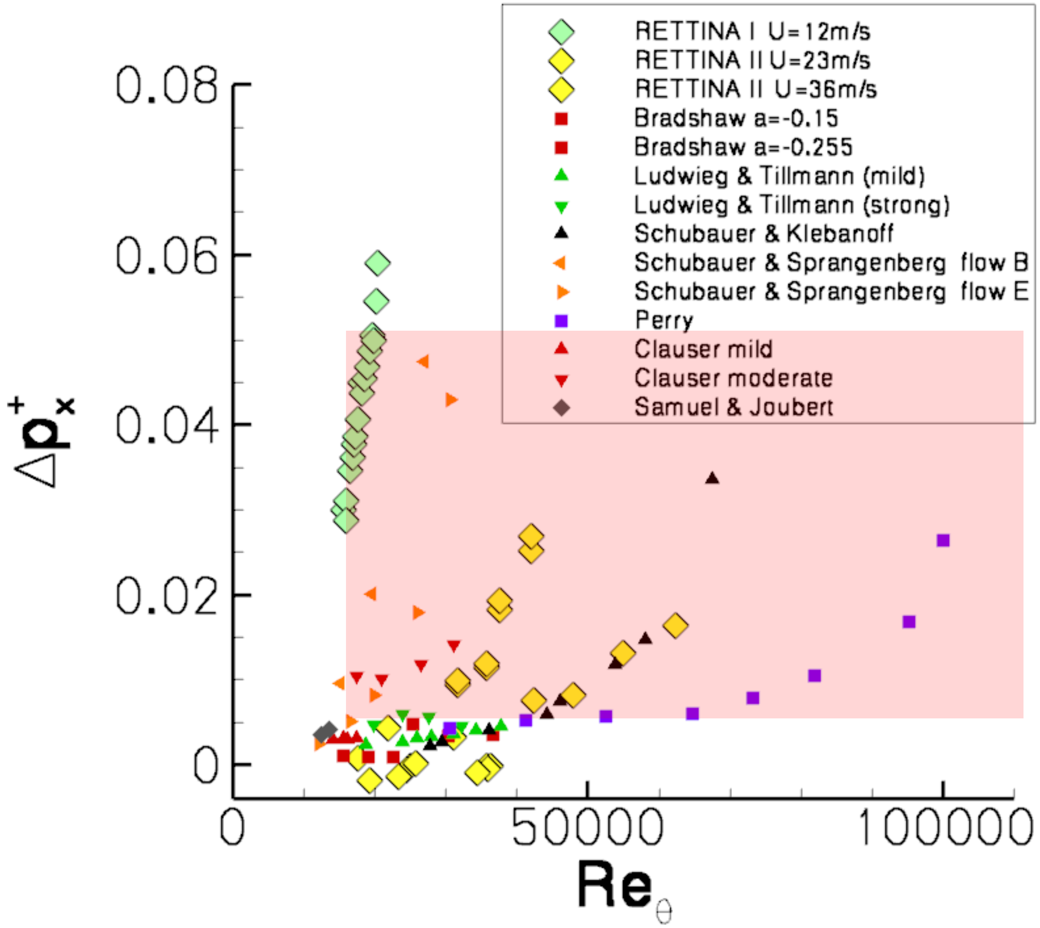
Data base set-up: The parameter space

Exp I.
Assess PIV at moderate Re



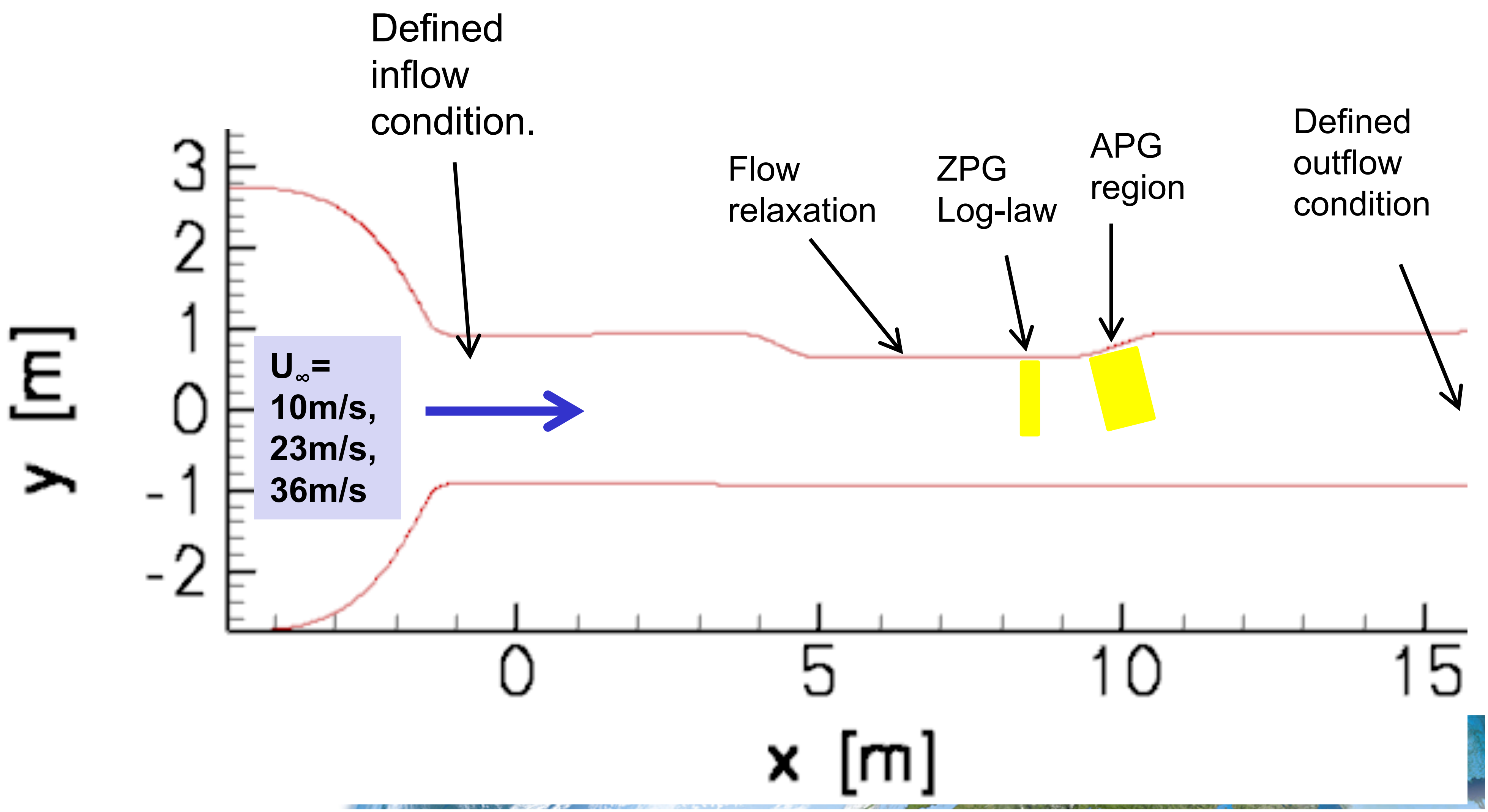
Data base set-up: The parameter space

Exp II.
Higher Re

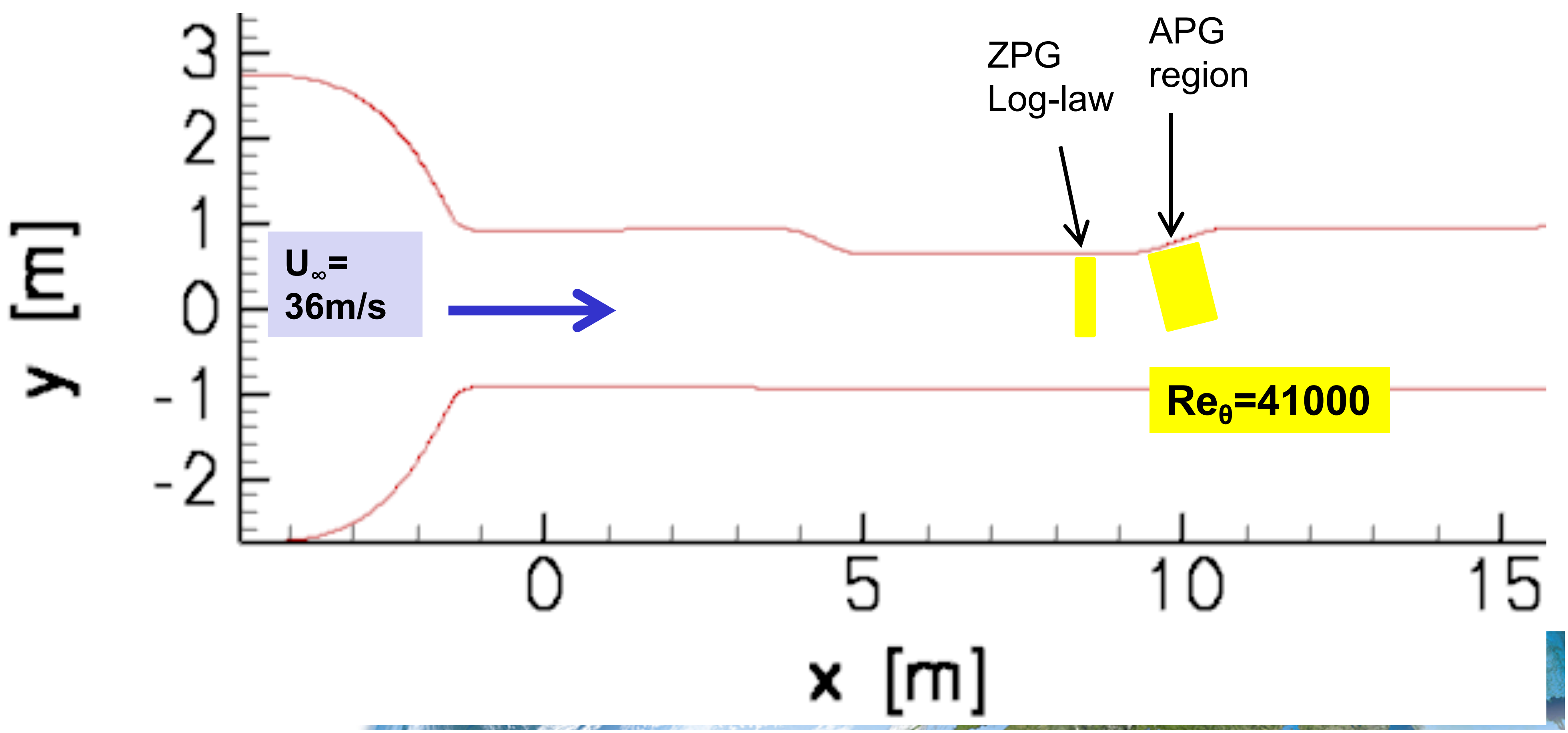


Experiment #2 (2015): RETTINA II exp. (DLR + UniBw Munich)

Funded by DLR institute AS and by DFG

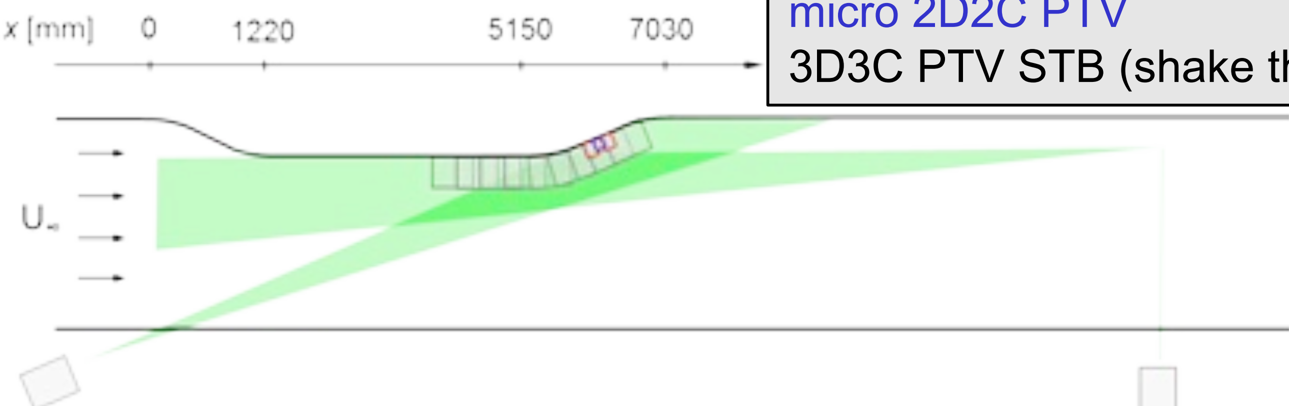


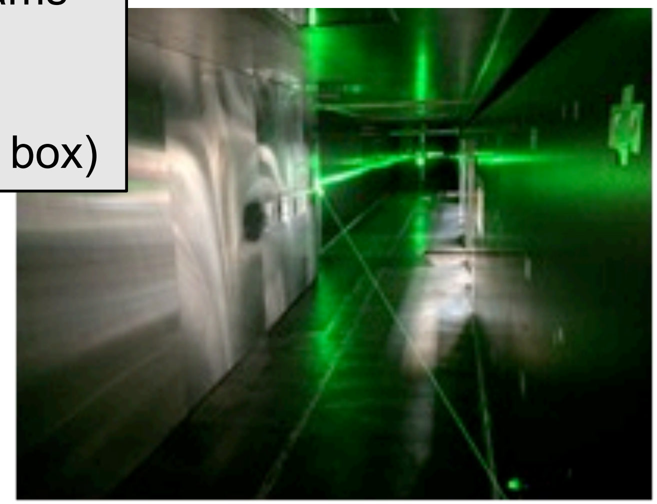
Experiment #2 (2015): RETTINA II exp. (DLR + UniBw Munich)

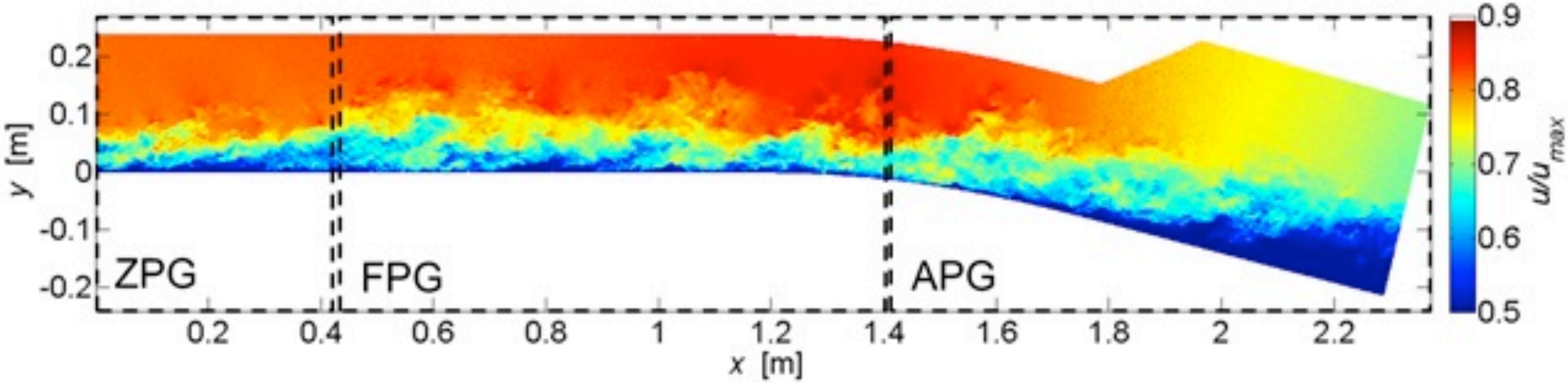


Experiment #2 (2015): RETTINA II exp. (DLR + UniBw Munich)

Large-scale 2D2C-PIV 9 cams
2D3C PIV
micro 2D2C PTV
3D3C PTV STB (shake the box)







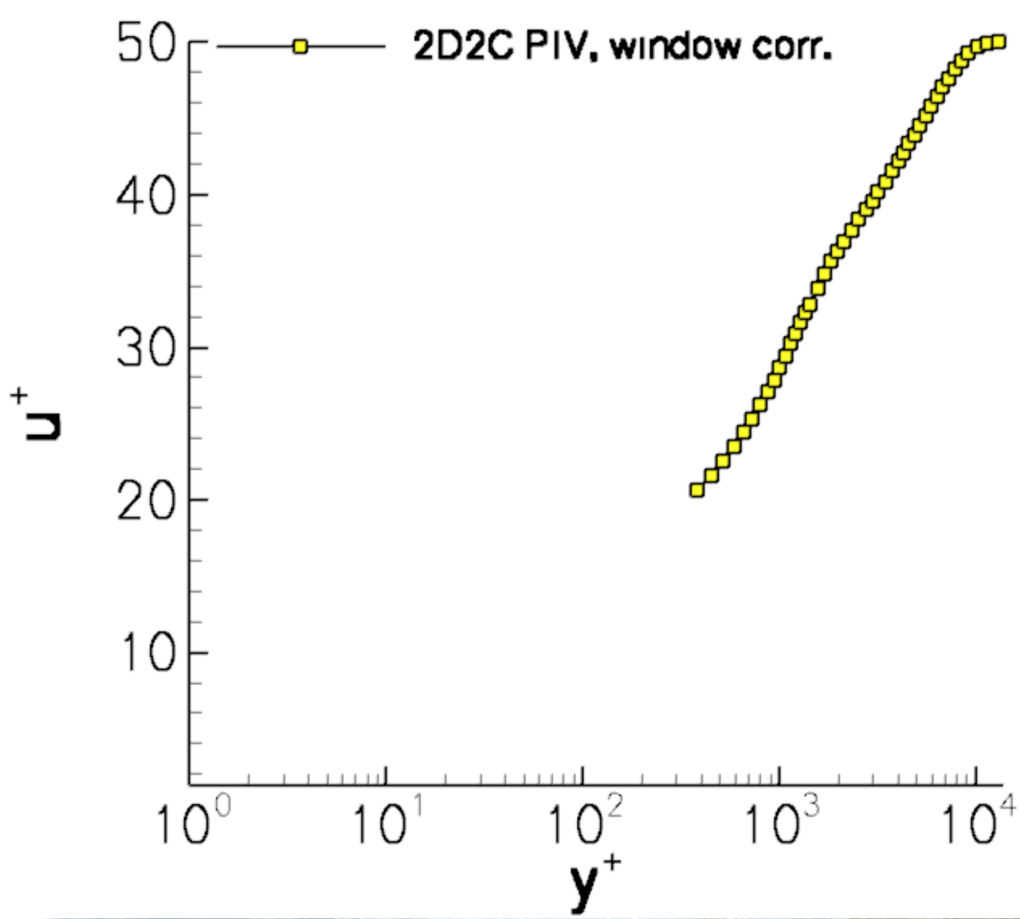
Experiment #2 (2015): RETTINA II exp. (DLR + UniBw Munich)

Challenge: Large range of scale of turbulent boundary layer

Solution approach: Multi-resolution multi-camera PIV

U=36m/s
Re_θ=41000

ν/u_τ = 20µm
δ_{99} = 0.2m



DLR

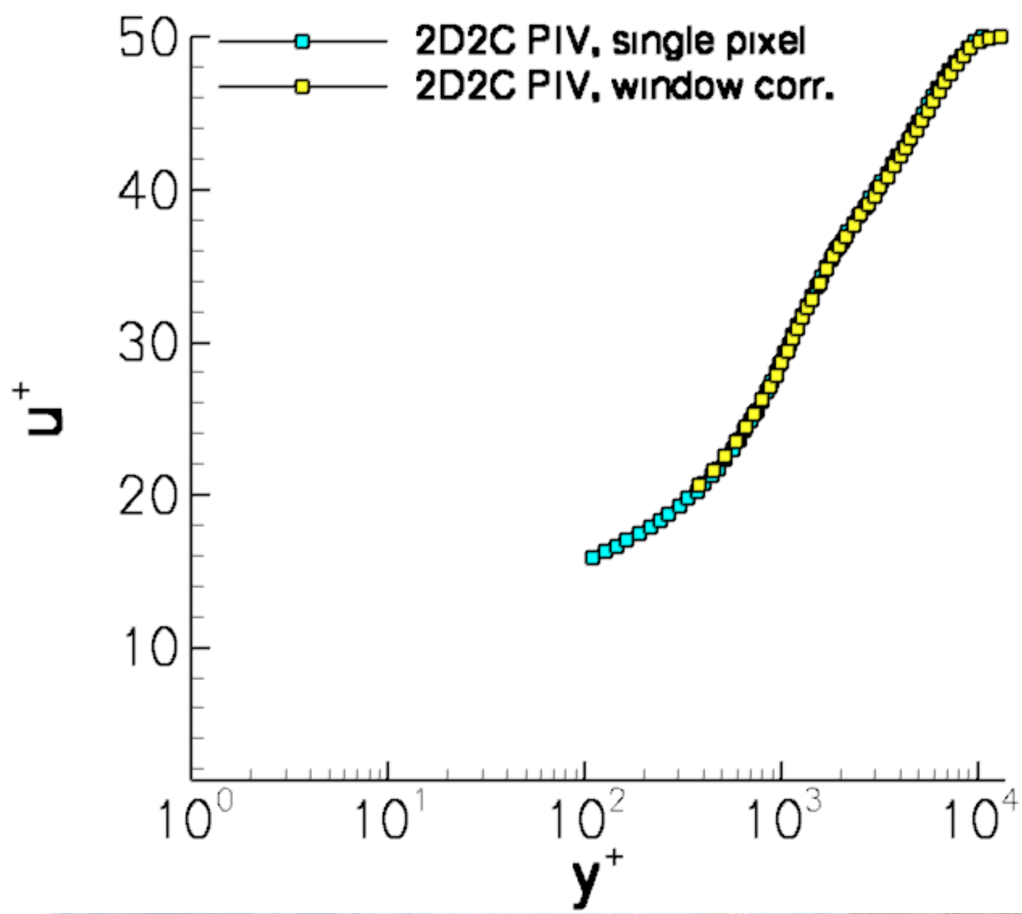
Experiment #2 (2015): RETTINA II exp. (DLR + UniBw Munich)

Challenge: Large range of scale of turbulent boundary layer

Solution approach: Multi-resolution multi-camera PIV

U=36m/s
Re_θ=41000

ν/u_τ = 20µm
δ_{99} = 0.2m



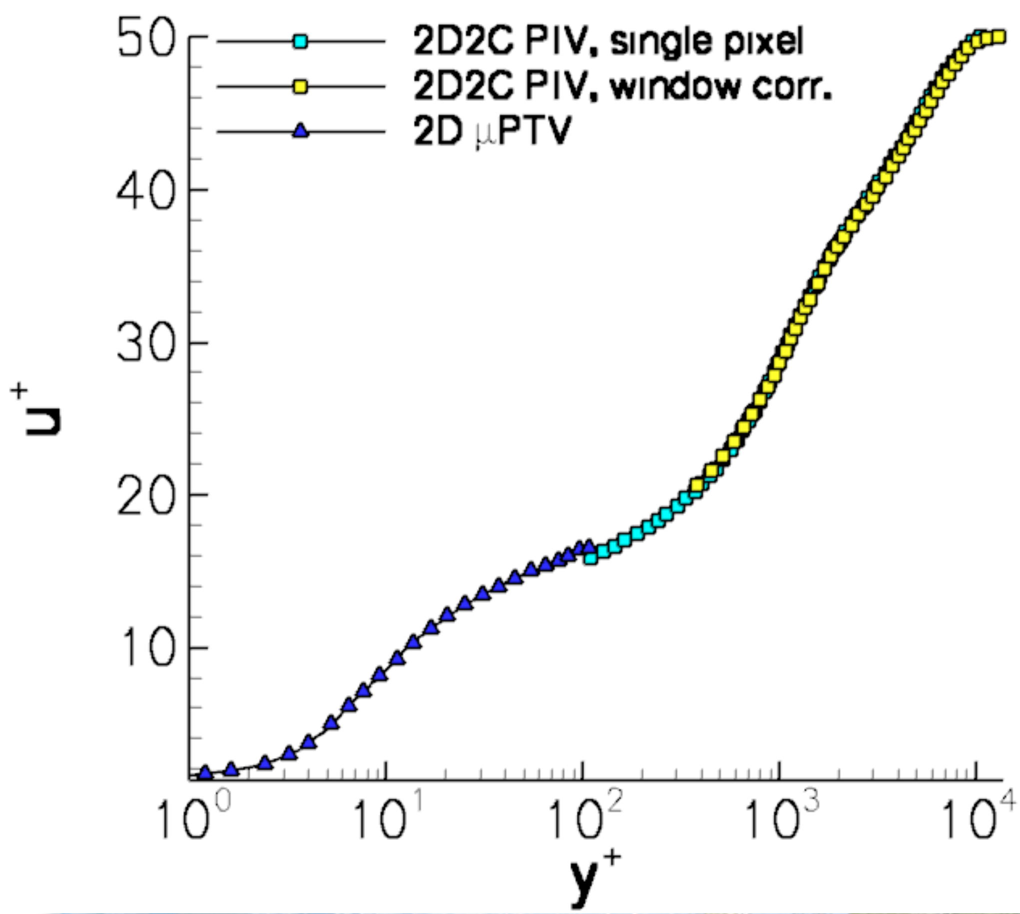
Experiment #2 (2015): RETTINA II exp. (DLR + UniBw Munich)

Challenge: Large range of scale of turbulent boundary layer

Solution approach: Multi-resolution multi-camera PIV

U=36m/s
Re_θ=41000

ν/u_τ = 20µm
δ_{99} = 0.2m



DLR

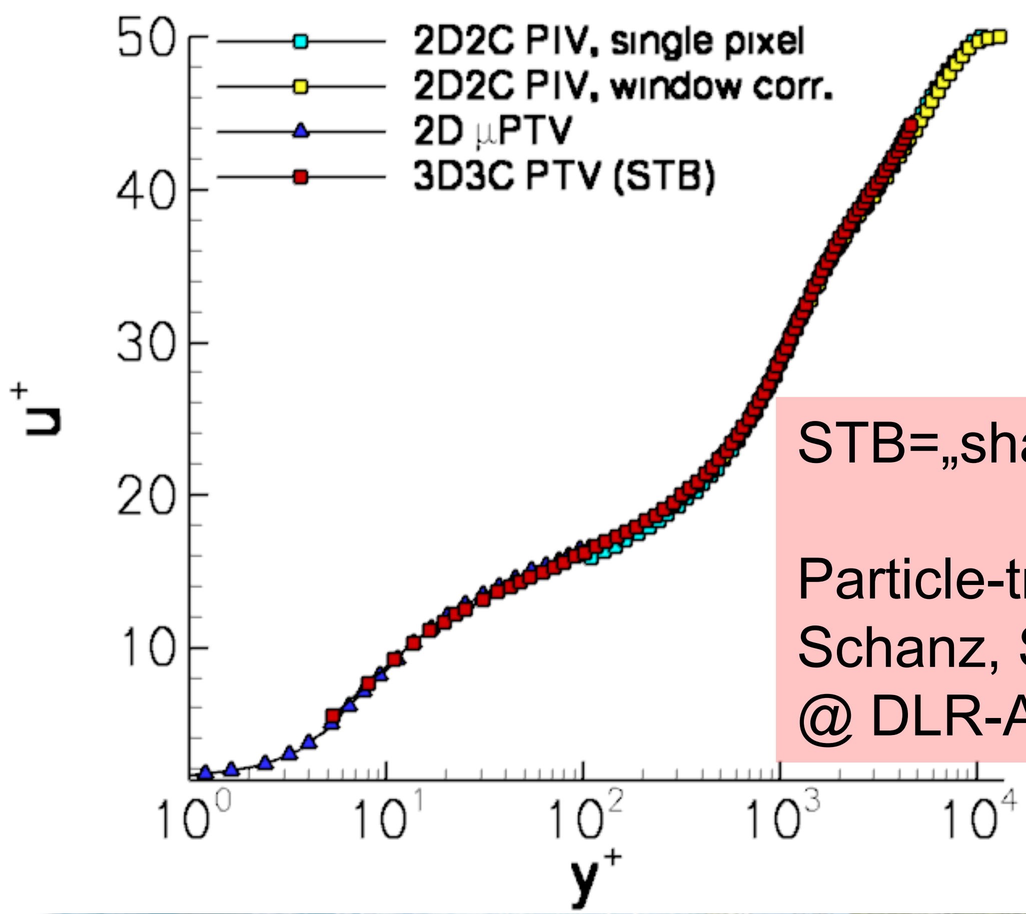
Experiment #2 (2015): RETTINA II exp. (DLR + UniBw Munich)

Challenge: Large range of scale of turbulent boundary layer

Solution approach: Multi-resolution multi-camera PIV

U=36m/s
Re_θ=41000

ν/u_τ = 20µm
δ_{99} = 0.2m



STB=„shake-the-box“

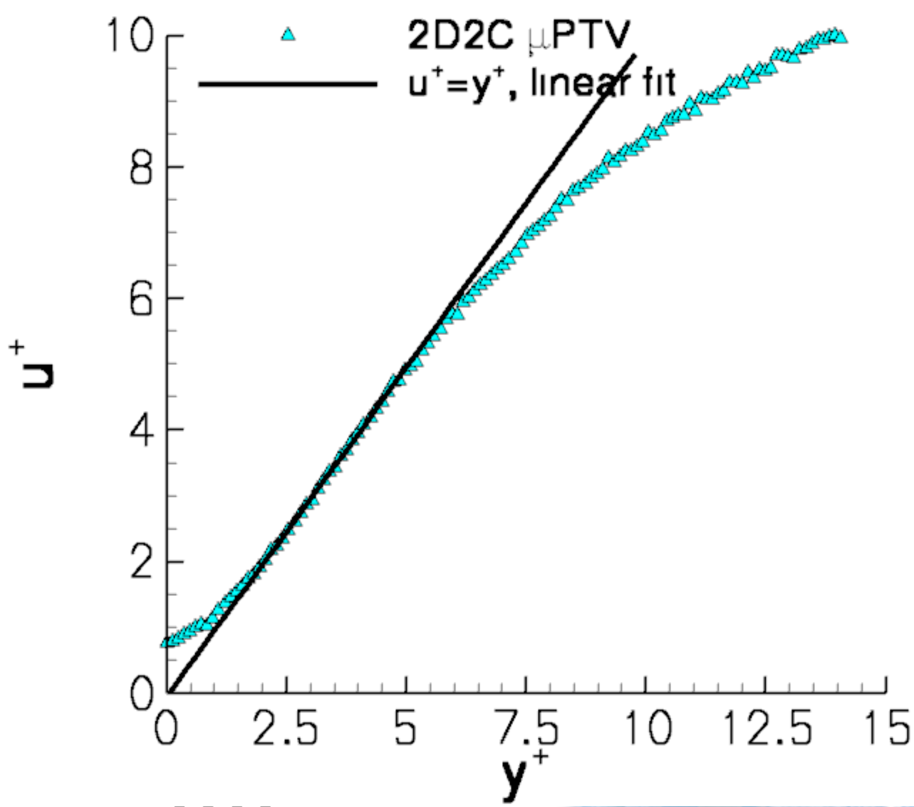
Particle-tracking-approach
Schanz, Schröder, Novara
@ DLR-AS

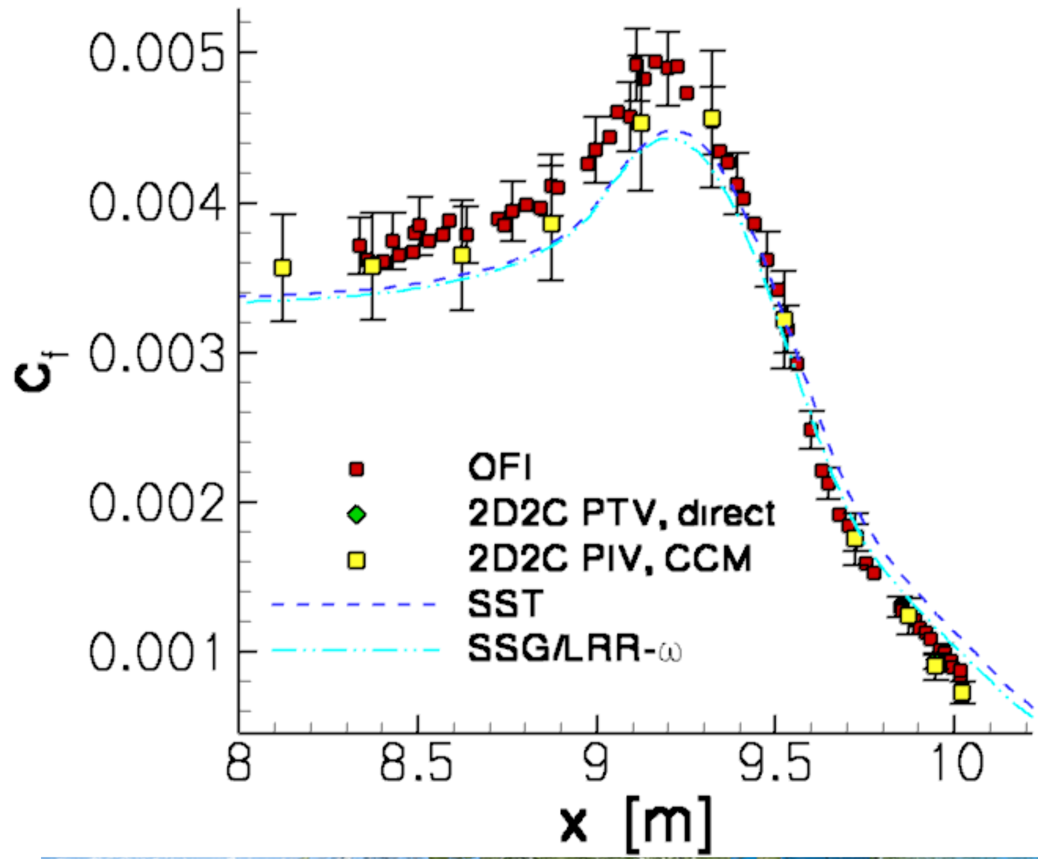
Measurement technique. Skin friction

Clauser chart (y+-range depends on ZPG, FPG, APG)

μPTV: Viscous sublayer mean velocity

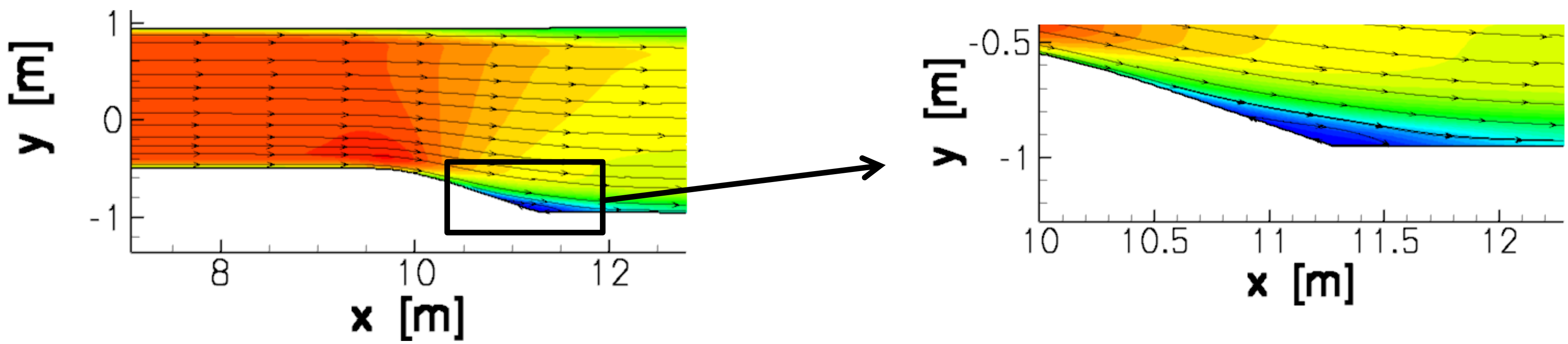
Oil film interferometry

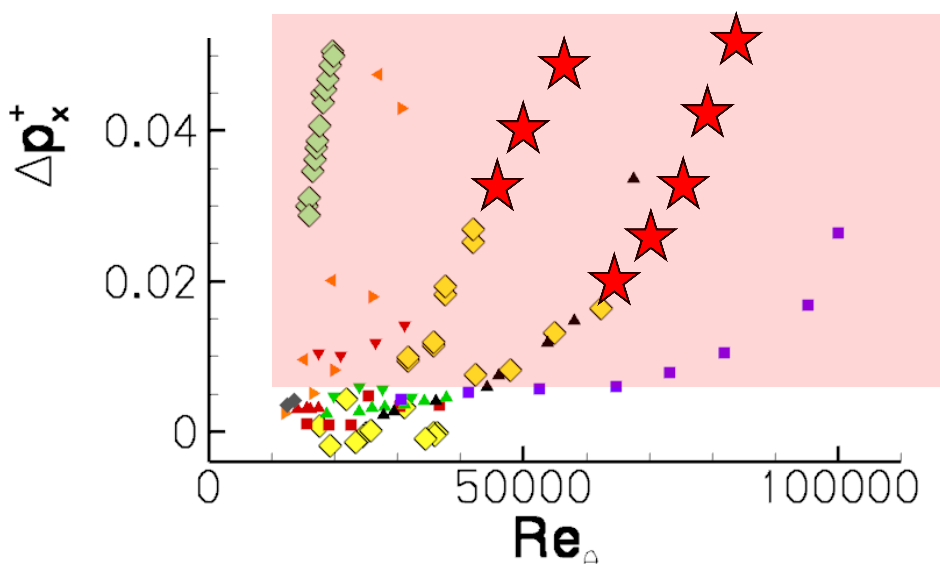




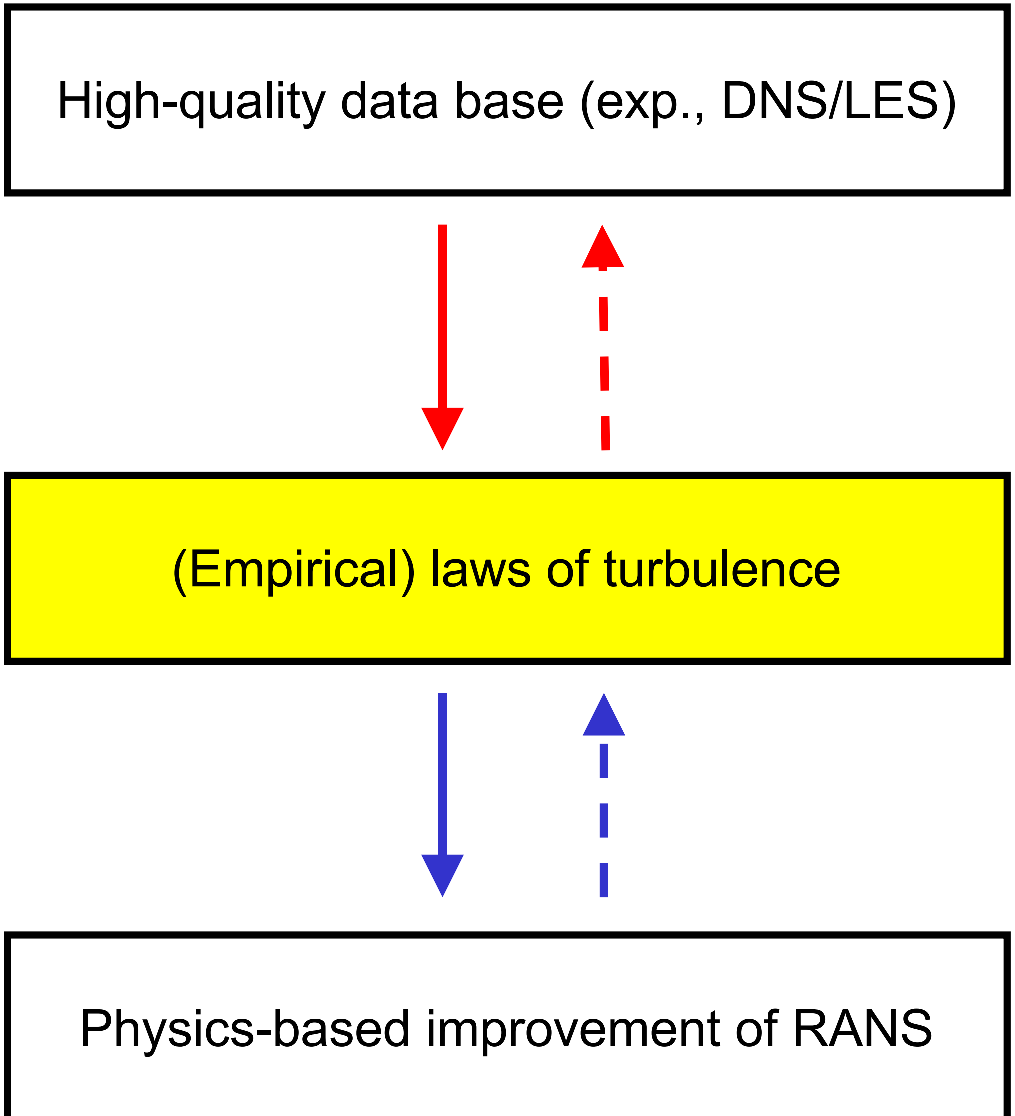
Experiment #3 (August 2017) within DLR internal project

- Mild separation and reattachment



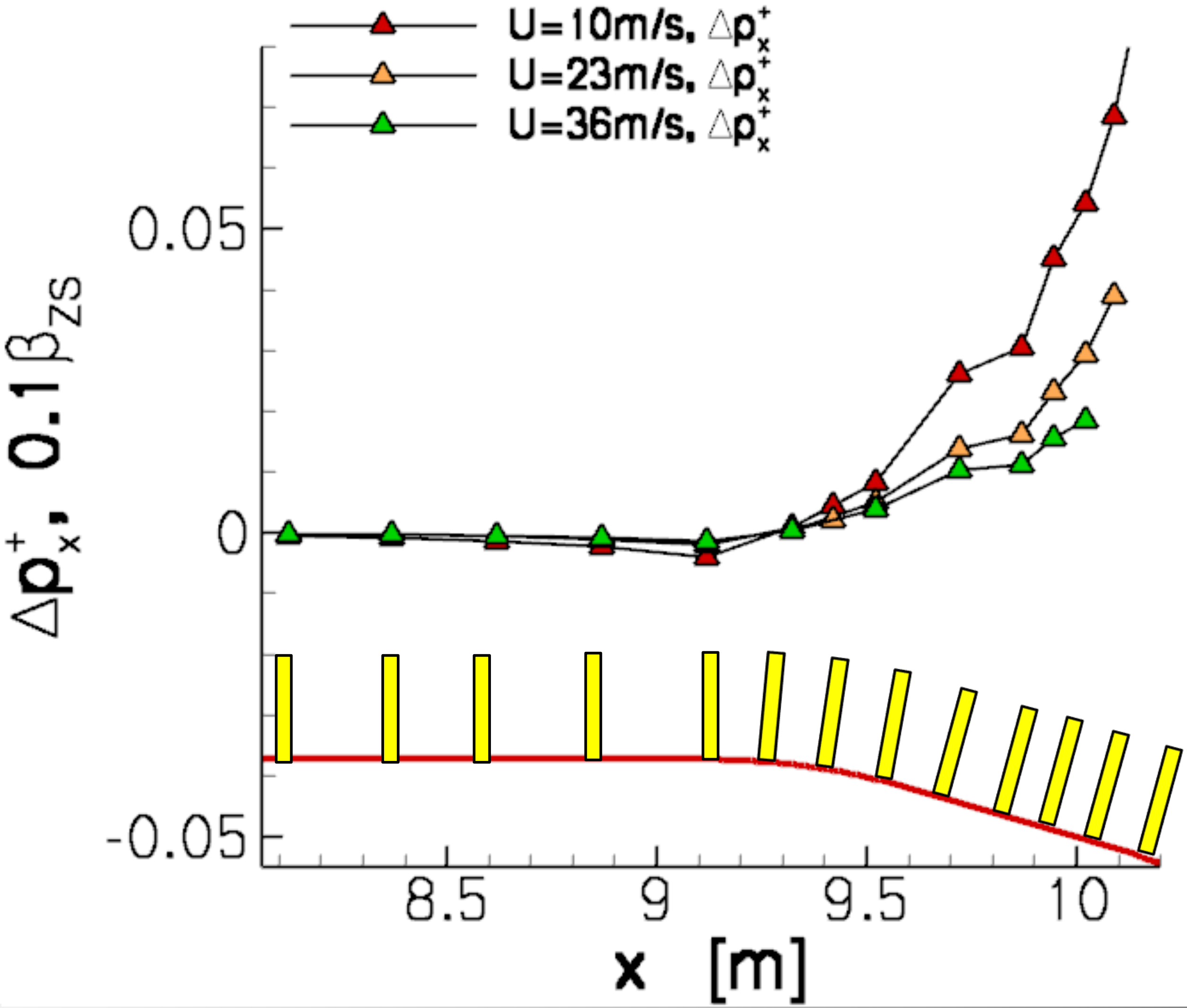


DLR strategy for RANS model improvement



Empirical wall law at APG

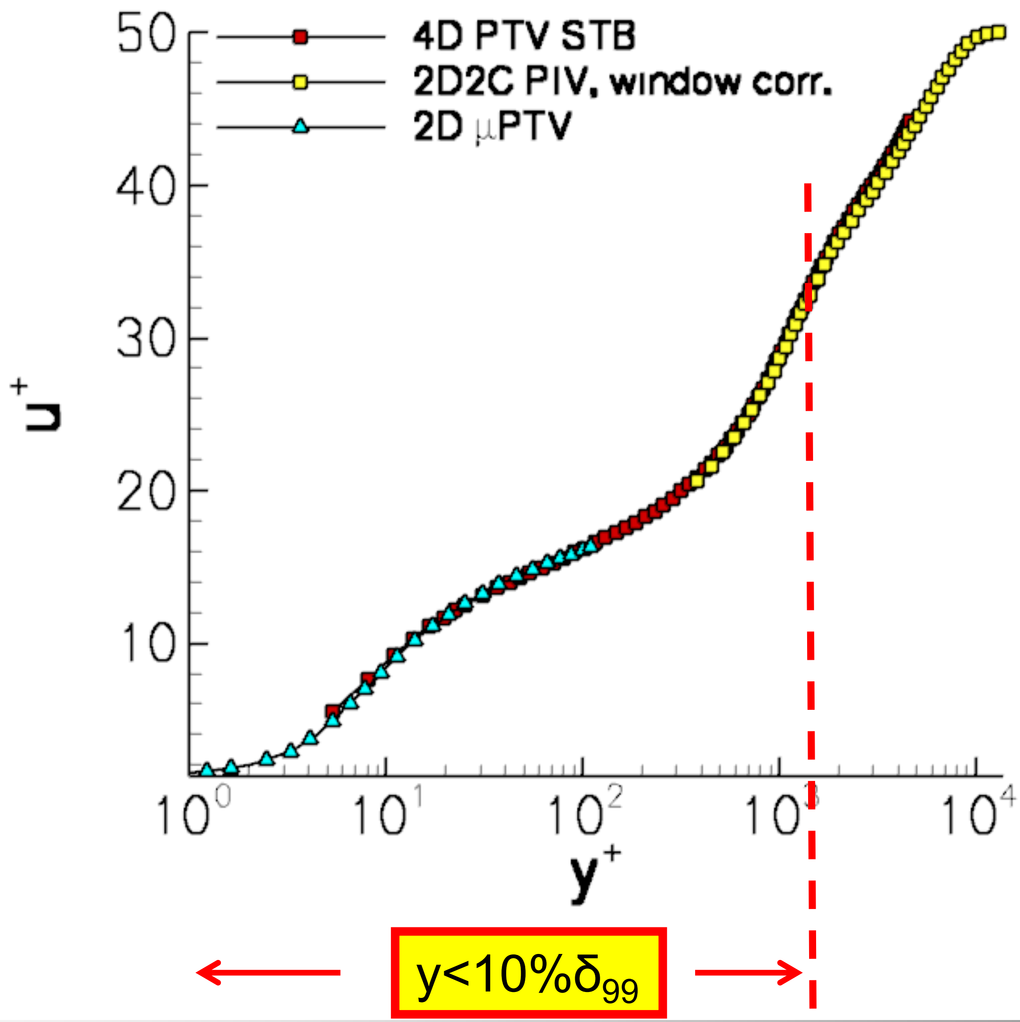
- Mean velocity profiles
- 2D2C Reynolds stresses



Empirical wall law at APG

Aim: Empirical wall-law at APG for $y<0.1\delta_{99}$

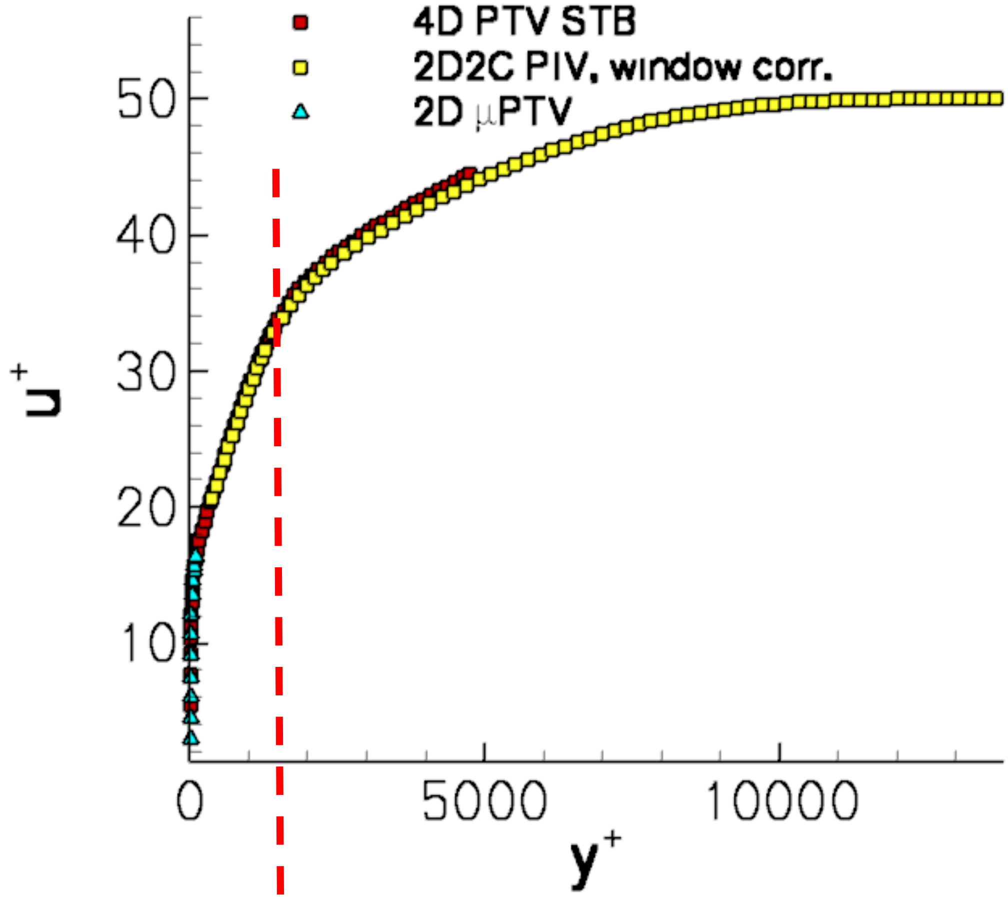
U=36m/s
$\Delta p_x^+ = 0.015$



Empirical wall law at APG

Aim: Empirical wall-law at APG for $y<0.1\delta_{99}$

U=36m/s
$\Delta p_x^+=0.015$

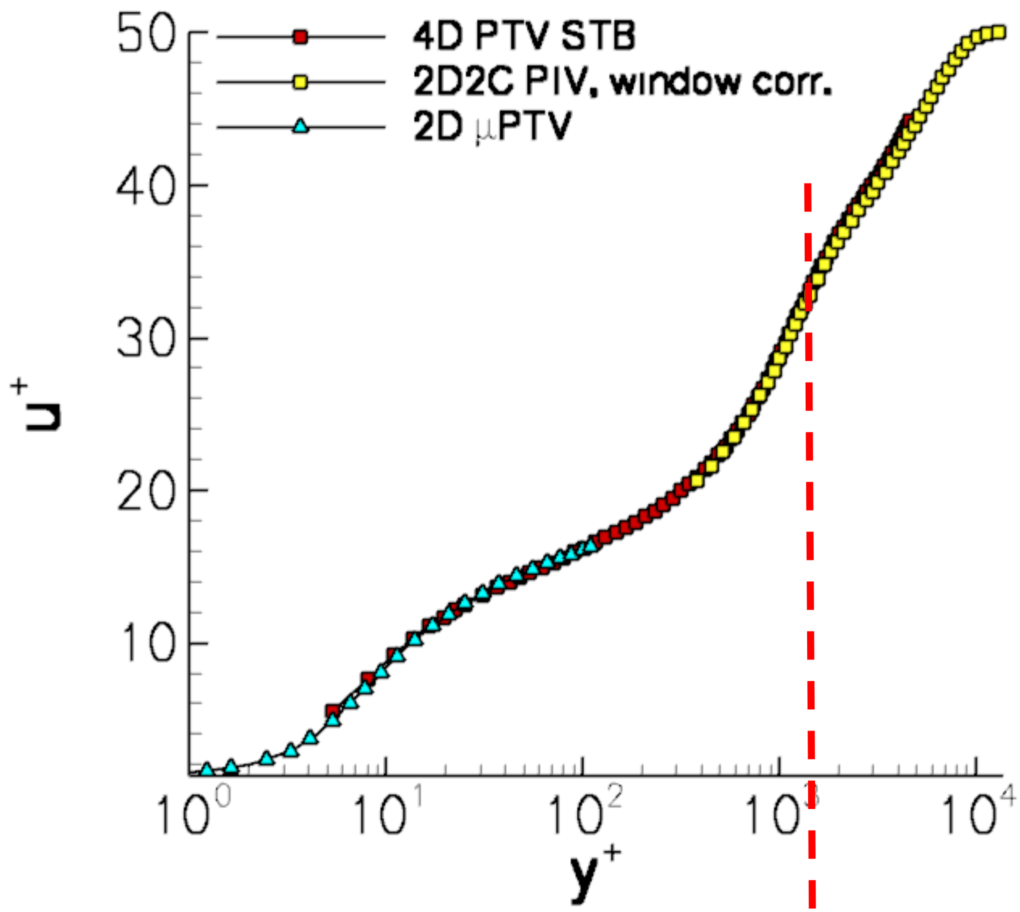


$y<10\%\delta_{99}$

Empirical wall law at APG

Hypothesis #1: Resilience of a small log-region

U=36m/s
$\Delta p_x^+=0.015$

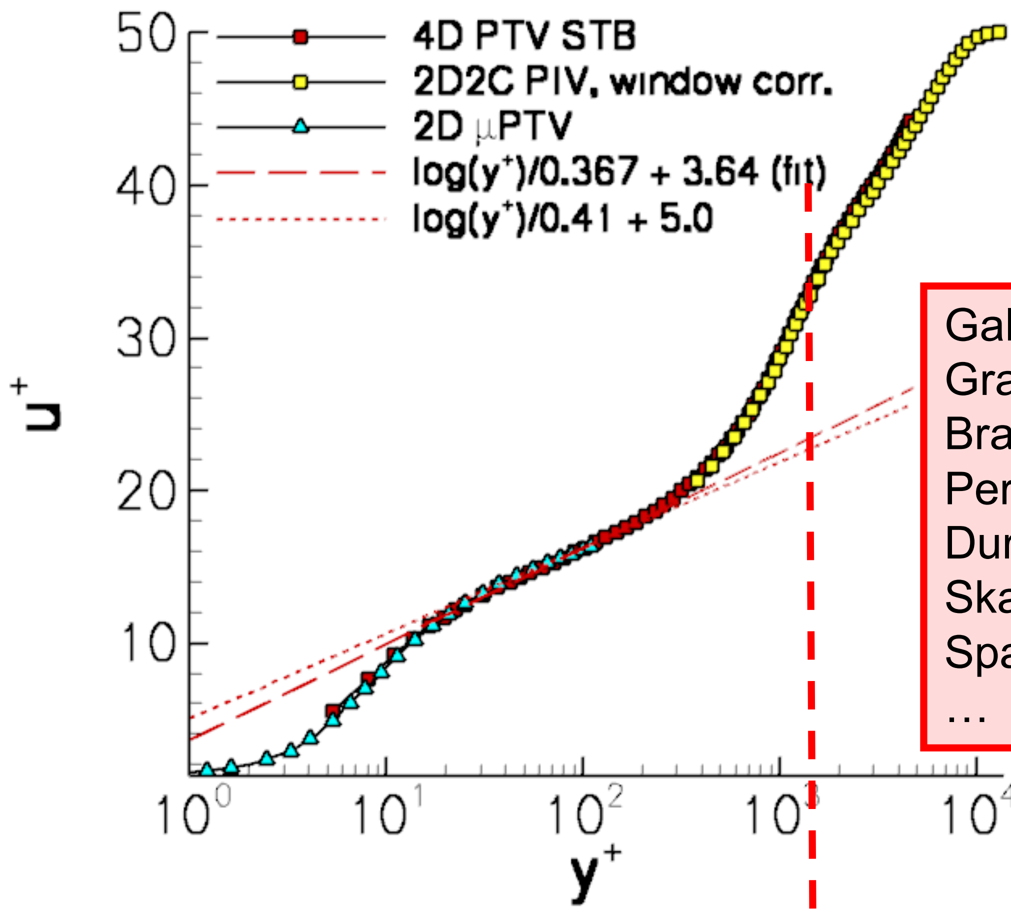


$y<0.1\delta_{99}$

Empirical wall law at APG

Hypothesis #1: Resilience of a small log-region

U=36m/s
$\Delta p_x^+=0.015$



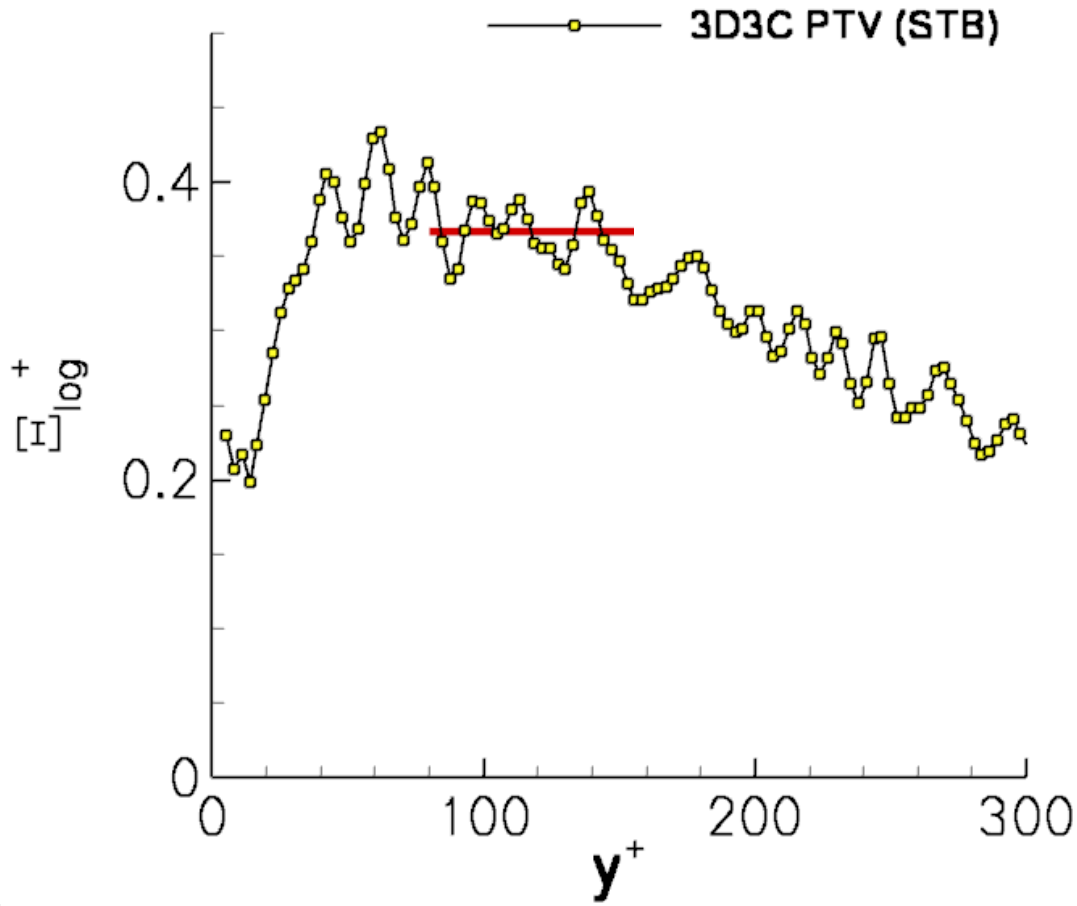
Galbraith et al. (1977),
Granville (1985),
Bradshaw (1995),
Perry & Schofield (1973),
Durbin & Belcher (1992)
Skare & Krogstad (1994)
Spalart & Coleman
…

$y<0.1\delta_{99}$

Results and analysis

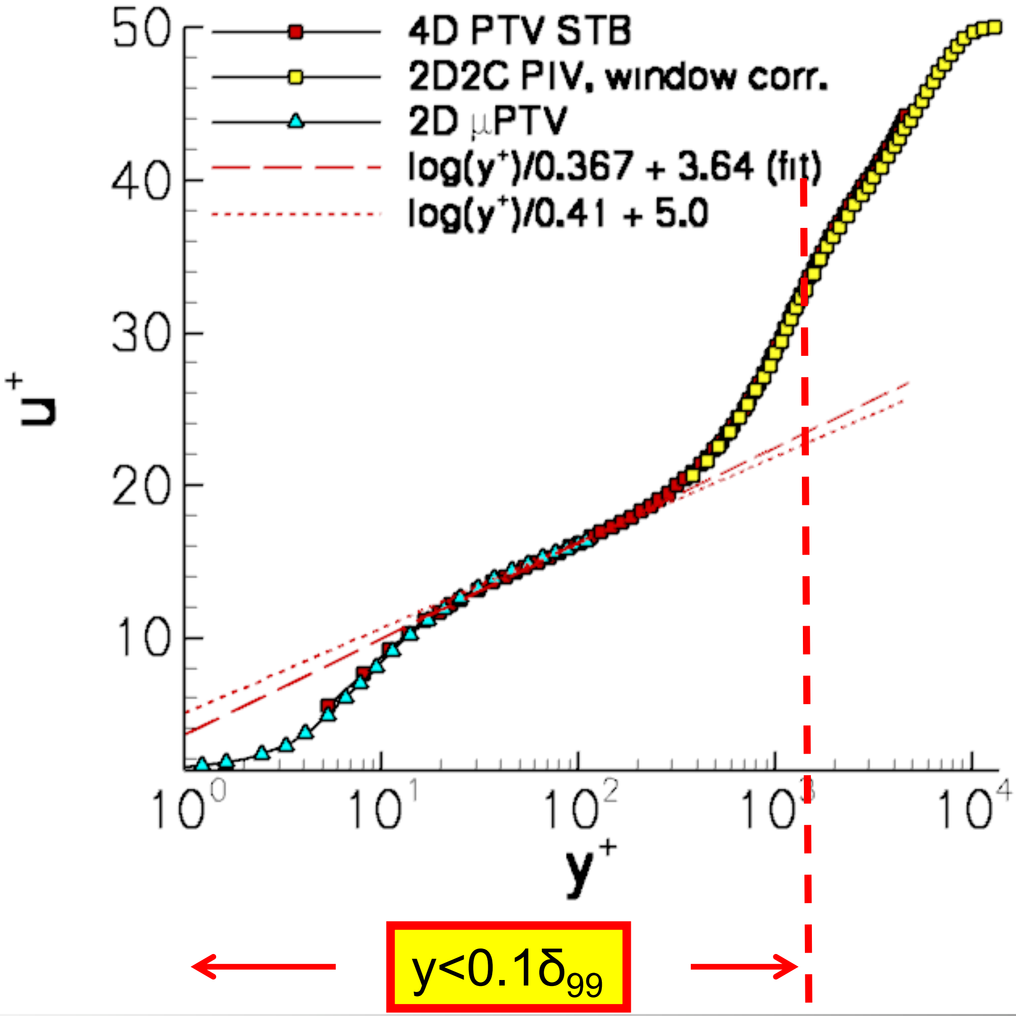
$$u^+_{\log} = \frac{1}{K_i} \log(y^+) + B_i$$

$$\Xi^{-1} = y^+ \frac{du^+}{dy^+} = \frac{1}{K_i}$$



Empirical wall law at APG

U=36m/s
$\Delta p_x^+=0.015$



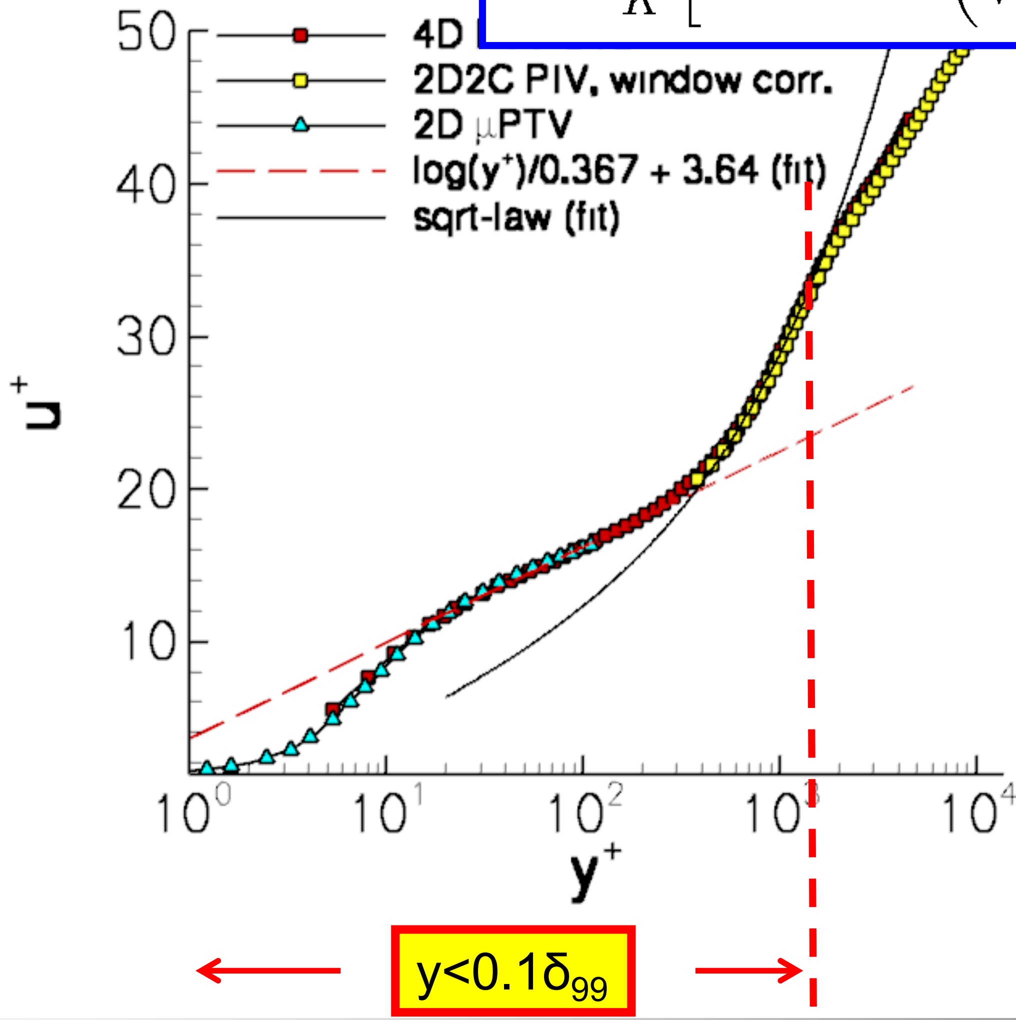
Empirical wall law at APG

Hypothesis #2: square-root (sqrt)-law above the log-region

$$u^+ = \frac{1}{K}\left[\log(y^+) + 2\left(\sqrt{1+\Delta p_x^+ y^+} - 1\right)\right] + \tilde{B}$$

U=36m/s

Δp_x^+=0.015



Szablewski (1954),
Townsend (1961),
McDonald (1969),
Brown & Joubert
…

Results and analysis

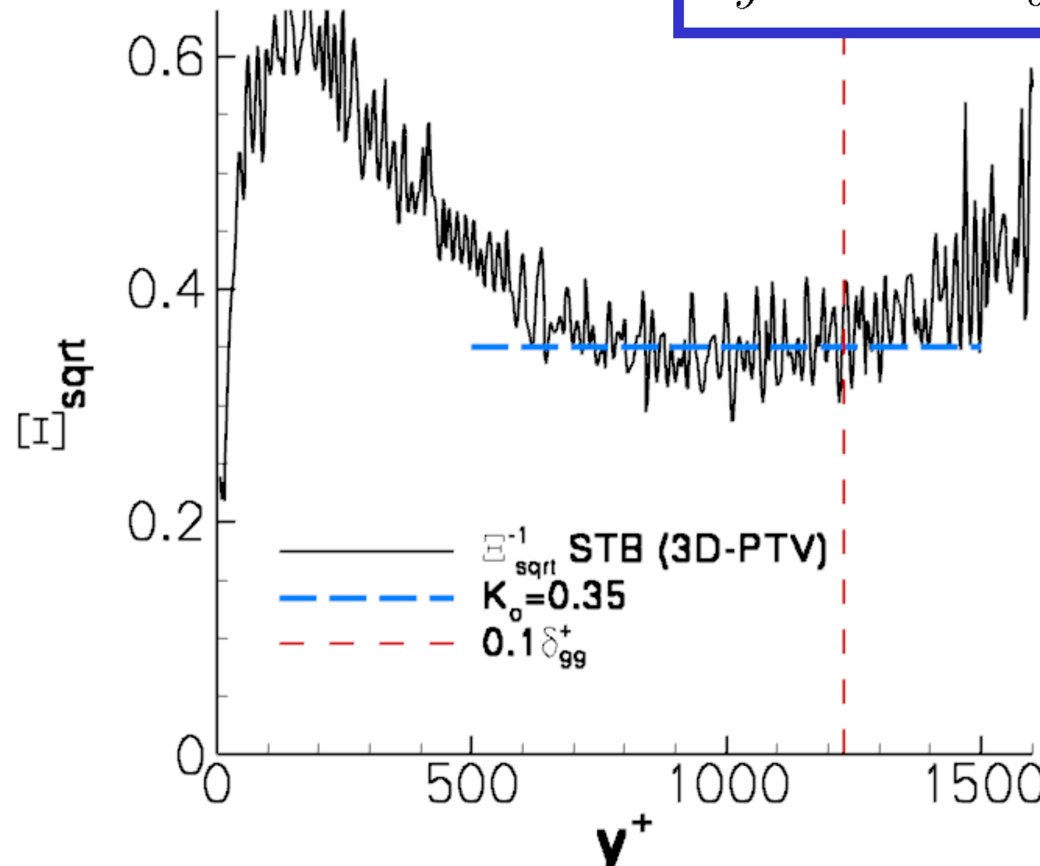
Hypothesis #2: square-root (sqrt)-law above the log-region

$$\frac{\mathrm{d}u^+}{\mathrm{d}y^+} = \frac{\sqrt{1+\lambda \Delta p_x^+ y^+}}{K_o y^+}$$

U=36m/s

Re_θ=41000

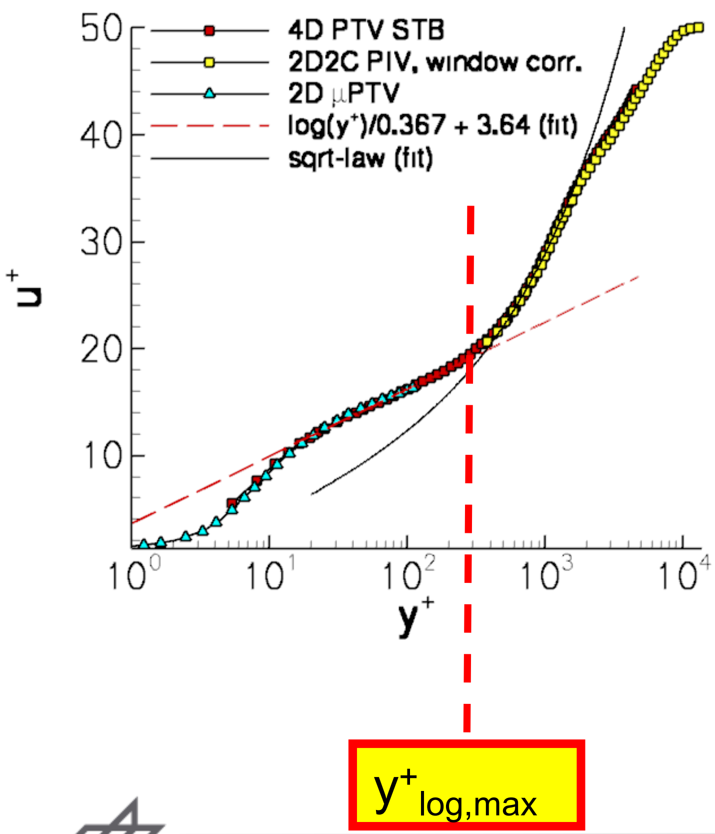
Δp_x^+=0.015



$y<0.1\delta_{99}$

Results and analysis

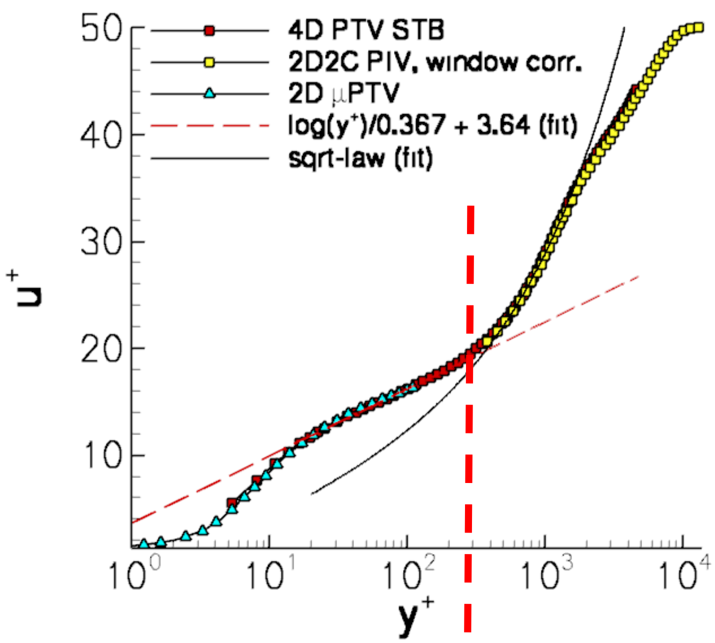
Composite profile Brown & Joubert 1969



Results and analysis

Hypothesis #3: transition from log-law to sqrt-law depends on Δp_x^+
(probably more complex …)

Composite profile Brown & Joubert 1969

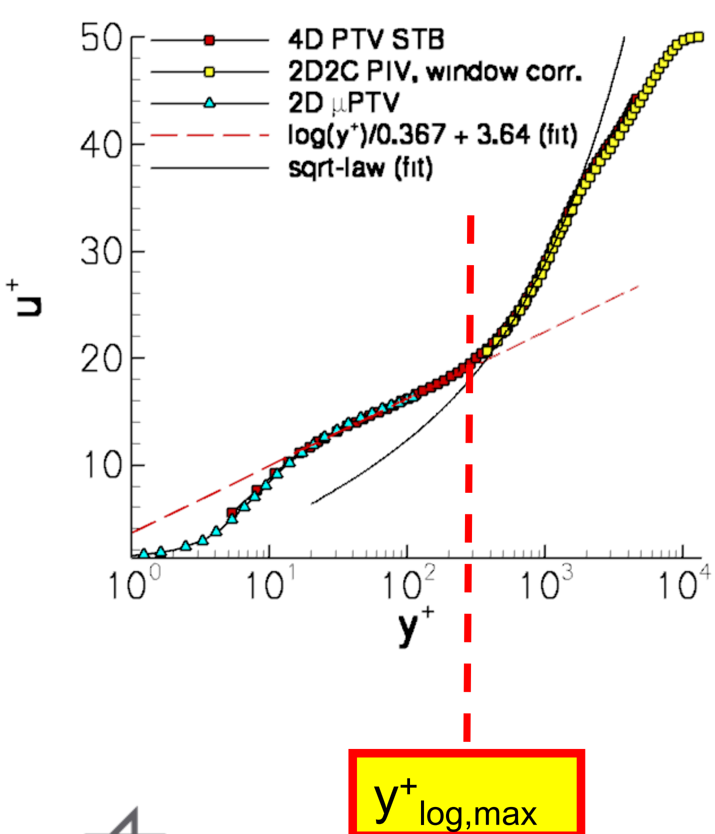


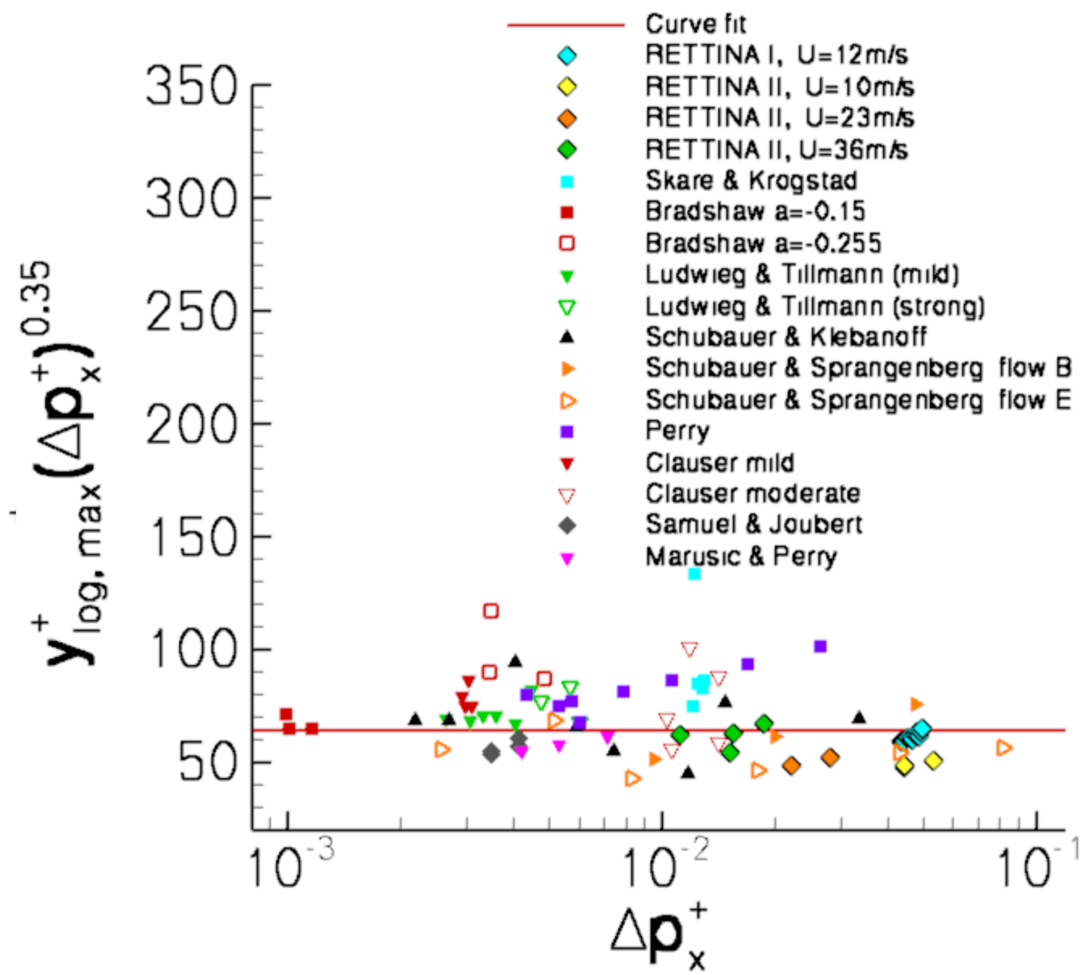
$y^+_{log,max}$

Results and analysis

Hypothesis #3: transition from log-law to sqrt-law depends on Δp_x^+
(probably more complex …)

Composite profile Brown & Joubert 1969

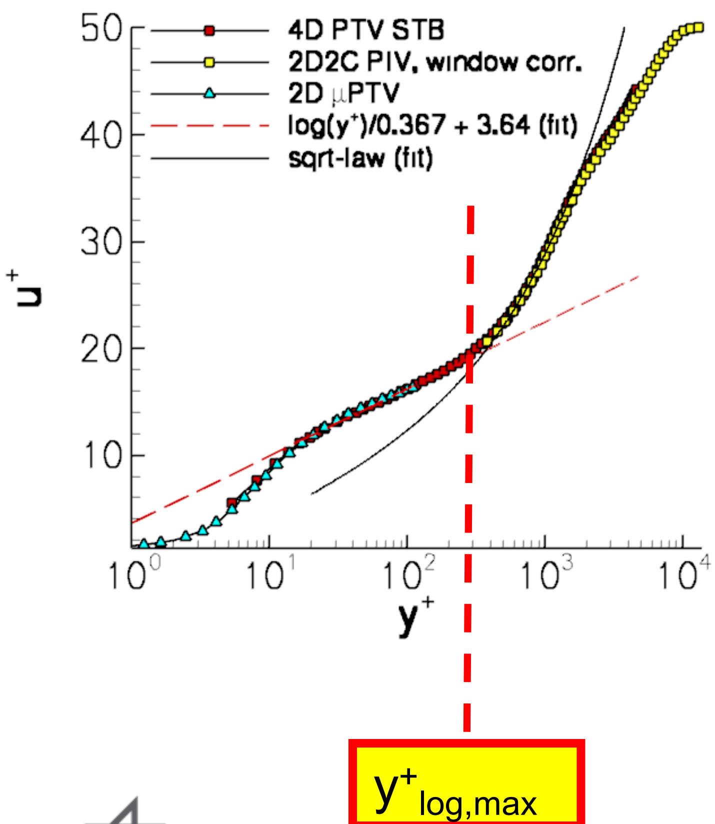


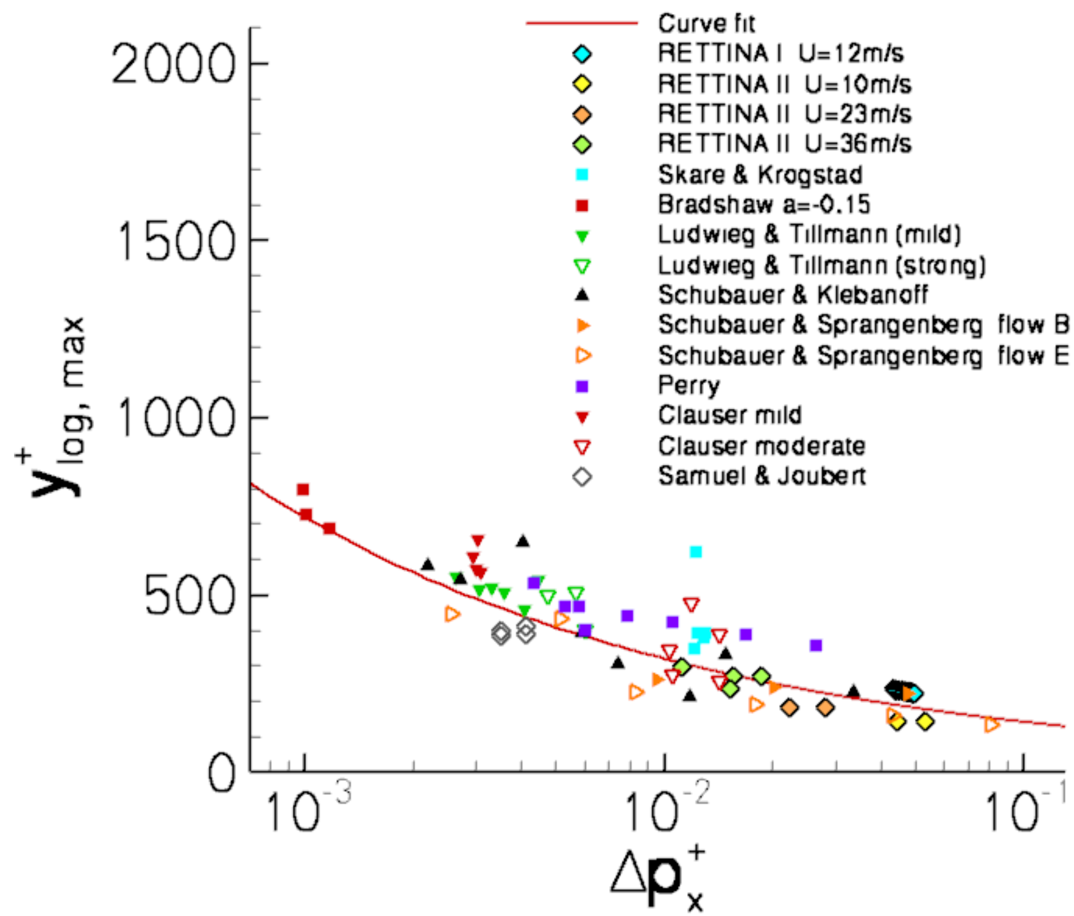


Results and analysis

Hypothesis #3: transition from log-law to sqrt-law depends on Δp_x^+
(probably more complex …)

Composite profile Brown & Joubert 1969



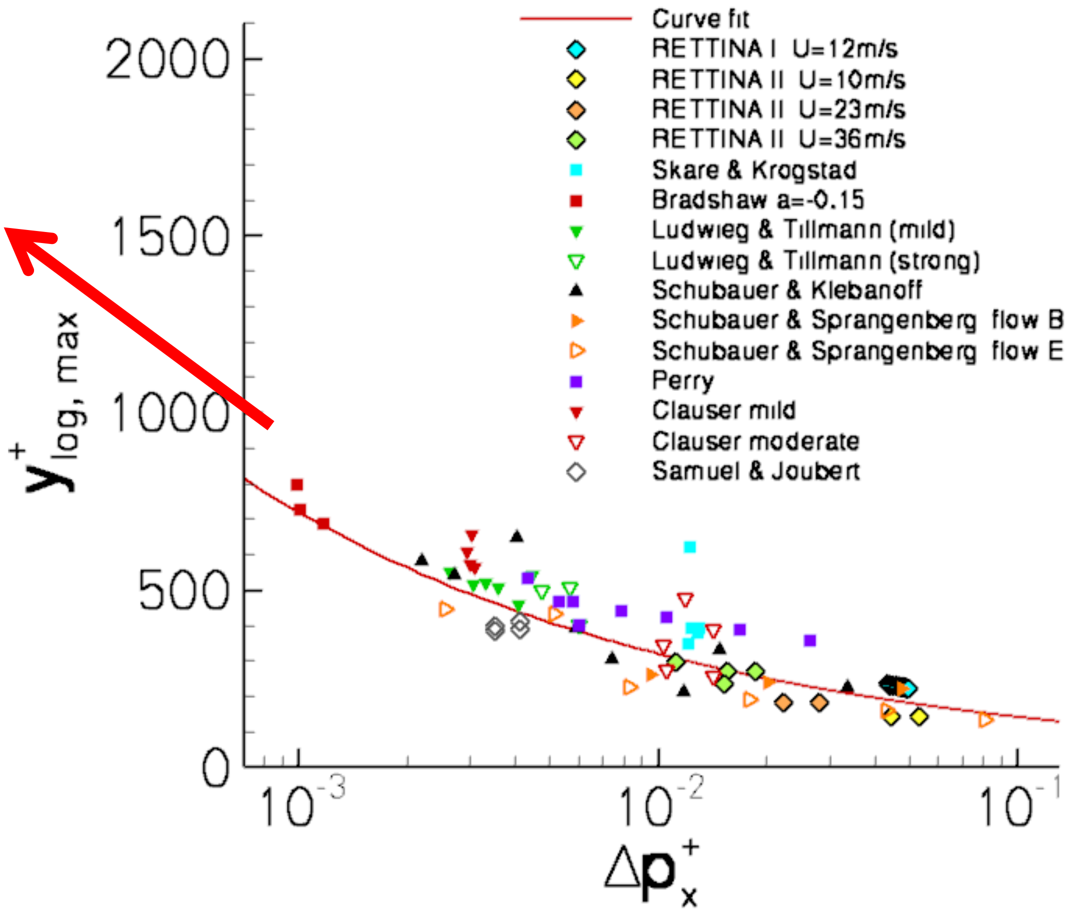


Results and analysis

Hypothesis #3: transition from log-law to sqrt-law depends on Δp_x^+
(probably more complex …)

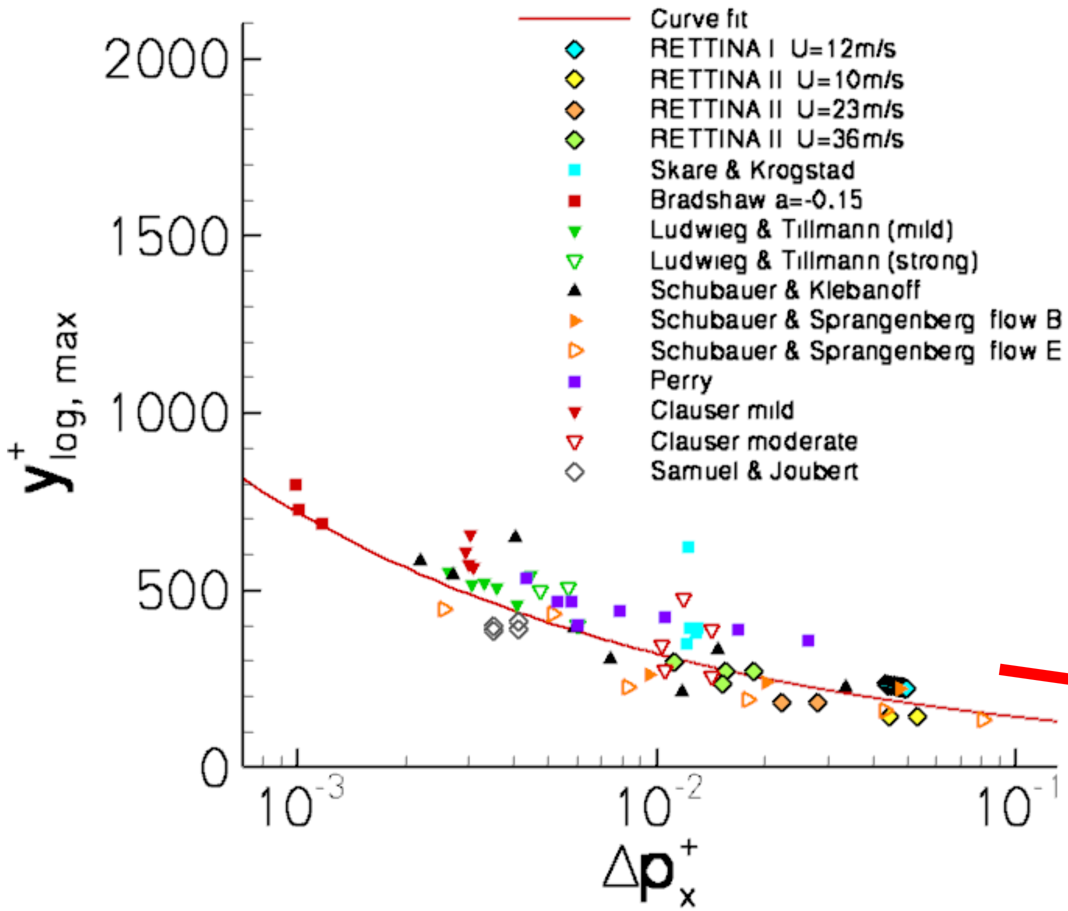
ZPG
$\Delta p_x^+ \rightarrow 0$

Pure log-law



Results and analysis

Hypothesis #3: transition from log-law to sqrt-law depends on Δp_x^+
(probably more complex …)



Separation
$\Delta p_x^+ \longrightarrow \infty$

Pure sqrt-law
(Stratford)

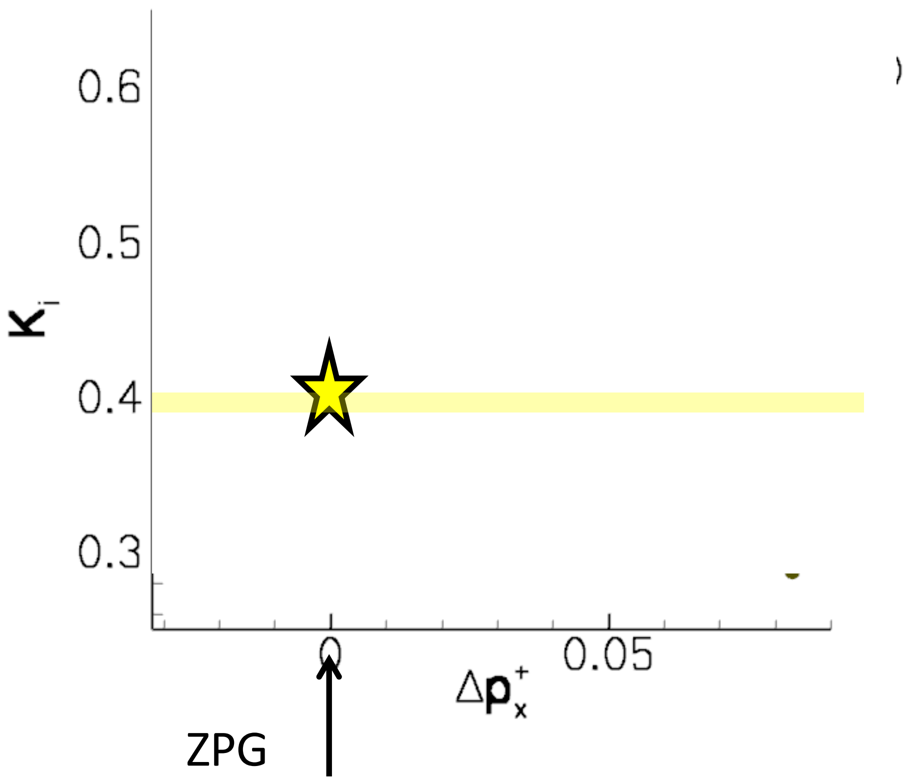
Empirical wall law at APG

Hypothesis #4: decrease of log-law slope parameter K_i at APG

- Slope coefficient K

$$u^+_{\log} = \frac{1}{\boxed{K_i}} \log(y^+) + B_i$$

„von Karman constant“ κ=0.40+/-0.1



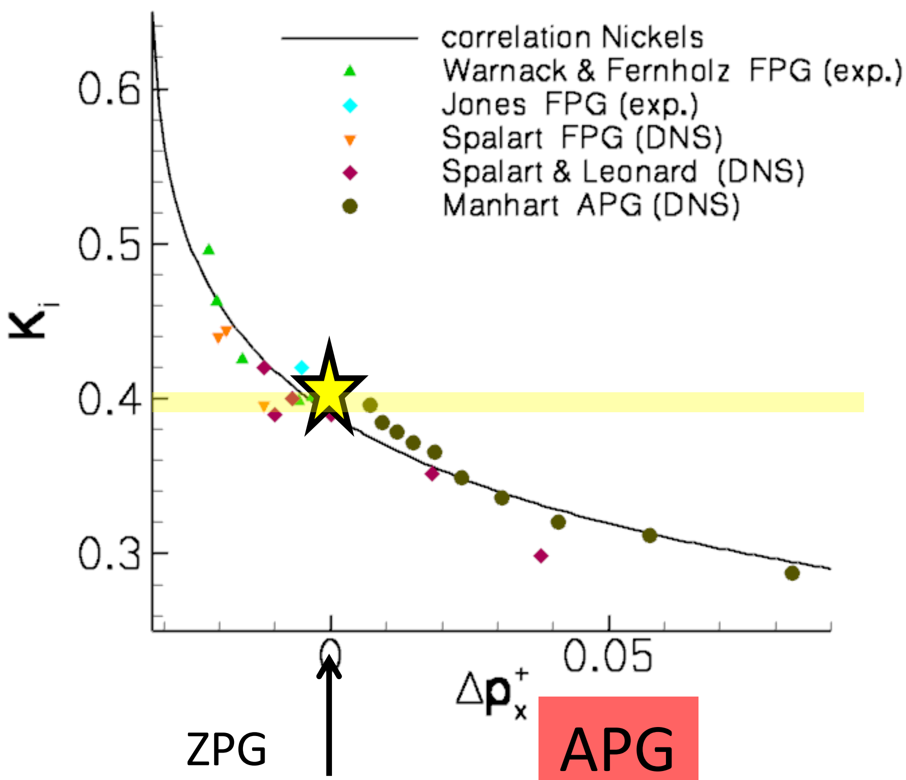
Empirical wall law at APG

Hypothesis #4: decrease of log-law slope parameter K_i at APG

- Slope coefficient K

$$\boxed{u^+_{\log} = \frac{1}{\boxed{K_i}} \log(y^+) + B_i}$$

„von Karman constant“ κ=0.40+/-0.1



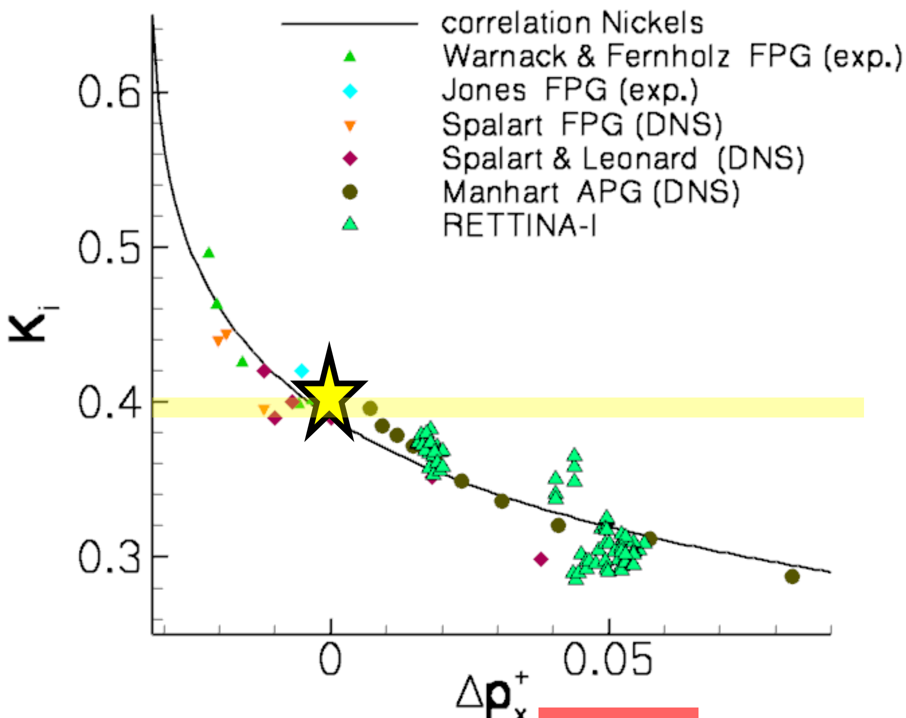
Empirical wall law at APG

Hypothesis #4: decrease of log-law slope parameter K_i at APG

- Slope coefficient K

$$u^+_{\log} = \frac{1}{\boxed{K_i}} \log(y^+) + B_i$$

„von Karman constant“ κ=0.40+/-0.1



APG

DLR

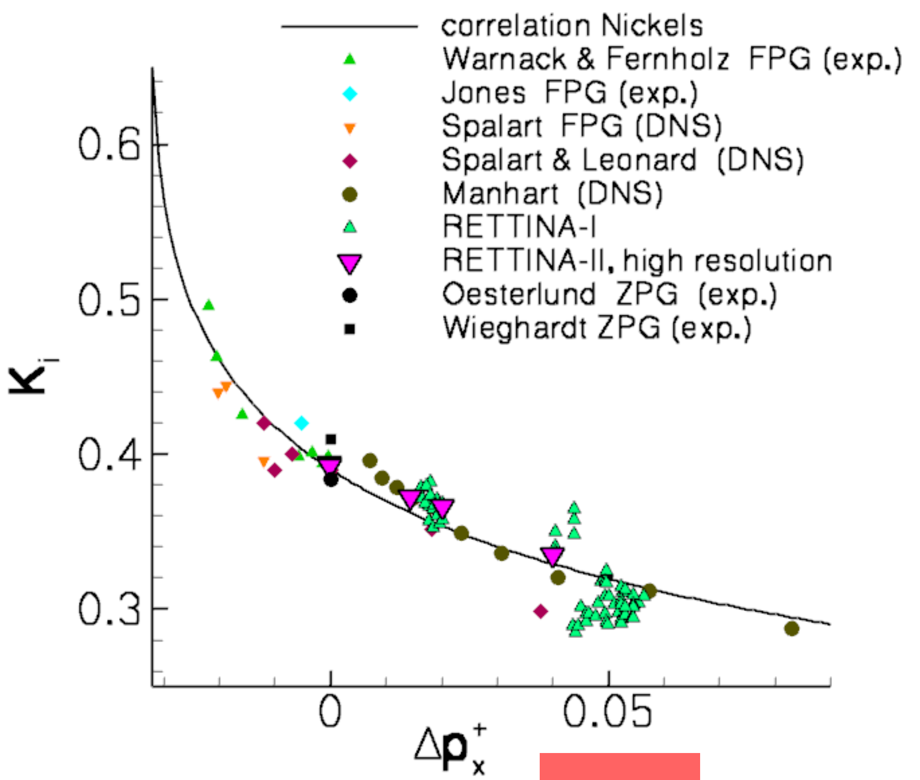
Empirical wall law at APG

Hypothesis #4: decrease of log-law slope parameter K_i at APG

- Slope coefficient K

$$u^+_{\log} = \frac{1}{\boxed{K_i}} \log(y^+) + B_i$$

„von Karman constant“ κ=0.40+/-0.1



APG

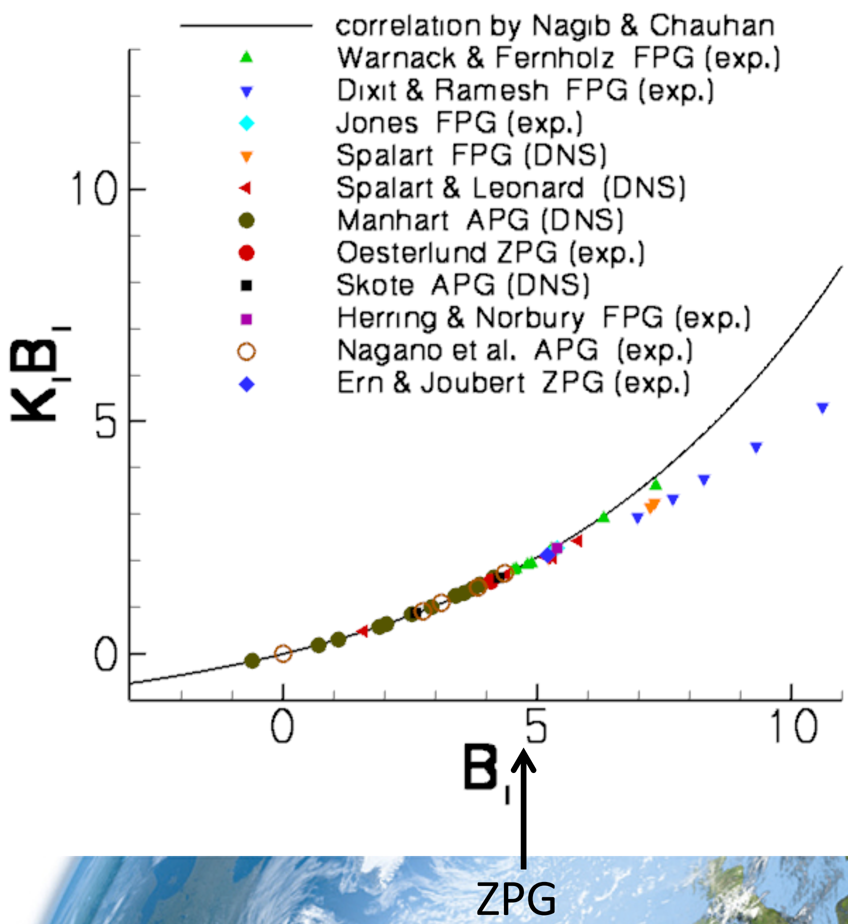
DLR

Empirical wall law at APG

Hypothesis #4: decrease of log-law slope parameter K_i at APG

- Slope coefficient K

$$u^+_{\log} = \frac{1}{\boxed{K_i}} \log(y^+) + \boxed{B_i}$$

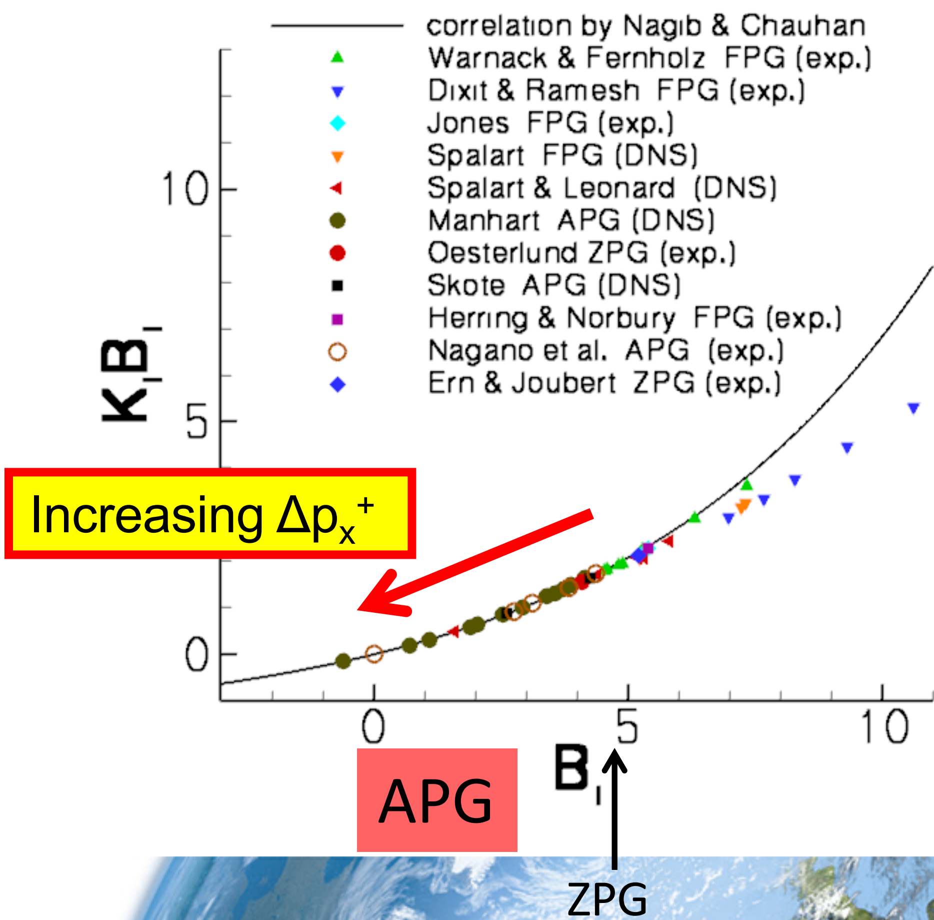


Empirical wall law at APG

Hypothesis #4: decrease of log-law slope parameter K_i at APG

- Slope coefficient K

$$u^+_{\log} = \frac{1}{\boxed{K_i}} \log(y^+) + \boxed{B_i}$$

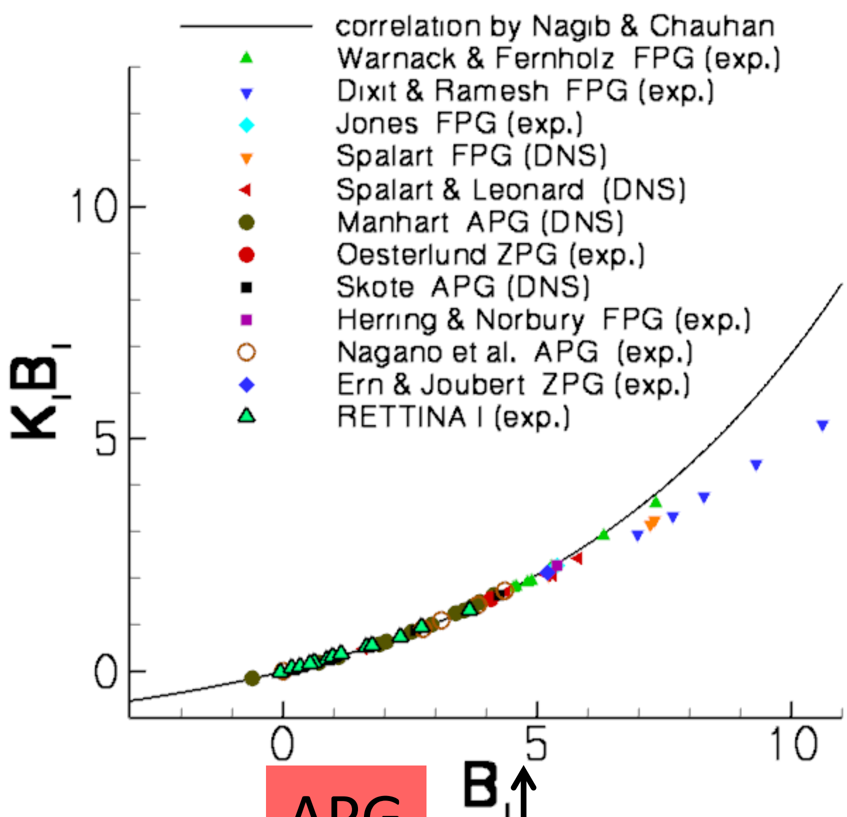


Empirical wall law at APG

Hypothesis #4: decrease of log-law slope parameter K_i at APG

- Slope coefficient K

$$u^{+}_{\log} = \frac{1}{\boxed{K_i}} \log(y^{+}) + \boxed{B_i}$$



APG

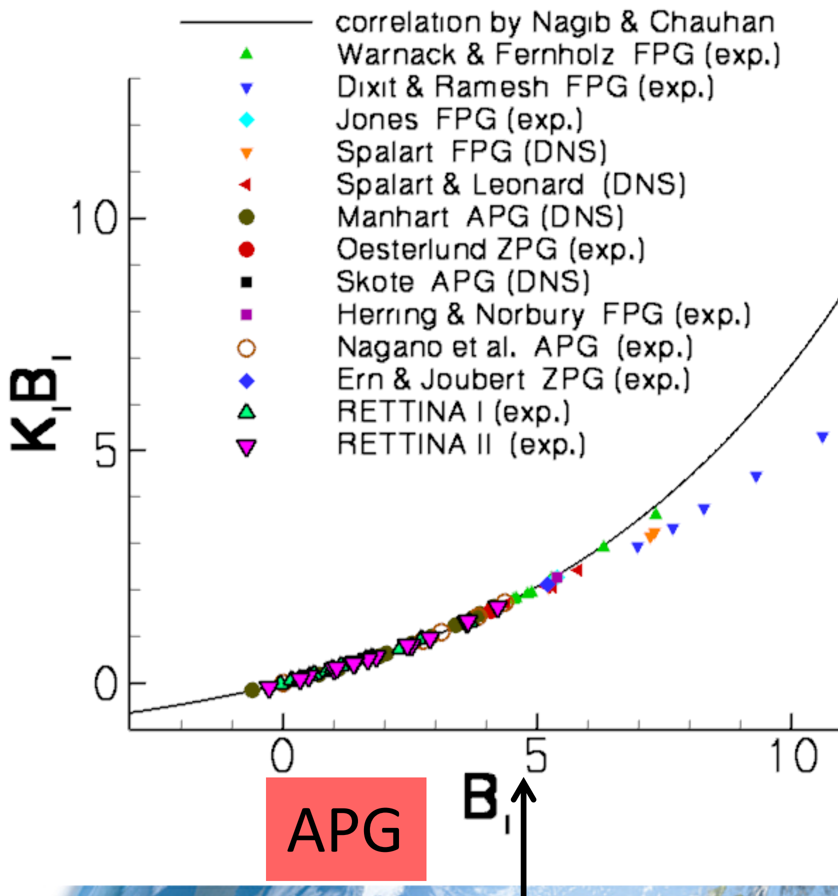
ZPG

Empirical wall law at APG

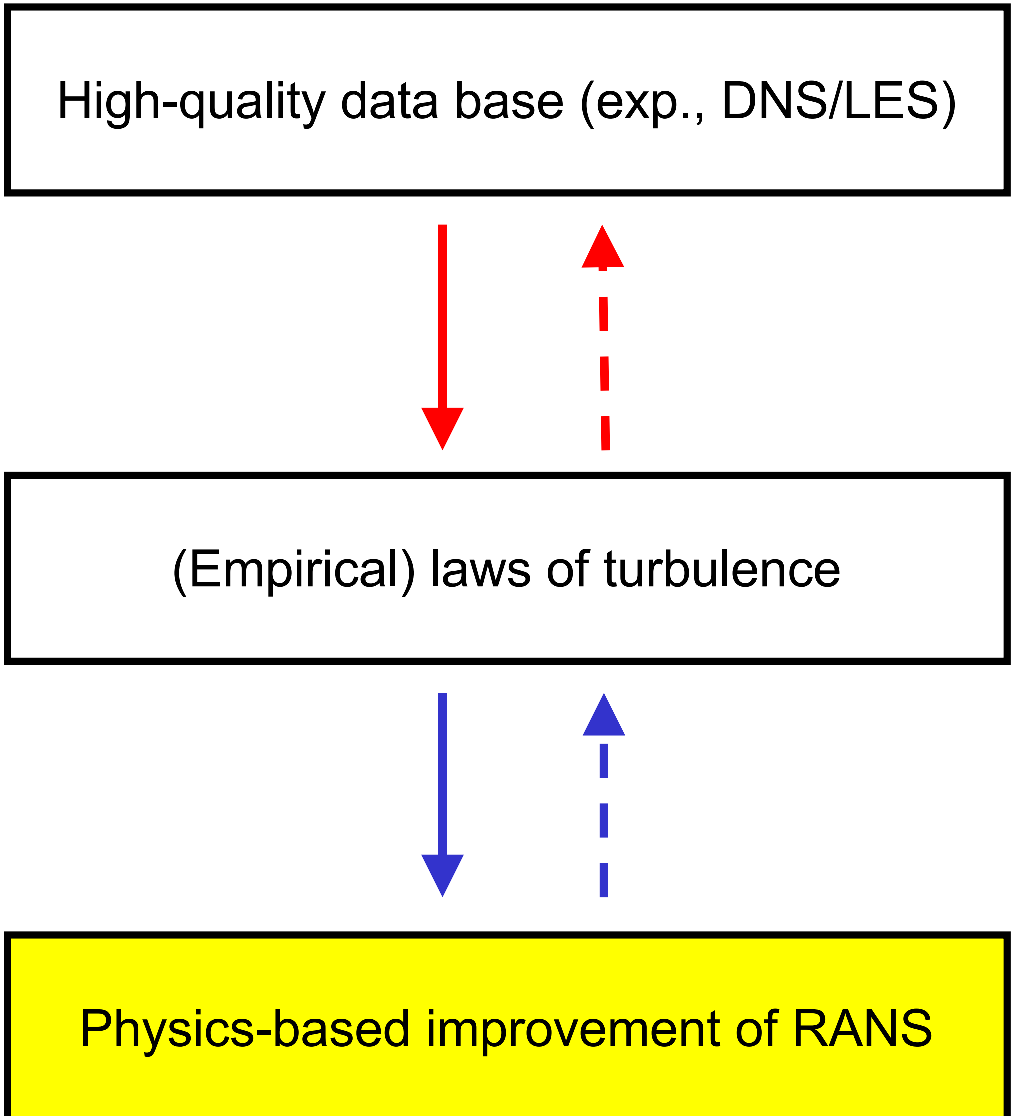
Hypothesis #4: decrease of log-law slope parameter K_i at APG

- Slope coefficient K

$$\boxed{u^+_{\log} = \frac{1}{\boxed{K_i}}\log(y^+) + \boxed{B_i}}$$



DLR strategy for RANS model improvement



DLR strategy for RANS model improvement

High-quality data base (exp., DNS/LES)

(Empirical) laws of turbulence

Physics-based improvement of RANS

Assumptions:
- Blended log-law + sqrt-law
- linear shear stress

DLR strategy for RANS model improvement

High-quality data base (exp., DNS/LES)

(Empirical) laws of turbulence

Physics-based improvement of RANS

Assumptions:
- Blended log-law + sqrt-law
- linear shear stress

Blending for eddy visc. ν_t

Idea: A composite wall-law for the turbulent viscosity

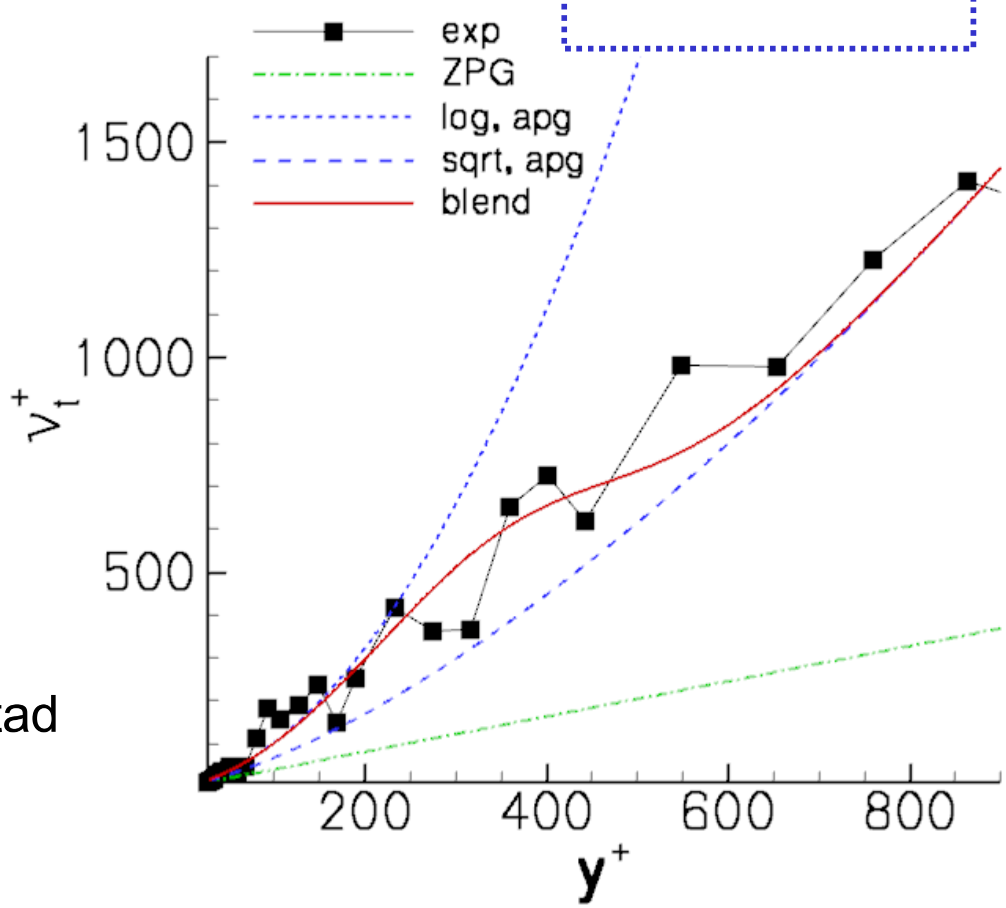
log-law at APG (Galbraith et al. 1977)

$$\kappa y^+ (1 + \lambda \Delta p_x^+ y^+)$$

sqrt-law at APG

$$\kappa y^+ \sqrt{1 + \lambda \Delta p_x^+ y^+}$$

$$\frac{-\overline{u'v'}^+}{\mathrm{d}u^+/\mathrm{d}y^+}$$



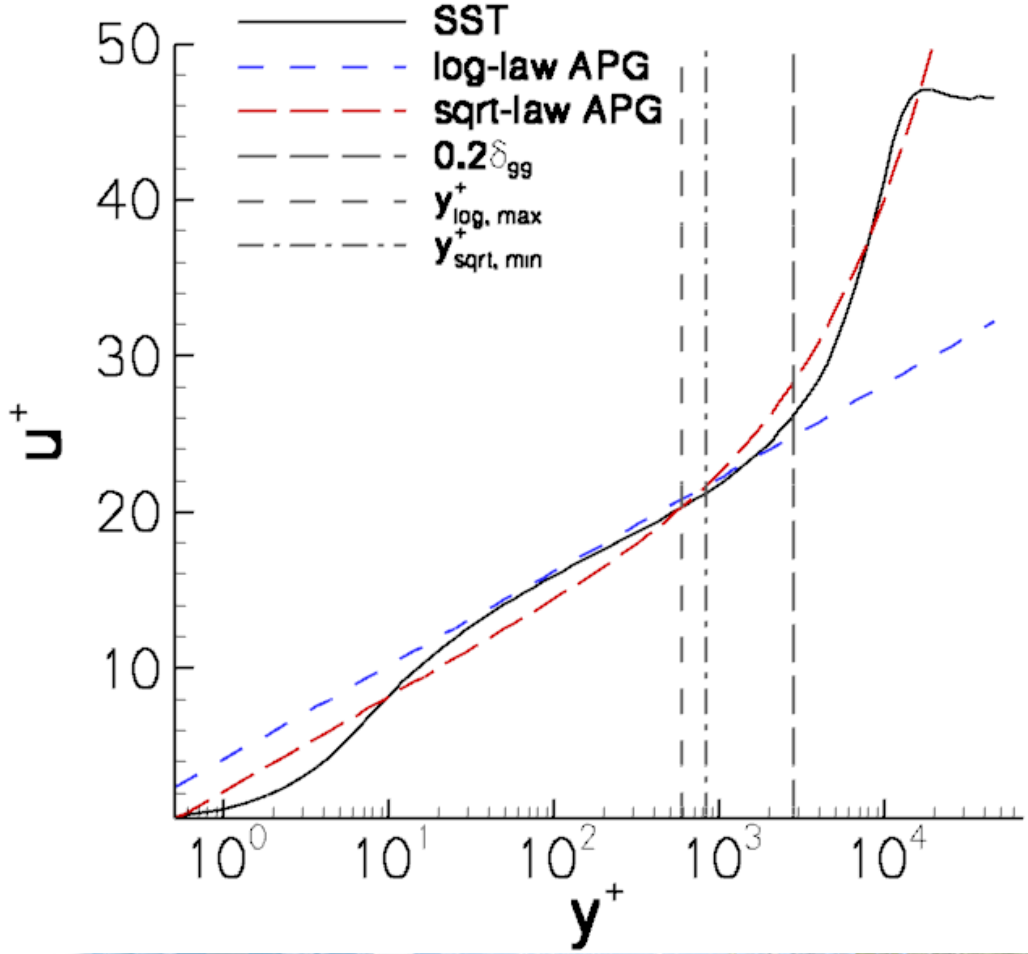
$\Delta p_x^+ = 0.012$

Exp. by Skare & Krogstad

DLR

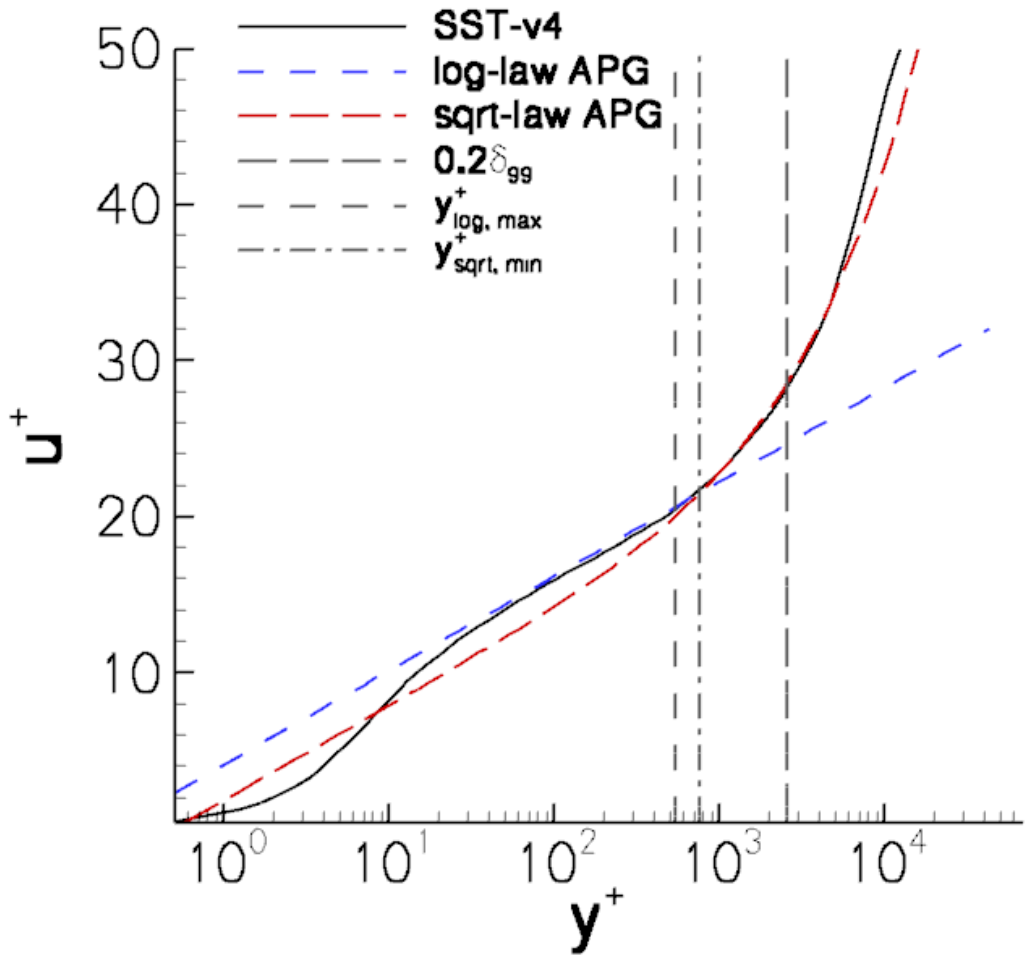
RANS modification

- HGR01 airfoil at Rec=25Mio, incidence angle α=10°



RANS modification

- HGR01 airfoil at Rec=25Mio, incidence angle α=10°



Idea #1: New terms with Δp_x^+-dependent local blending

- Modifications should be activated only in parts of the boundary layer

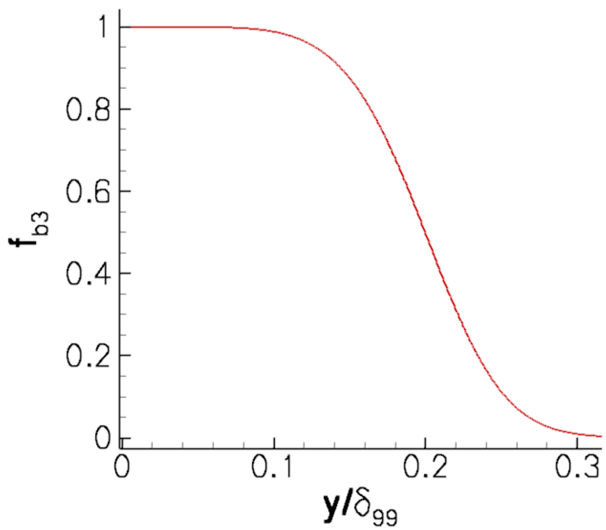
Pressure diffusion at APG (Rao & Hassan)

$$-D^+_{k,p} = -\frac{\mathrm{d}}{\mathrm{d}y^+}\left[-\sigma_{k,P}\,\nu_t^+\,\frac{\overline{u'v'}^+}{k^+}\,\Delta p_x^+\right]$$

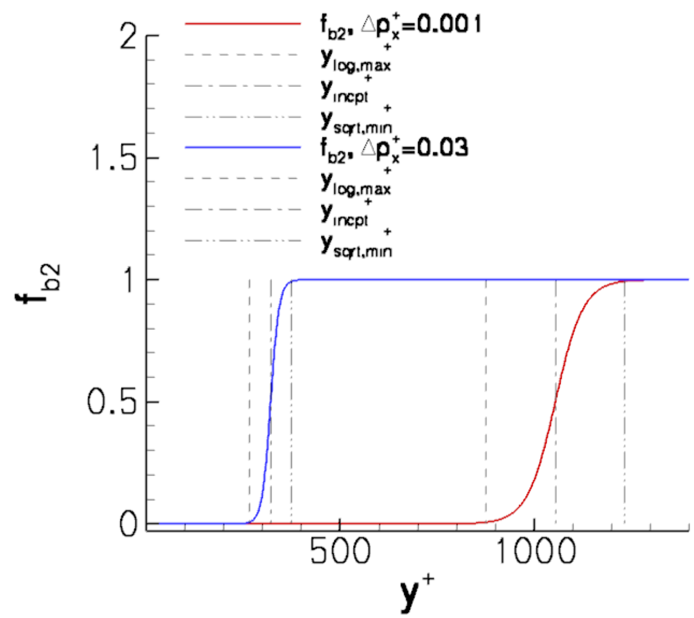
$$\frac{\partial k}{\partial t} + \vec{\nabla}\cdot(\vec{U}k) - D_{k,t} - f_{b1} f_{b3} D_{k,p} = P_k - \epsilon_k$$

$$\frac{\partial \omega}{\partial t} + \vec{\nabla}\cdot(\vec{U}\omega) - D_{\omega,t} - f_{b2} f_{b3} D_{\omega,p} = P_\omega - \epsilon_\omega + f_{b2} f_{b3} D_{cd,\omega}$$

Blending for $y<0.15\delta_{99}$



Activate only in the sqrt-region



DLR

Idea #2: RANS model coefficients sensitized to pressure gradient Δp_x^+

Coefficient γ controls the slope of the log-law

$$\frac{\partial(k)}{\partial t} + \vec{\nabla}\cdot(\vec{u}k) - \vec{\nabla}\cdot\Big(\left(\nu + \nu_t\sigma_k\right)\vec{\nabla}k\Big) = \tilde{\mathscr{P}} - \beta_k k\omega$$

$$\frac{\partial(\omega)}{\partial t} + \vec{\nabla}\cdot(\vec{u}\omega) - \vec{\nabla}\cdot\Big(\left(\nu + \nu_t\sigma_\omega\right)\vec{\nabla}\omega\Big) = \frac{\gamma}{\nu_t}\mathscr{P} - \beta_\omega\omega^2$$

$$\gamma = \frac{\beta_\omega}{\beta_k} - \frac{\sigma_\omega\kappa^2}{\sqrt{\beta_k}}$$

Standard calibration is for ZPG
κ=0.41 = const

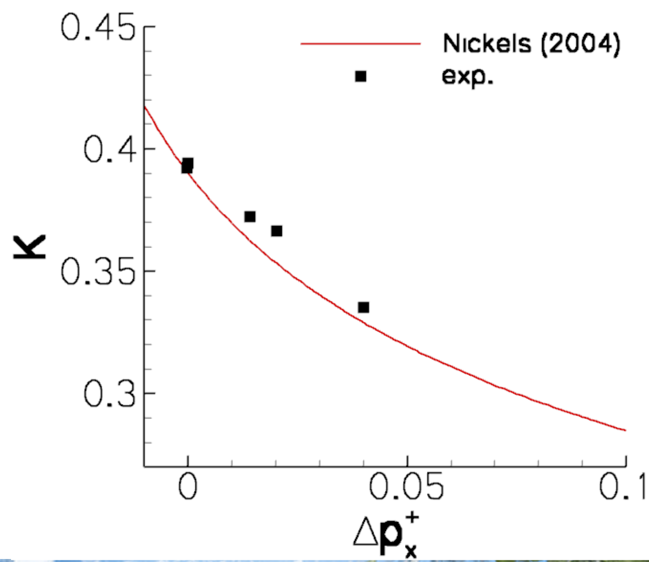
Idea #2: RANS model coefficients sensitized to pressure gradient Δp_x^+

- The RANS model coefficient which controls the log-law slope (“Karman-constant”) becomes a function depending on local flow

$$\frac{\partial(k)}{\partial t} + \vec{\nabla}\cdot(\vec{u}k) - \vec{\nabla}\cdot\Big((\nu+\nu_t\sigma_k)\,\vec{\nabla}k\Big) = \tilde{\mathscr{P}} - \beta_k k\omega$$

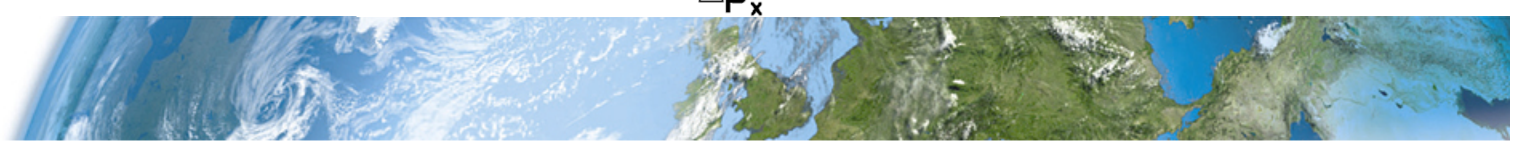
$$\frac{\partial(\omega)}{\partial t} + \vec{\nabla}\cdot(\vec{u}\omega) - \vec{\nabla}\cdot\Big((\nu+\nu_t\sigma_\omega)\,\vec{\nabla}\omega\Big) = \frac{\gamma}{\nu_t}\tilde{\mathscr{P}} - \beta_\omega\omega^2$$

$$\gamma = \frac{\beta_\omega}{\beta_k} - \frac{\sigma_\omega\kappa^2}{\sqrt{\beta_k}}$$



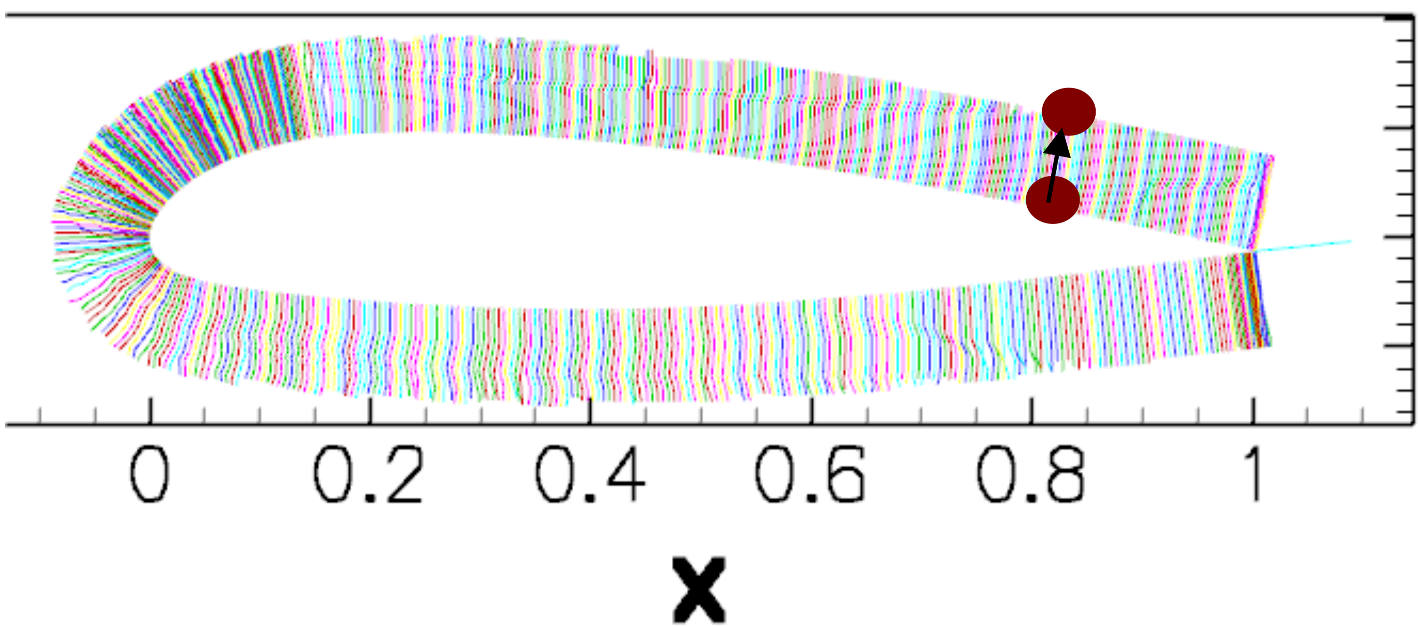
$$K_{\mathrm{i}} = \frac{\kappa_0}{\sqrt{1+\Delta p_x^+ y_c^+}}$$

DLR



Data structure of wall-normal lines for Δp_x^+

- Extension of unstructured flow solver TAU
  - Wall-normal lines
  - Method to determine δ_{99}, δ^*, θ, H_{12}
  - Coding efforts vs. Improvements in predictive accuracy



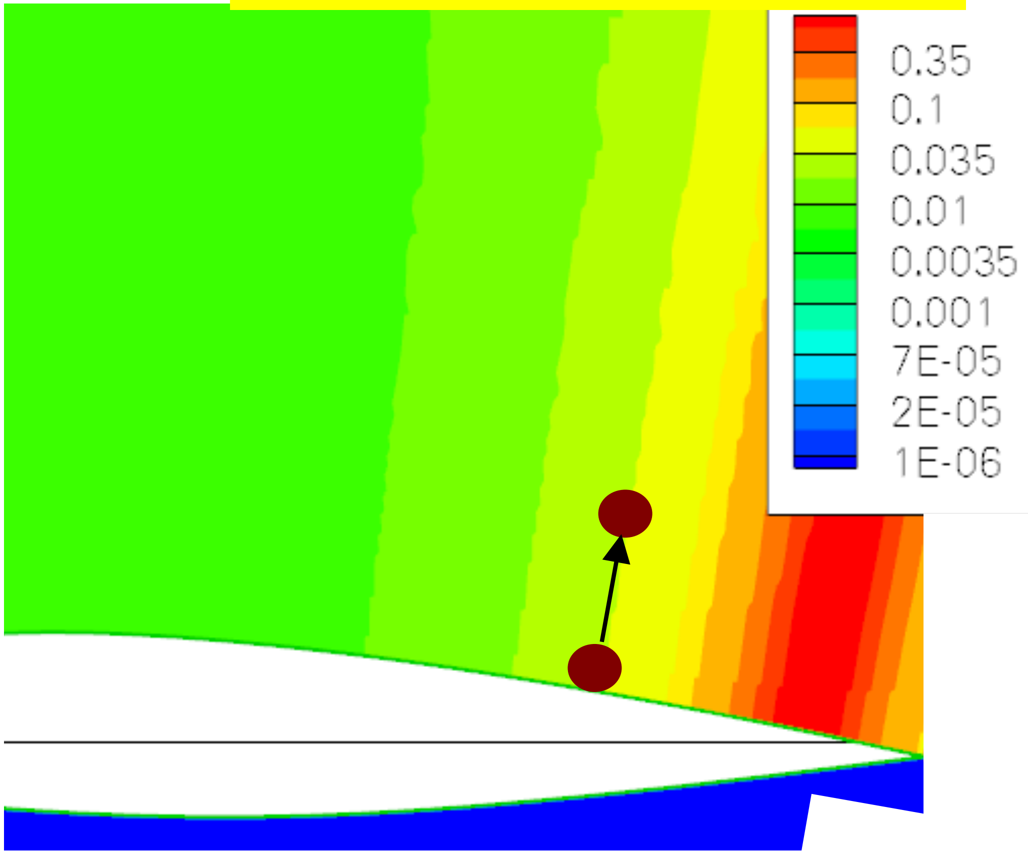
Field point

Surface point
Δp_x^+ from dp/dx and u_τ
δ_{99}, δ^*, θ, H_{12}

DLR

RANS model sensitized to pressure gradient Δp_x^+

Field distribution of Δp_x^+



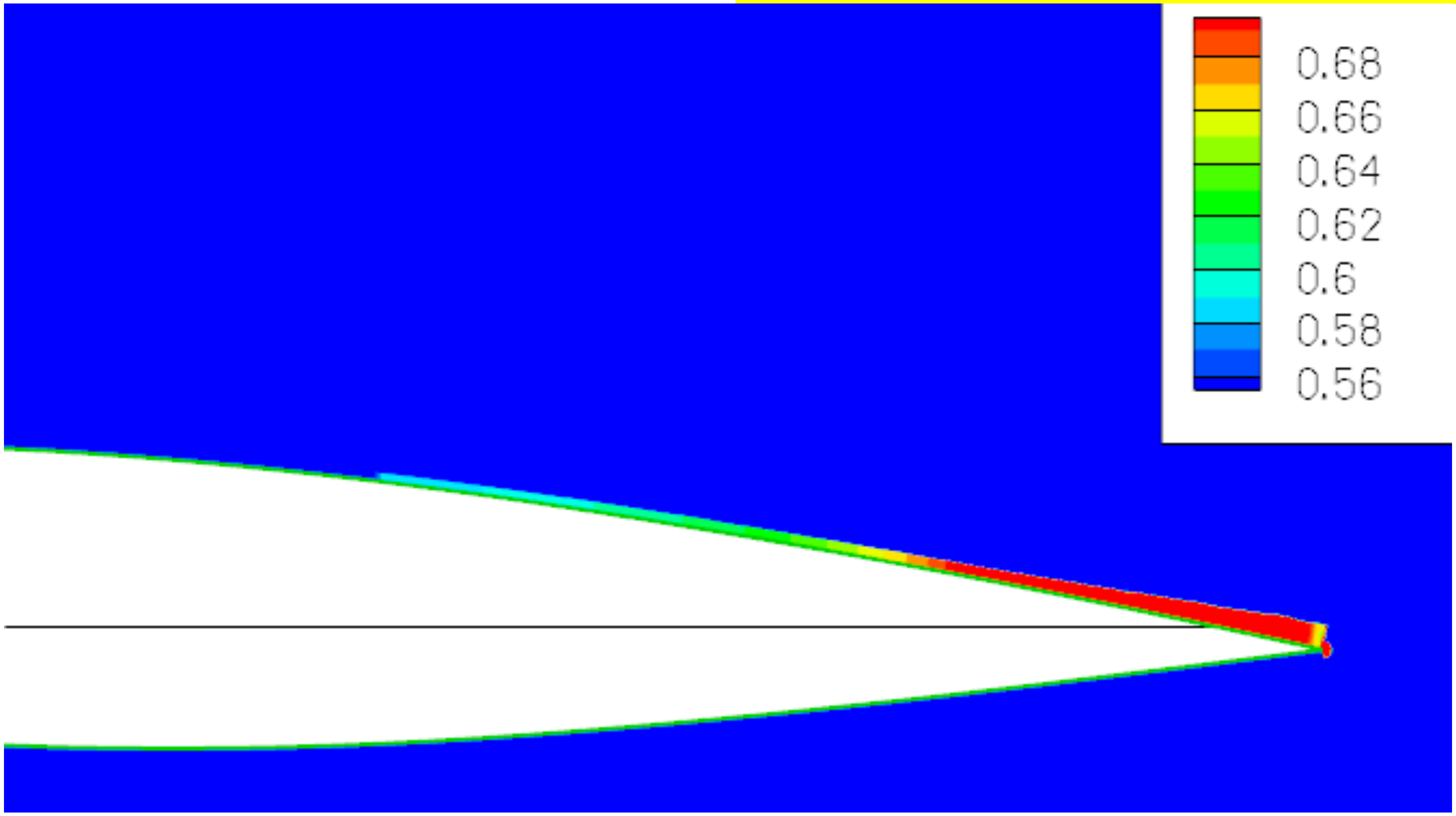
Map to corresponding field points

Surface quantities Δp_x^+ from dp/dx and u_τ

DLR

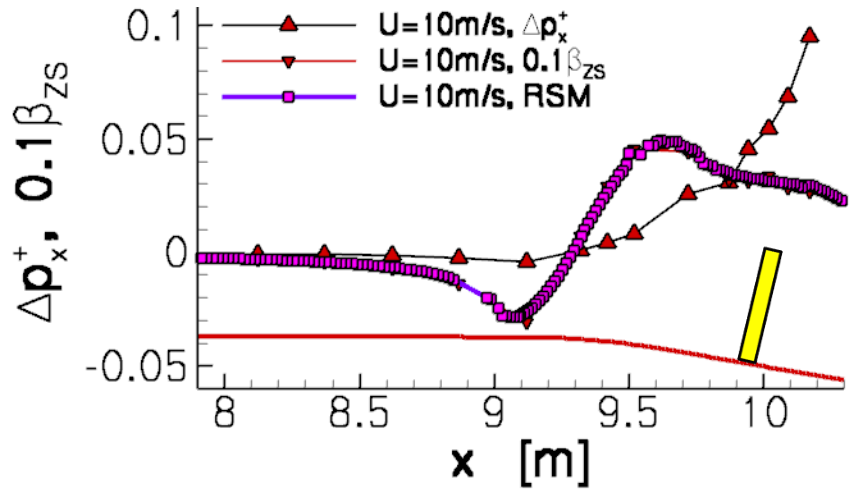
RANS model sensitized to pressure gradient Δp_x^+

RANS model coefficient **γ** becomes a function of Δp_x^+

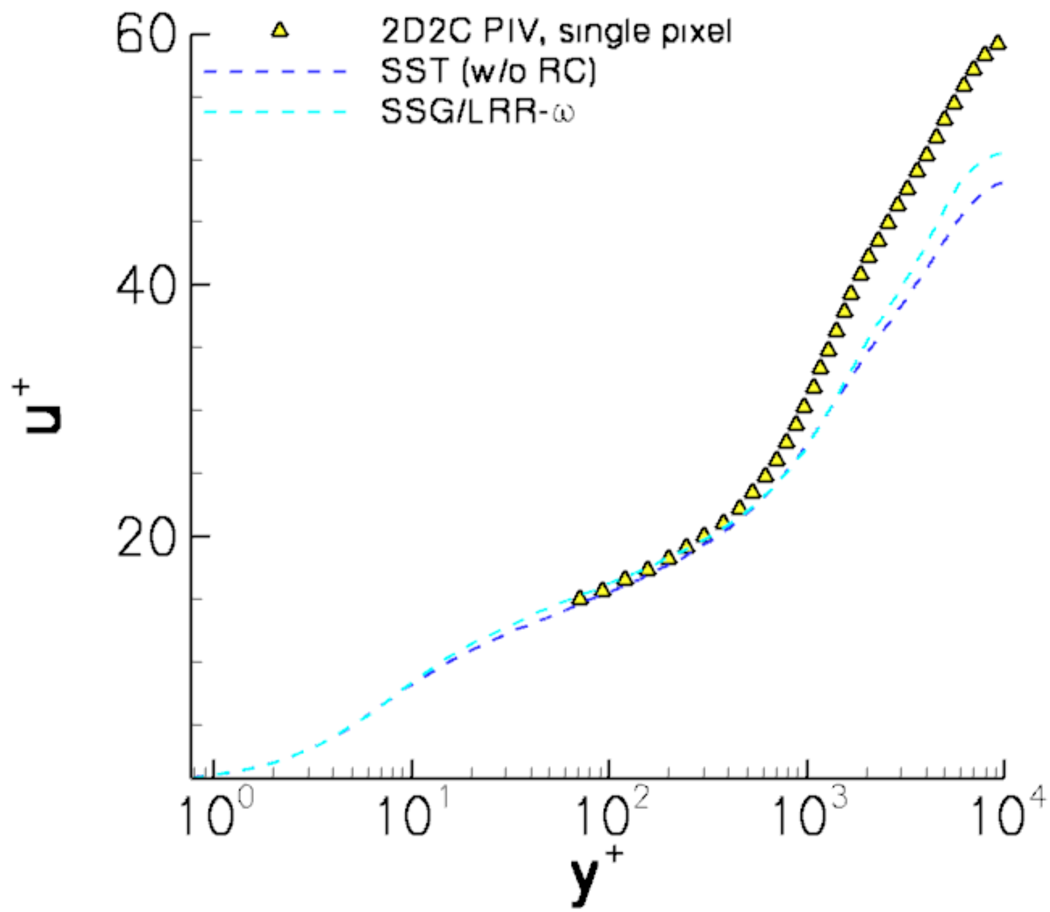


DLR

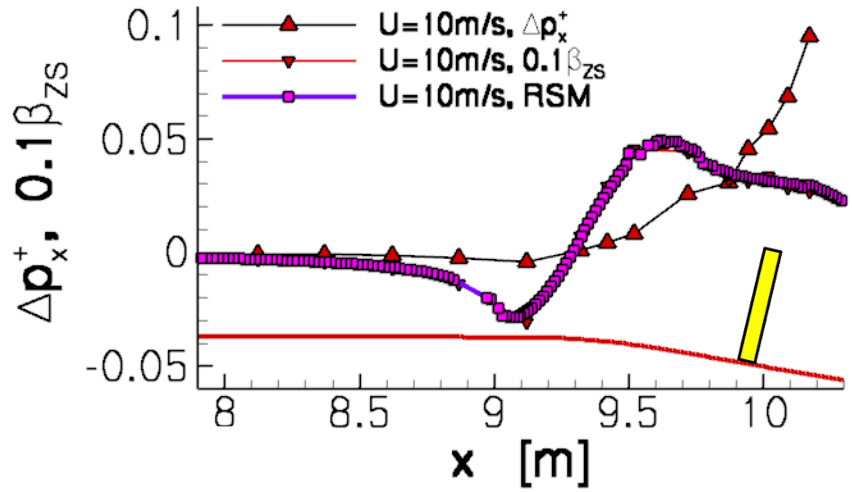
Validation of modified RANS



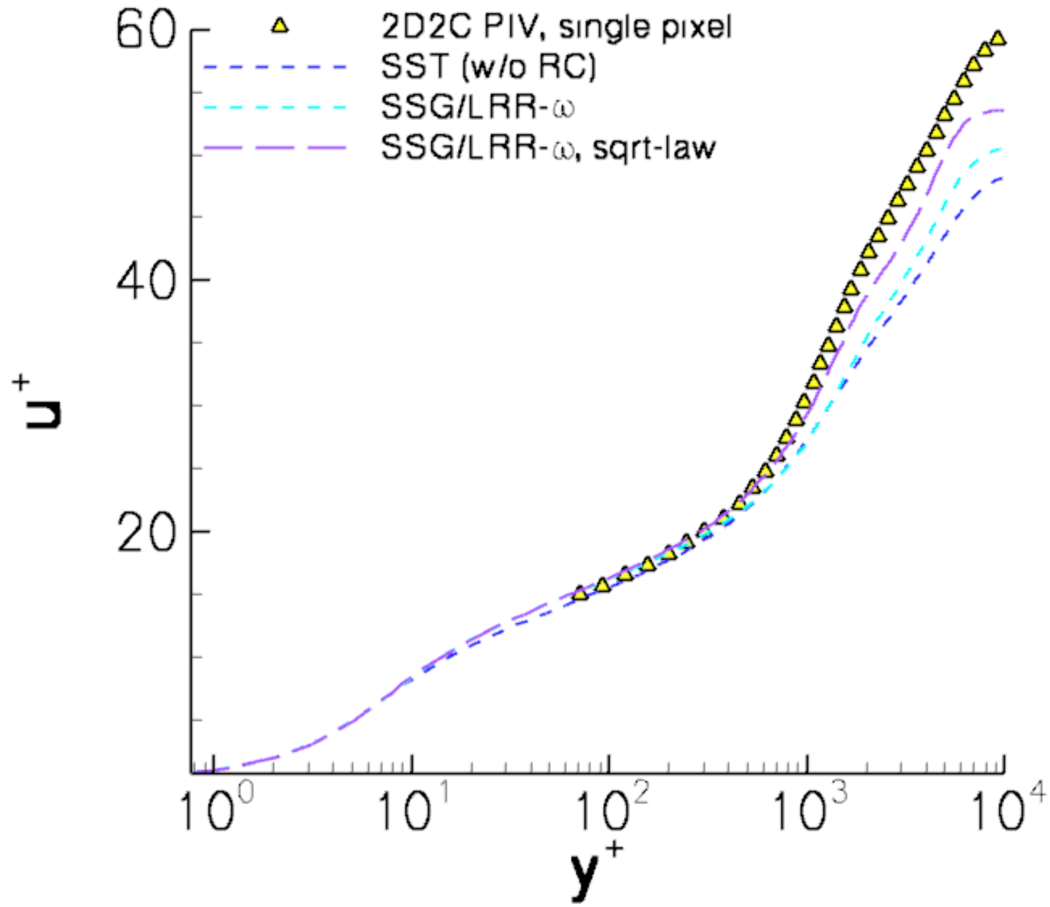
U=36m/s



Validation of modified RANS

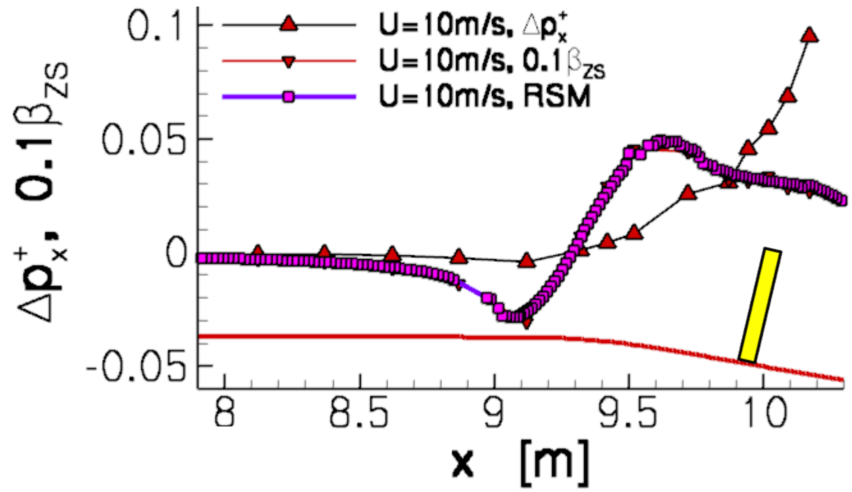


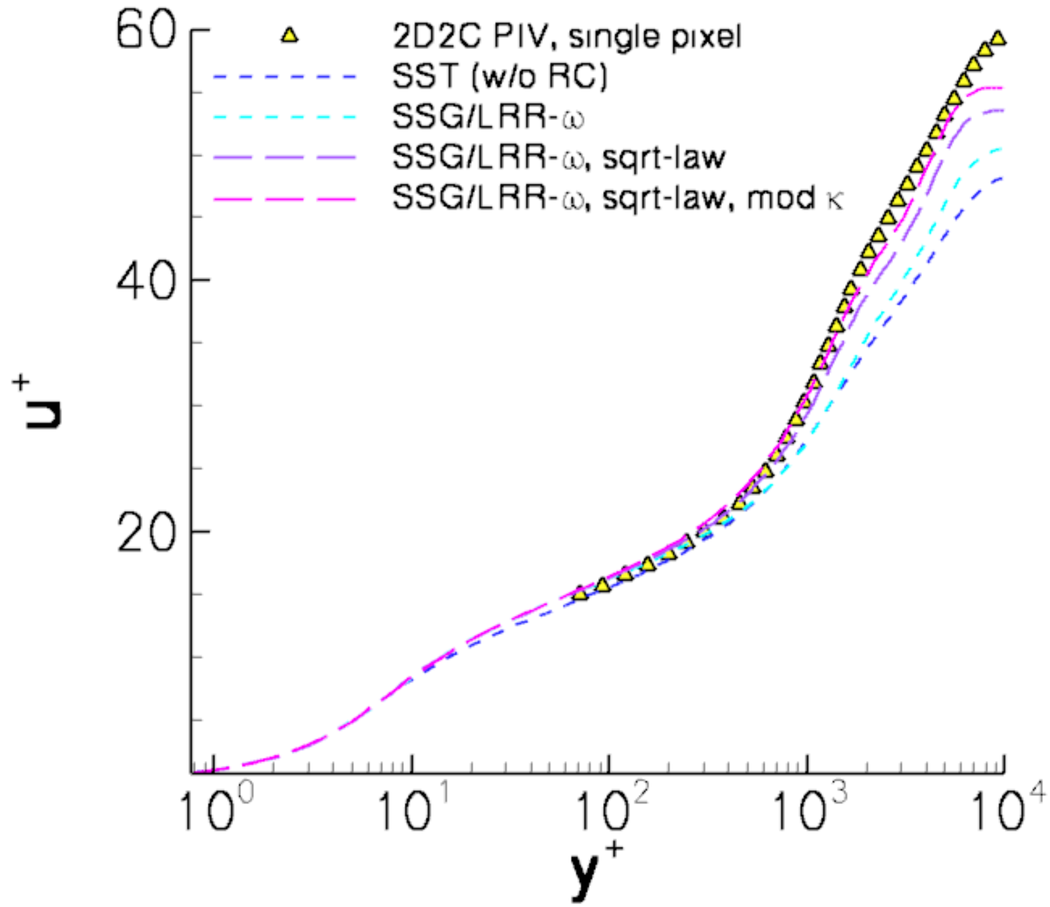
U=36m/s



Validation of RANS

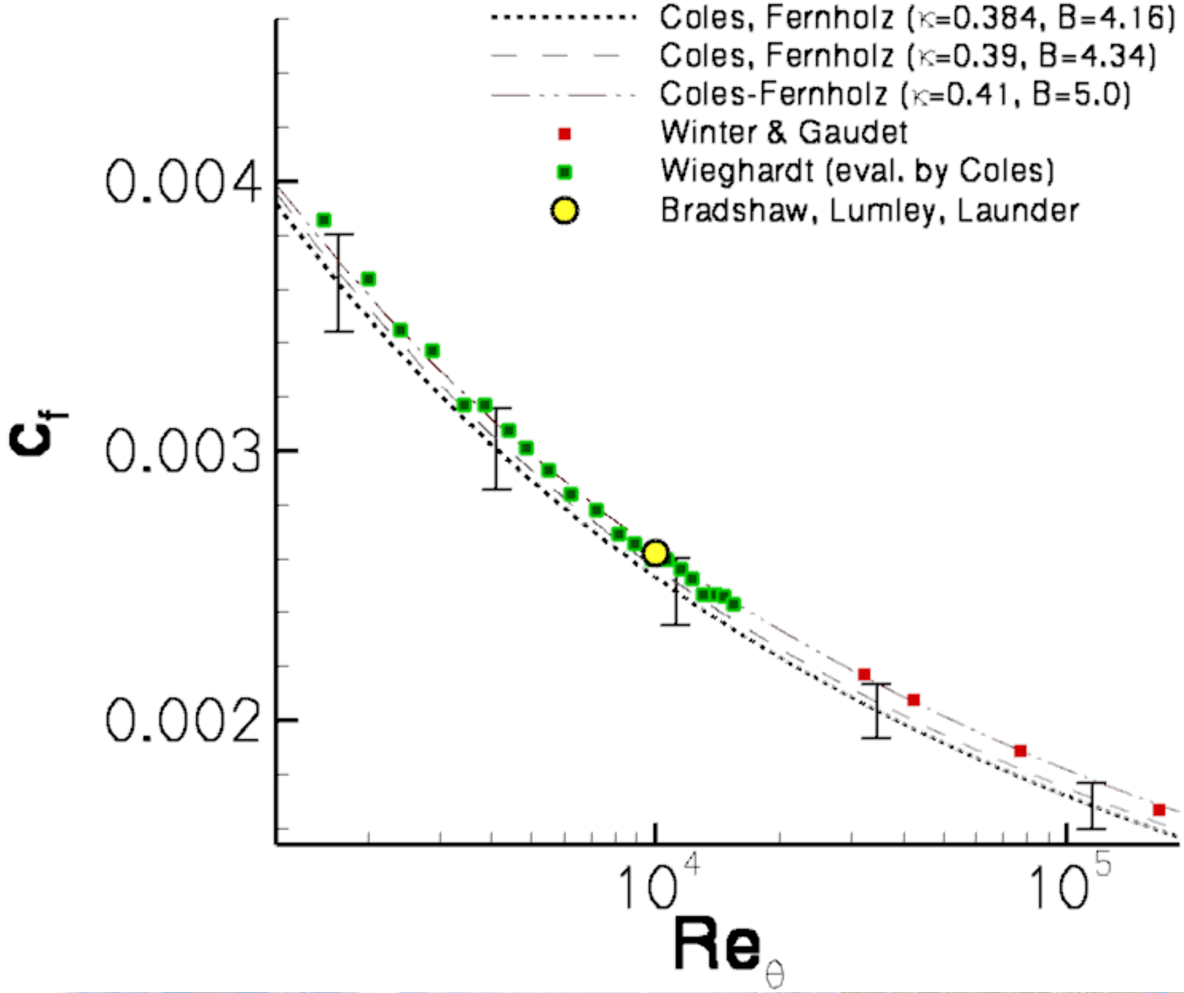
U=36m/s



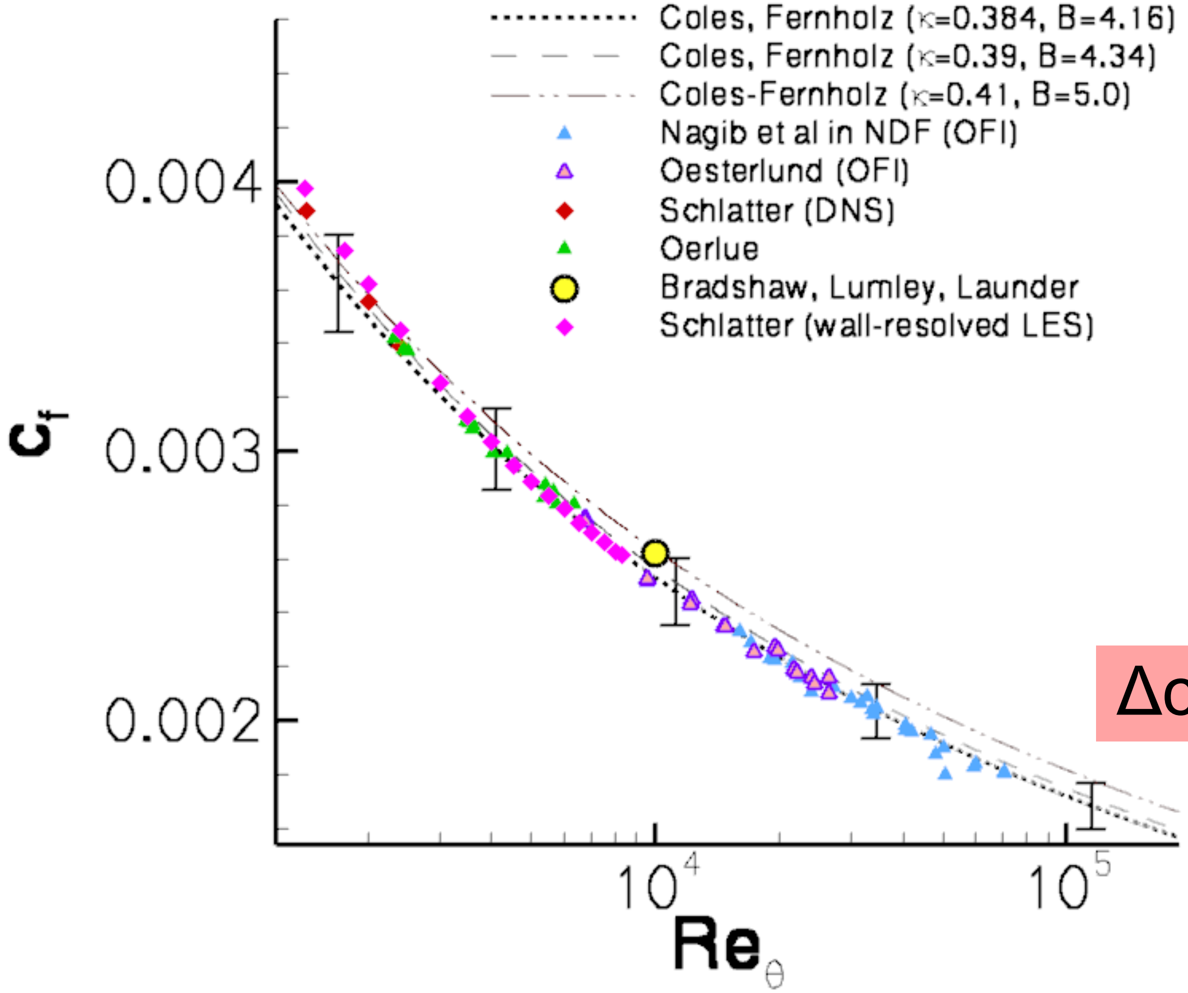


Questions to the audience

Turbulent Boundary Layer at ZPG



Turbulent Boundary Layer at ZPG



Δcf=5%!

Conclusions

Conclusions

- DLR strategy: Physics based improvement of RANS using new laws of turbulence
    - First step: Wall law at APG
    - Composite wall law at APG (Brown & Joubert)
        - Small log-region around y^{+}~100
        - Sqrt-law region above log-region
    - Machine learning for further tuning of first theoretical ideas

- Optimism to improve RANS models for aerodynamic flows during next decades
    - Great potential for future research
    - DNS/LES at relevant Re possible
        - New smart DNS flows (e.g., work of Spalart & Coleman, Soria & Jimenez)
    - Improved measurement techniques (e.g. advances in PIV/PTV)

DLR

End of the presentation

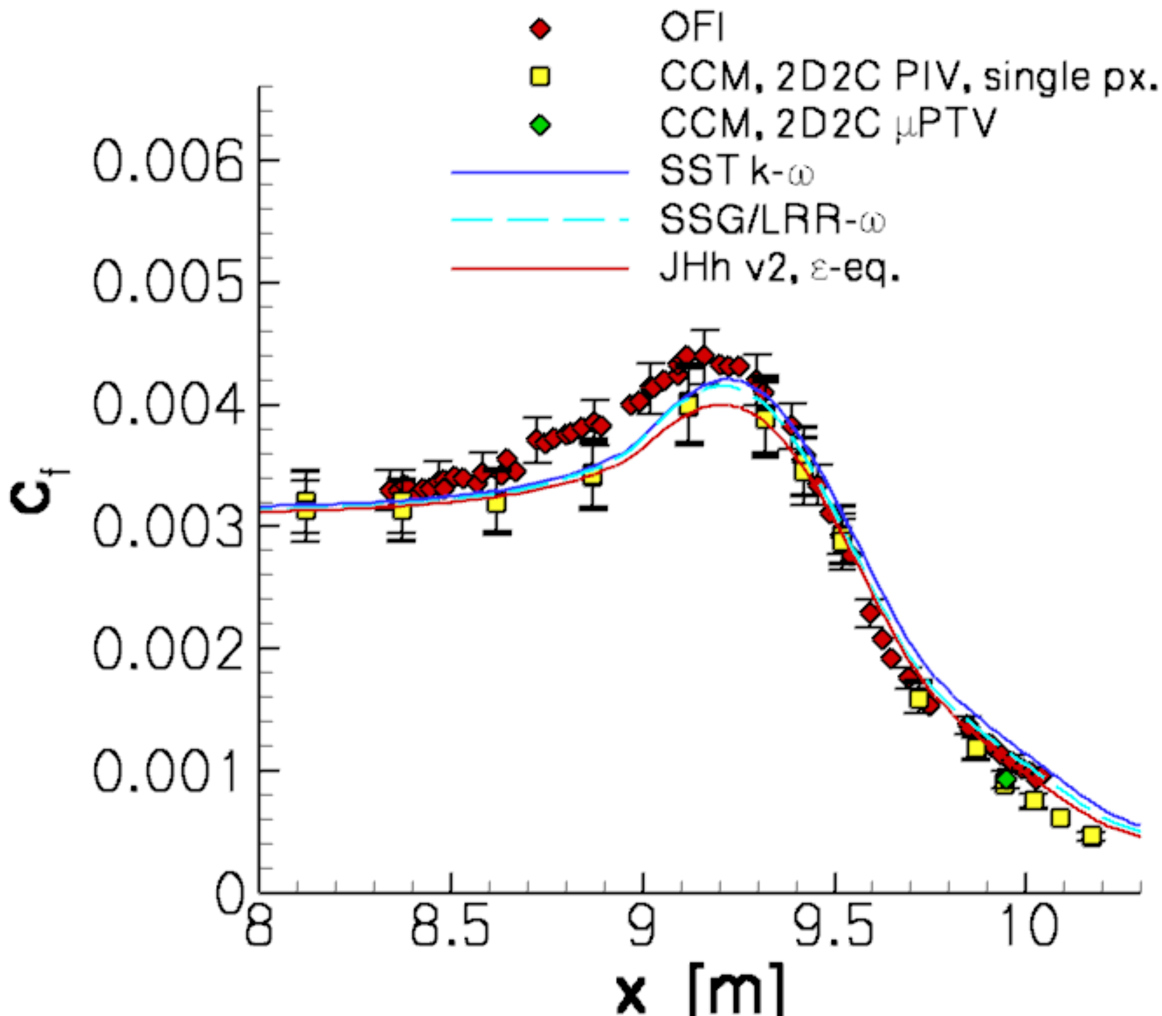
- Experimental data shown in cooperation with
  - Daniel Schanz, Matteo Novara, Erich Schülein, Andreas Schröder DLR AS
  - Nico Reuther, Nicolas Buchmann, Rainer Hain, Christian Cierpka, Christian Kähler, UniBw München

- Funding of measurement campaign by DFG within „Investigation of turbulent boundary layers with pressure gradient at high Reynolds numbers with high resolution multi-camera techniques“

DLR

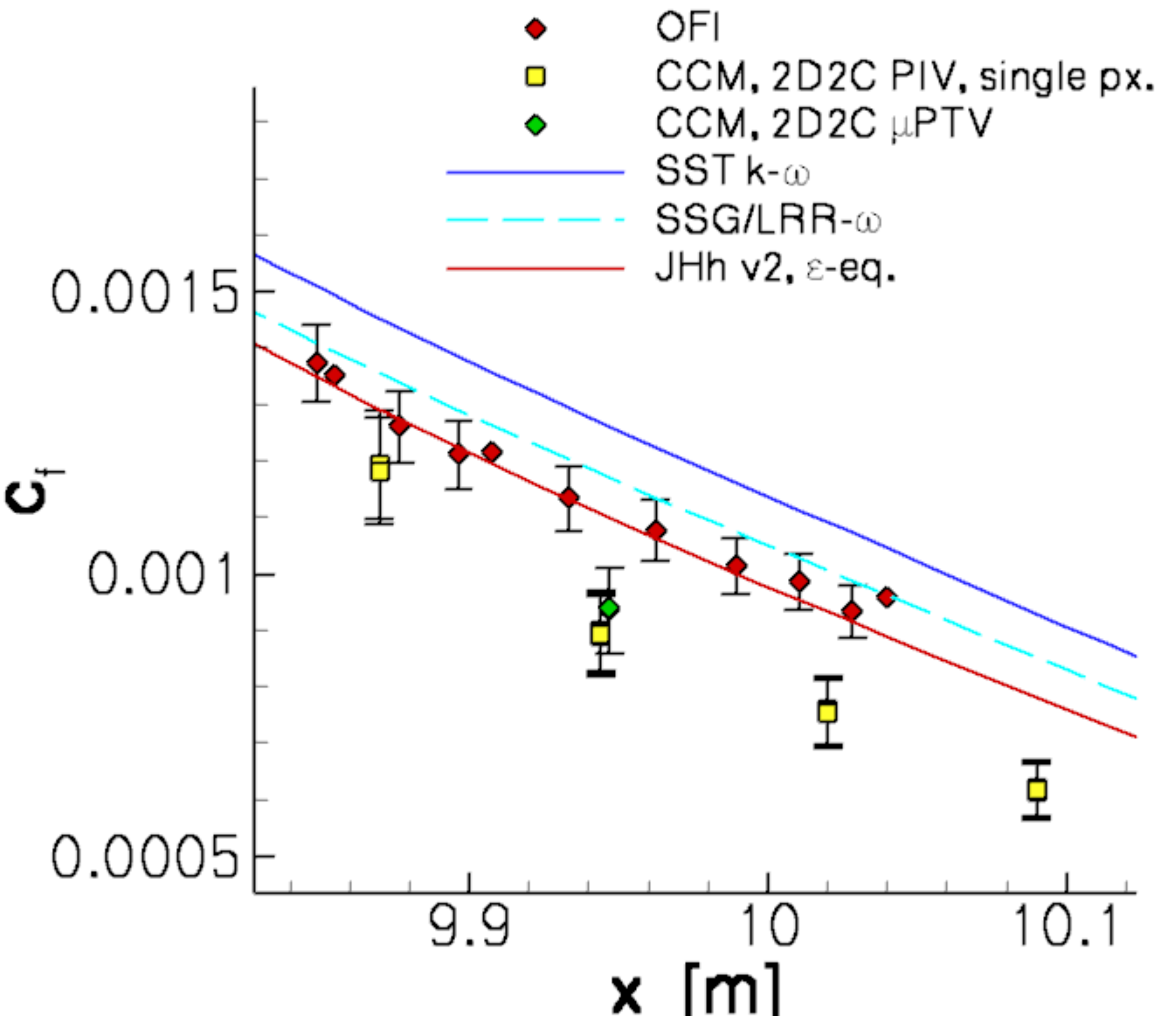
Validation RETTINA II

U=36m/s



Validation RETTINA II

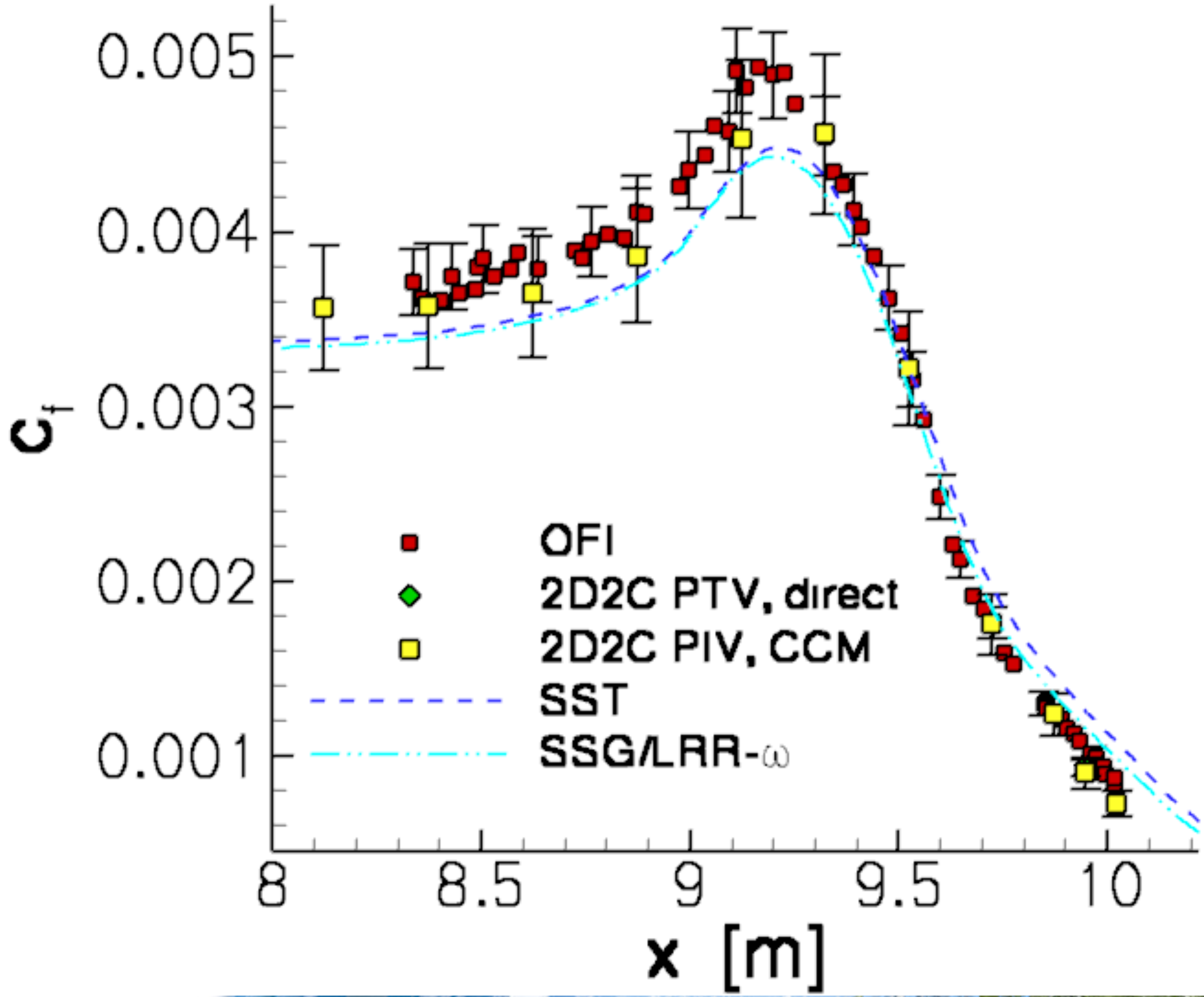
U=36m/s



DLR

Validation RETTINA II

U=23m/s



Measuring technique

Modification of the k-omega model

- Modified **consistent** model

$$-D_{k,t}^{+} - D_{k,p}^{+} = P_k^{+} - \epsilon_k^{+}$$

$$-D_{\omega,t}^{+} - D_{\omega,p}^{+} = P_\omega^{+} - \epsilon_\omega^{+} + D_{cd,\omega}^{+}$$

- Modification of the k-equation:
  - **Pressure diffusion term (Rao & Hassan)**

$$-D_{k,p}^{+} = -\frac{\mathrm{d}}{\mathrm{d}y^{+}}\left[-\sigma_{k,P}\nu_t^{+}\frac{\overline{u'v'}^{+}}{k^{+}}\Delta p_x^{+}\right]$$

- Modification of the ω-equation:
  - **Pressure diffusion term**

$$-D_{\omega,p}^{+} = -\frac{\omega^{+}}{k^{+}}D_{k,p}^{+}$$

  - ***Negative* cross-diffusion term**

$$D_{cd,\omega}^{+} = \frac{2\sigma_{\omega 2}}{\omega}\frac{\mathrm{d}k^{+}}{\mathrm{d}y^{+}}\frac{\mathrm{d}\omega^{+}}{\mathrm{d}y^{+}}$$

DLR

Consistency with wall-law

- Summary of boundary layer analysis

	Consistency with log-law at APG	Consistency with sqrt-law at APG
k-equation	No	No
ω-equation	Yes	No

$$-D_{k,t}^{+} = P_{k}^{+} - \epsilon_{k}^{+}$$

$$-D_{\omega,t}^{+} = P_{\omega}^{+} - \epsilon_{\omega}^{+}$$

Consistency with wall-law

- Summary of boundary layer analysis

	Consistency with log-law at APG	Consistency with sqrt-law at APG
Modified k-equation	Yes	Yes
Modified ω-equation	Yes	Yes

$$-D_{k,t}^{+} - D_{k,p}^{+} = P_k^{+} - \epsilon_k^{+}$$

$$-D_{\omega,t}^{+} - D_{\omega,p}^{+} = P_\omega^{+} - \epsilon_\omega^{+} + D_{cd,\omega}^{+}$$

Consistency with wall-law

- Summary of boundary layer analysis

	Blending active in log-law region at APG	Blending active in sqrt-law region at APG
Modified k-equation	Yes	Yes
Modified ω-equation	No	Yes

$$-D^+_{k,t} - \boxed{D^+_{k,p}} = P^+_k - \epsilon^+_k$$

$$-D^+_{\omega,t} - \boxed{D^+_{\omega,p}} = P^+_\omega - \epsilon^+_\omega \boxed{+ D^+_{cd,\omega}}$$

DLR

The ω-equation in the sqrt-law region

Following ideas by Rao & Hassan, Catris & Aupoix

$$-\frac{\mathrm{d}}{\mathrm{d}y^+}\left(\sigma_\omega \nu_t^+ \frac{\mathrm{d}\omega^+}{\mathrm{d}y^+}\right) = \gamma \left(\frac{\mathrm{d}u^+}{\mathrm{d}y^+}\right)^2 - \beta_\omega \left(\omega^+\right)^2$$

$$\Leftrightarrow \qquad -D_{\omega,t}^+ \quad = \quad P_\omega^+ \quad - \quad \epsilon_\omega^+$$

The ω-equation in the sqrt-law region

Following ideas by Rao & Hassan, Catris & Aupoix

$$\boxed{\kappa y^+ \sqrt{1+\alpha y^+}} \qquad \boxed{\frac{\sqrt{1+\alpha y^+}}{\kappa y^+}} \qquad \boxed{\frac{\sqrt{1+\alpha y^+}}{a_1 \kappa y^+}}$$

$$-\frac{\mathrm{d}}{\mathrm{d}y^+}\left(\sigma_\omega \nu_t^+ \frac{\mathrm{d}\omega^+}{\mathrm{d}y^+}\right) = \gamma \left(\frac{\mathrm{d}u^+}{\mathrm{d}y^+}\right)^2 - \beta_\omega \left(\omega^+\right)^2$$

$$\Leftrightarrow \qquad -D^+_{\omega,t} \quad = \quad P^+_\omega \quad - \quad \epsilon^+_\omega$$

The ω-equation in the sqrt-law region

$$\boxed{\kappa y^+ \sqrt{1+\alpha y^+}} \qquad \boxed{\frac{\sqrt{1+\alpha y^+}}{\kappa y^+}} \qquad \boxed{\frac{\sqrt{1+\alpha y^+}}{a_1 \kappa y^+}}$$

$$-\frac{\mathrm{d}}{\mathrm{d}y^+}\left(\sigma_\omega \nu_t^+ \frac{\mathrm{d}\omega^+}{\mathrm{d}y^+}\right) = \gamma\left(\frac{\mathrm{d}u^+}{\mathrm{d}y^+}\right)^2 - \beta_\omega \left(\omega^+\right)^2$$

$$\Leftrightarrow \qquad -D^+_{\omega,t} \quad = \quad P^+_\omega \quad - \quad \epsilon^+_\omega$$

$$\Leftrightarrow \qquad -\frac{\sigma_\omega}{a_1}\frac{1}{(y^+)^2} \neq \frac{1}{\kappa^2}\left(\gamma - \frac{\beta_\omega}{a_1^2}\right)\frac{1+\alpha y^+}{(y^+)^2}$$

The ω-equation in the sqrt-law region

$$-\frac{\mathrm{d}}{\mathrm{d}y^+}\left(\sigma_\omega \overset{\boxed{\kappa y^+\sqrt{1+\alpha y^+}}}{\nu_t^+}\frac{\mathrm{d}\omega^+}{\mathrm{d}y^+}\right) = \gamma\left(\overset{\boxed{\frac{\sqrt{1+\alpha y^+}}{\kappa y^+}}}{\frac{\mathrm{d}u^+}{\mathrm{d}y^+}}\right)^2 - \beta_\omega\left(\overset{\boxed{\frac{\sqrt{1+\alpha y^+}}{a_1\kappa y^+}}}{\omega^+}\right)^2$$

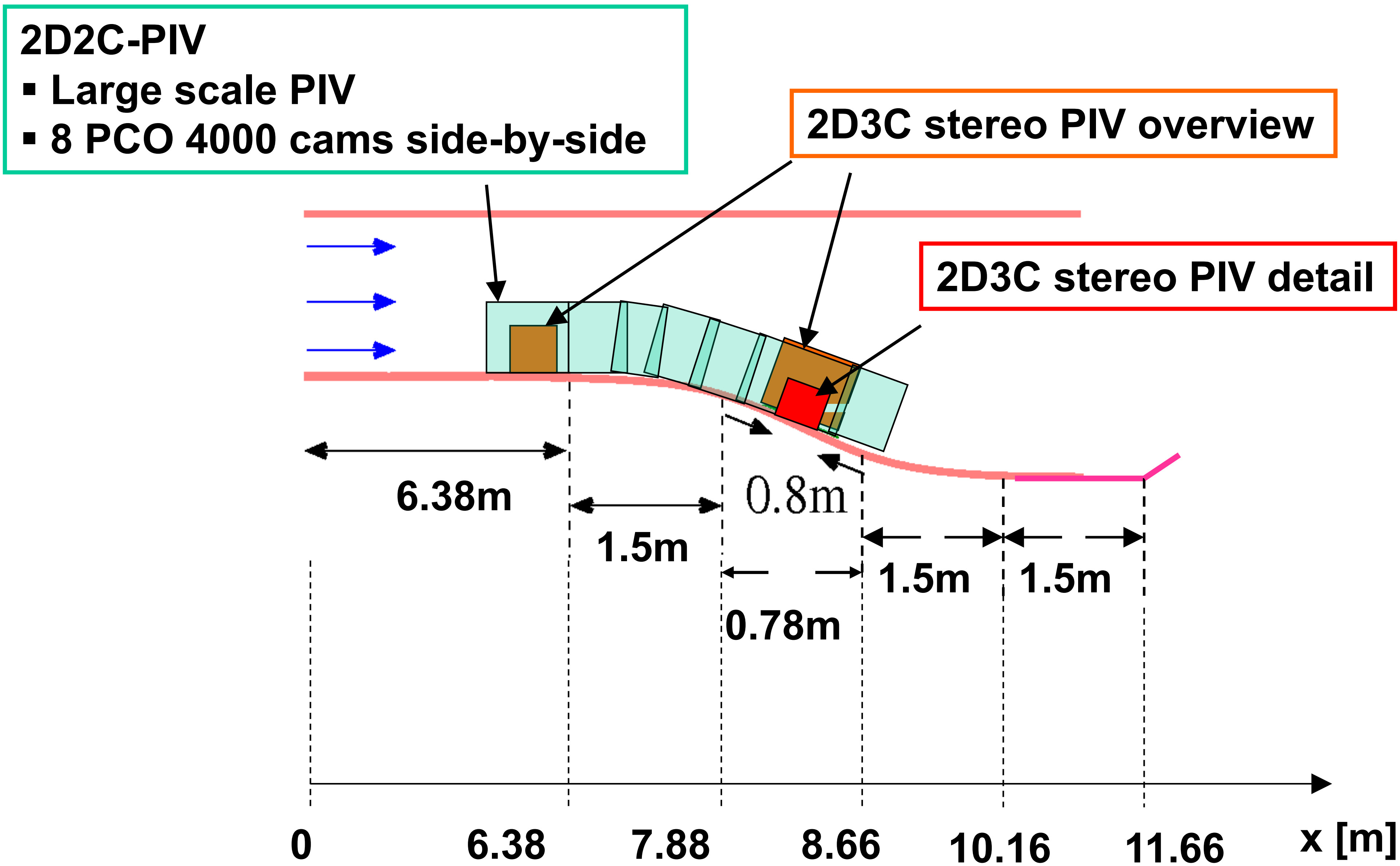
$$\Leftrightarrow \qquad -D^+_{\omega,t} \quad = \quad P^+_\omega \quad - \quad \epsilon^+_\omega$$

$$\Leftrightarrow \qquad -\frac{\sigma_\omega}{a_1}\frac{1}{(y^+)^2} \neq \frac{1}{\kappa^2}\left(\gamma - \frac{\beta_\omega}{a_1^2}\right)\frac{1+\alpha y^+}{(y^+)^2}$$

Conclusion: The ω-equation is **not consistent** with the assumed solution in the sqrt-law region at APG

DLR

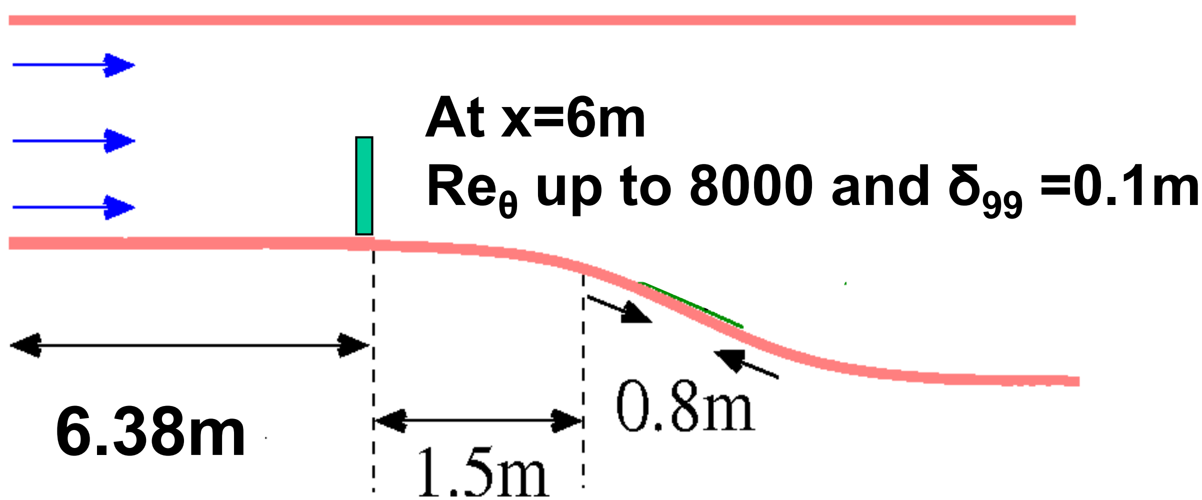
Measurement technique combining different PIV systems



Requirements/Design criteria on a new flow experiment

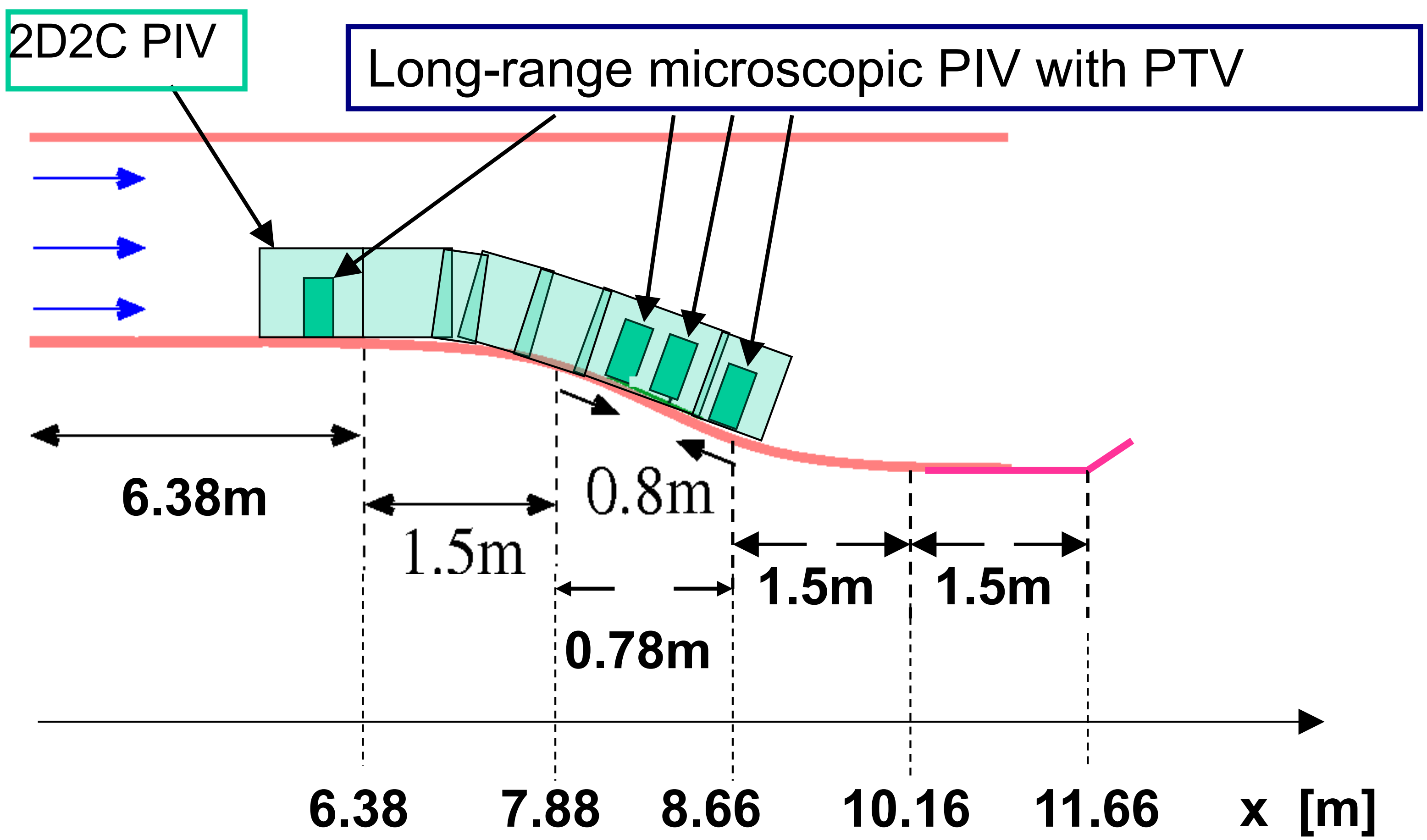
1. Large Re_θ for sufficiently thick overlap region
2. Log-law established before entering the APG-section
3. Thick BL at low flow velocity so that PIV in viscous sublayer possible
4. Obtain large values for Δp_x^+ , i.e. $\Delta p_x^+ > 0.06$

U_∞= 9m/s and
U_∞= 12m/s



Design of the test case using RANS-CFD with the DLR TAU code by DLR

Data for mean velocity down to y^+=1 using LR-PIV with PTV



Particle-tracking velocimetry algorithm (PTV) applied to the LR-PIV data see also Kähler et al., Exp. Fluids (2012)

Boundary layer theory.

An approximative model for the shear stress

Turbulent boundary layer equation

- Consider the equation for wall-parallel mean velocity component U

$$\nu\frac{\partial^2 U}{\partial y^2} - \frac{\partial \overline{u'v'}}{\partial y} = \frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}x} + U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} - \cancel{\nu\frac{\partial^2 U}{\partial x^2}} + \cancel{\frac{\partial \overline{u'u'}}{\partial x}}$$

- Integration in wall-normal direction from y‘=0 to wall-distance y

$$\frac{\tau}{\rho} = \nu\frac{\partial U}{\partial y} - \overline{u'v'} = \frac{\tau_w}{\rho} + \frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}x}y + \int_{y'=0}^{y} U\frac{\partial U}{\partial x}\mathrm{d}y' + \int_{y'=0}^{y} V\frac{\partial U}{\partial y'}\mathrm{d}y'$$

- Or in viscous inner units

$$\tau^+ = \frac{\partial U^+}{\partial y^+} - \overline{u'v'}^+ = 1 + \Delta p_x^+ y^+ + I_{cu}^+(y^+) + I_{cv}^+(y^+)$$

Review: Modeling the total shear stress (van den Berg 1973)

- Aim: Relate the total shear stress and the mean velocity profile for U

$$\tau^+ = \frac{\partial U^+}{\partial y^+} - \overline{u'v'}^+ = 1 + \Delta p_x^+ y^+ + I_{cu}^+(y^+) + I_{cv}^+(y^+)$$

- by approximating the integrated convective (or: mean inertial) terms

$$I_{cu}^+(y^+) = \frac{1}{u_\tau^2}\int_{y'=0}^{y} U\frac{\partial U}{\partial x}\mathrm{d}y' \,, \quad I_{cv}^+(y^+) = \frac{1}{u_\tau^2}\int_{y'=0}^{y} V\frac{\partial U}{\partial y'}\mathrm{d}y'$$

- Van den Berg makes the modeling assumption that

$$U(x,y) = u_\tau(x)\, f(y^+(x,y)) \,, \quad y^+(x,y) = \frac{u_\tau(x) y}{\nu}$$

DLR

C²A²S²E

Review: Modeling the total shear stress (van den Berg 1973)

- Modeling assumption $$U(x,y) = u_\tau(x)\, f(y^+(x,y)) \;, \quad y^+(x,y) = \frac{u_\tau(x) y}{\nu}$$
- Then

$$\frac{\partial U}{\partial x} = \frac{\mathrm{d}u_\tau}{\mathrm{d}x}\left[f + y^+ \frac{\mathrm{d}f}{\mathrm{d}y^+} \right]$$

- For the V-velocity and using the continuity equation

$$V = -\int_0^y \frac{\partial U}{\partial x}\mathrm{d}y = -\frac{\mathrm{d}u_\tau}{\mathrm{d}x}\int_0^y \left[f + y\frac{\mathrm{d}f}{\mathrm{d}y}\right]\mathrm{d}y = -\frac{\mathrm{d}u_\tau}{\mathrm{d}x}\int_0^y \frac{\mathrm{d}}{\mathrm{d}y}[yf]\,\mathrm{d}y = -\frac{\mathrm{d}u_\tau}{\mathrm{d}x} y f$$

- This gives for the mean inertial term

$$\int_0^y U\frac{\partial U}{\partial x}\mathrm{d}y + \int_0^y V\frac{\partial U}{\partial y}\mathrm{d}y = \nu\frac{\mathrm{d}u_\tau}{\mathrm{d}x}\int_0^{y^+} f^2\,\mathrm{d}y^+$$

Review: Modeling the total shear stress (van den Berg 1973)

- This gives the model by van den Berg (1973)

$$\tau^+ = 1 + \Delta p_x^+ y^+ + \frac{\nu}{u_\tau^2}\frac{\mathrm{d}u_\tau}{\mathrm{d}x}\int_0^{y^+} f^2 \mathrm{d}y^+$$

- Short notation:

$$\boxed{\tau^+ = 1 + \alpha y^+ + \beta I_1}$$

$$I_1 = \int_0^{y^+} f^2 \mathrm{d}y^+$$

- With
  - Pressure gradient parameter $\alpha \equiv \Delta p_x^+ = \frac{\nu}{\rho u_\tau^3}\frac{\mathrm{d}P}{\mathrm{d}x}$
  - Wall shear-stress gradient parameter $\beta = \frac{\nu}{u_\tau^2}\frac{\mathrm{d}u_\tau}{\mathrm{d}x}$

Extended modeling for the total shear stress

- Modeling assumption $U(x,y) = u_\tau(x)\, f(y^+(x,y), \alpha(x))$
- For the chain rule of differentiation we need

$$\frac{\partial y^+}{\partial y} = \frac{u_\tau}{\nu}\,, \quad \frac{\partial y^+}{\partial x} = \frac{y}{\nu}\frac{\mathrm{d}u_\tau}{\mathrm{d}x}\,, \quad \frac{\mathrm{d}\alpha}{\mathrm{d}x} = -\frac{3\nu}{\rho u_\tau^4}\frac{\mathrm{d}u_\tau}{\mathrm{d}x}\frac{\mathrm{d}P}{\mathrm{d}x} + \frac{\nu}{\rho u_\tau^3}\frac{\mathrm{d}^2 P}{\mathrm{d}x^2}$$

- Then we obtain, e.g.

$$\frac{\partial U}{\partial x} = \frac{u_\tau^2}{\nu}\beta\left[f + y^+\frac{\partial f}{\partial y^+}\right] + \frac{u_\tau^2}{\nu}(-3\alpha\beta + \alpha^*)\frac{\partial f}{\partial \alpha}$$

  - With an additional parameter taking into account d^2P/dx^2

$$\alpha = \frac{\nu}{\rho u_\tau^3}\frac{\mathrm{d}P}{\mathrm{d}x}\,, \quad \beta = \frac{\nu}{u_\tau^2}\frac{\mathrm{d}u_\tau}{\mathrm{d}x}\,, \quad \alpha^* = \frac{\nu}{u_\tau}\frac{\nu}{\rho u_\tau^3}\frac{\mathrm{d}^2 P}{\mathrm{d}x^2}\,, \quad \gamma = -3\alpha\beta + \alpha^*$$

Extended modeling for the total shear stress

- This gives the extended model

$$\boxed{\tau^+ = 1 + \alpha y^+ + \beta I_1 + \gamma I_2}$$

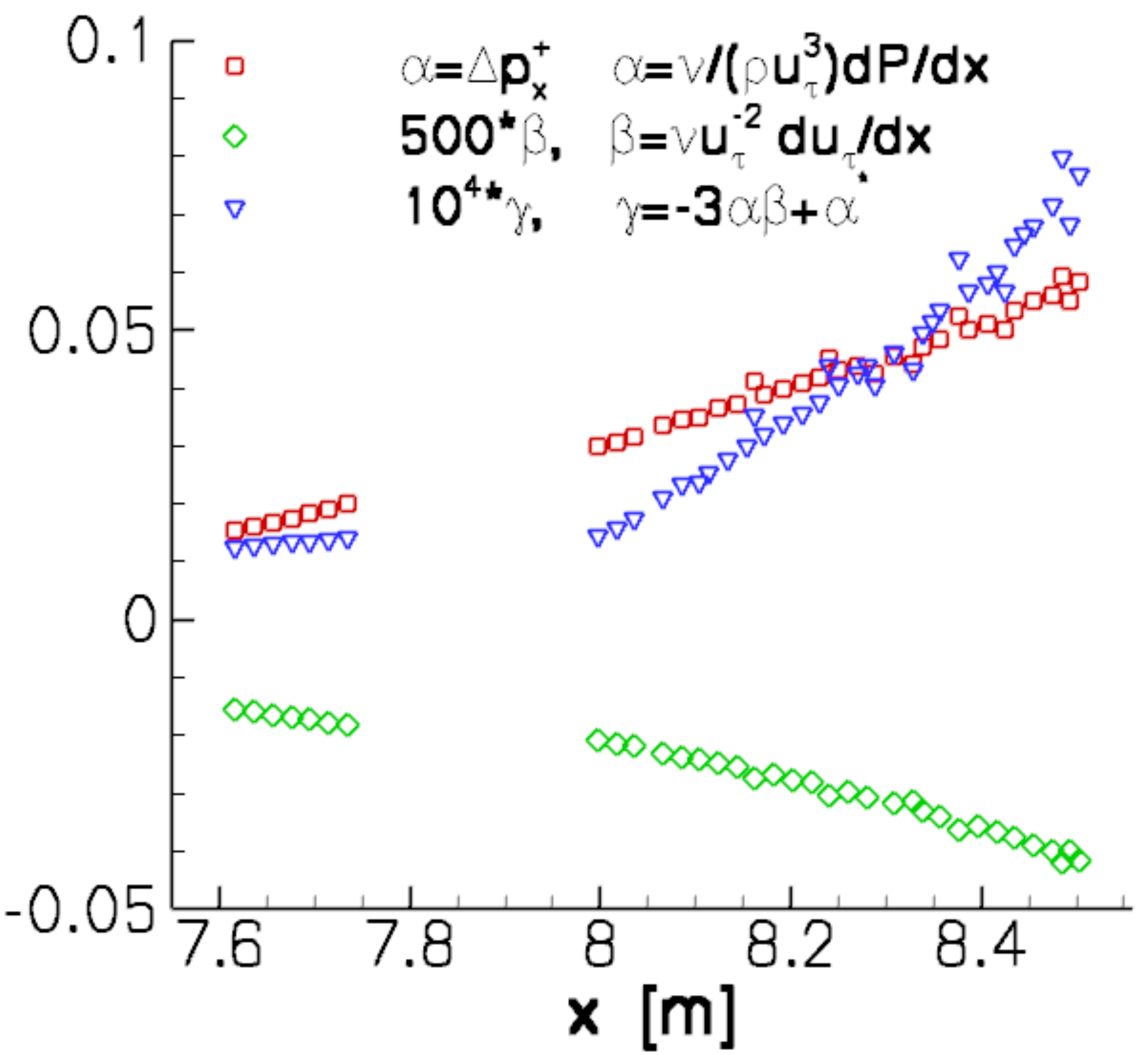
- With integrals

$$I_1 = \int_0^{y^+} f^2 \mathrm{d}y^+ \qquad I_2 = 2\int_0^{y^+} f \frac{\partial f}{\partial \alpha} \mathrm{d}y^+ - f \int_0^{y^+} \frac{\partial f}{\partial \alpha} \mathrm{d}y^+$$

- and with parameters
  - Pressure gradient parameter $$\alpha \equiv \Delta p_x^+ = \frac{\nu}{\rho u_\tau^3} \frac{\mathrm{d}P}{\mathrm{d}x}$$
  - Wall shear-stress gradient parameter $$\beta = \frac{\nu}{u_\tau^2} \frac{\mathrm{d}u_\tau}{\mathrm{d}x}$$
  - Cross parameter $$\gamma = -3\alpha\beta + \alpha^*$$
  - d²P/dx² -parameter $$\alpha^* = \frac{\nu}{u_\tau} \frac{\nu}{\rho u_\tau^3} \frac{\mathrm{d}^2 P}{\mathrm{d}x^2}$$

Characteristic non-dimensional parameters

- Comparison of the characteristic non-dimensional parameters



$$\gamma = -3\alpha\beta + \alpha^*$$

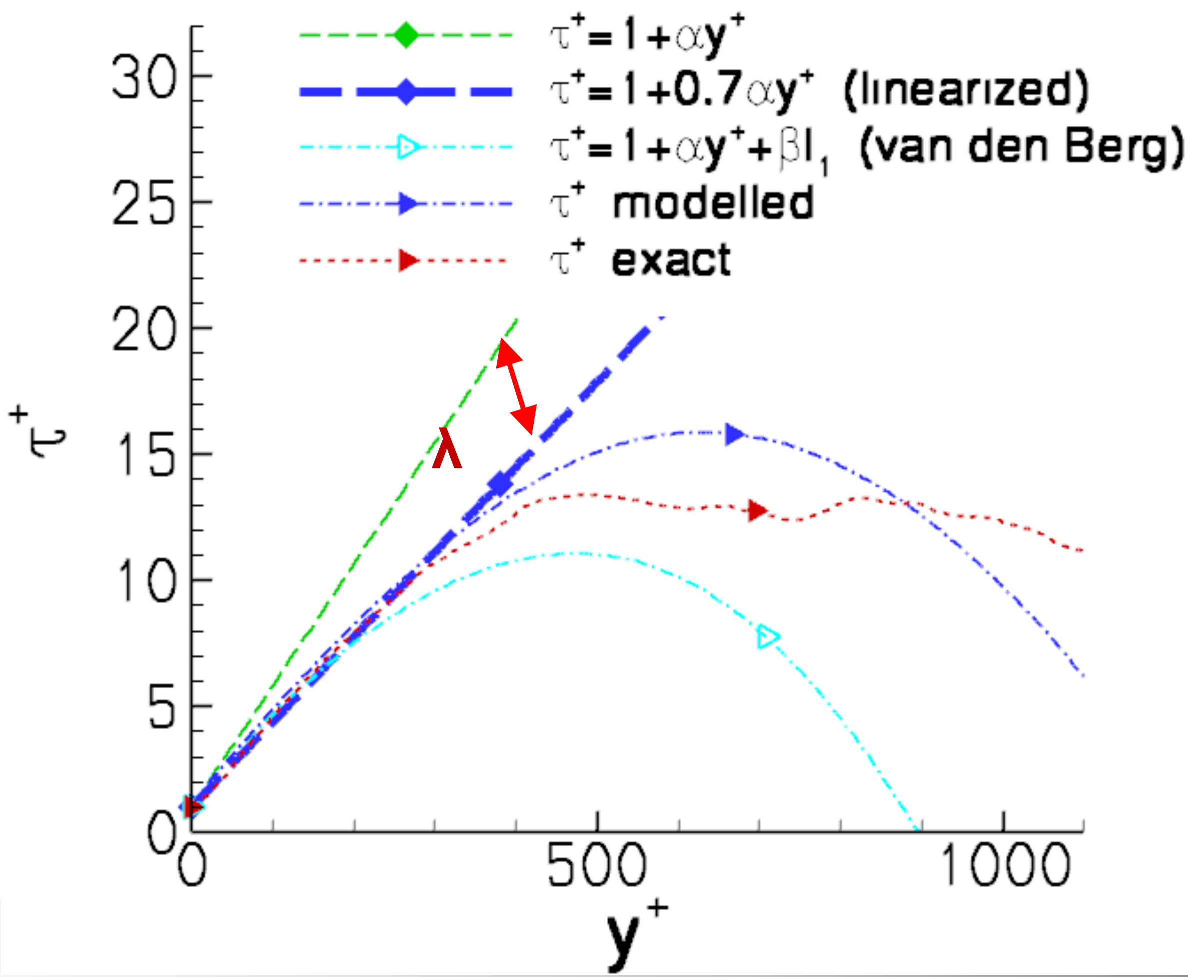
$$\alpha^* = \frac{\nu}{u_\tau} \frac{\nu}{\rho u_\tau^3} \frac{\mathrm{d}^2 P}{\mathrm{d}x^2}$$

$$\alpha \equiv \Delta p_x^+ = \frac{\nu}{\rho u_\tau^3} \frac{\mathrm{d}P}{\mathrm{d}x}$$

$$\beta = \frac{\nu}{u_\tau^2} \frac{\mathrm{d}u_\tau}{\mathrm{d}x}$$

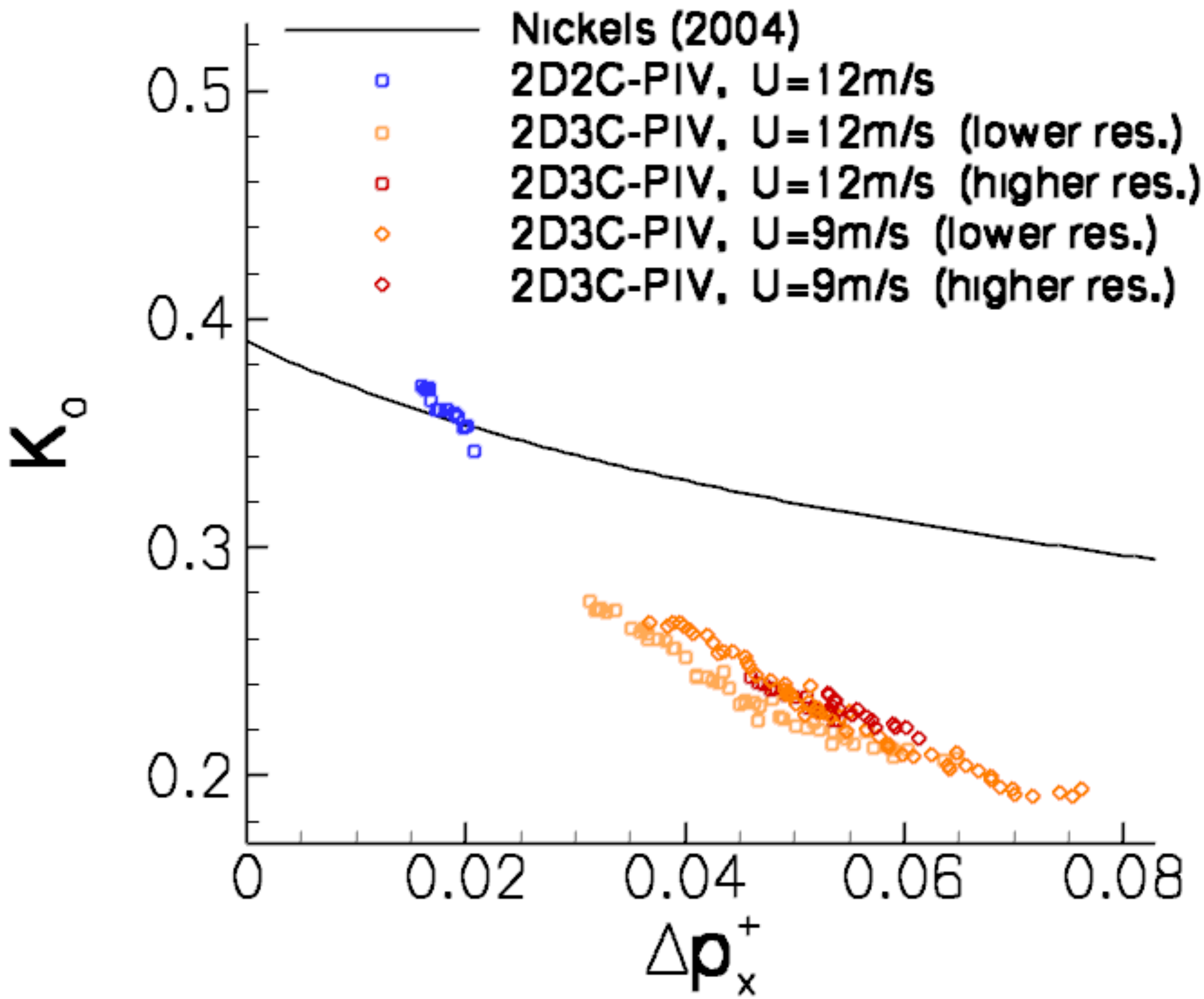
Turbulent shear stress in non-equilibrium flow

- Mean inertial terms cause a significant reduction of the turbulent shear stress $\tau^+=1+\Delta p_x^+ y^+$



Present exp.
at x=8.18m
Δp_x^+=0.0473

Systematic trend/quantitative formula for variation of K_o



Conclusion

Conclusion.

- Design of a flow experiment suitable to study the law-of-the-wall for flows at a substantial adverse pressure gradient
  - Sufficiently large overlap-layer due to large Re_θ
  - Significant adverse pressure gradient Δp_x^+ up to $\Delta p_x^+ = 0.065$

- Combination of different PIV techniques gives high-quality data set
  - Large number of velocity profiles
  - High quality gradients and slope diagnostic functions
  - Comparison of direct and indirect method for u_τ locally

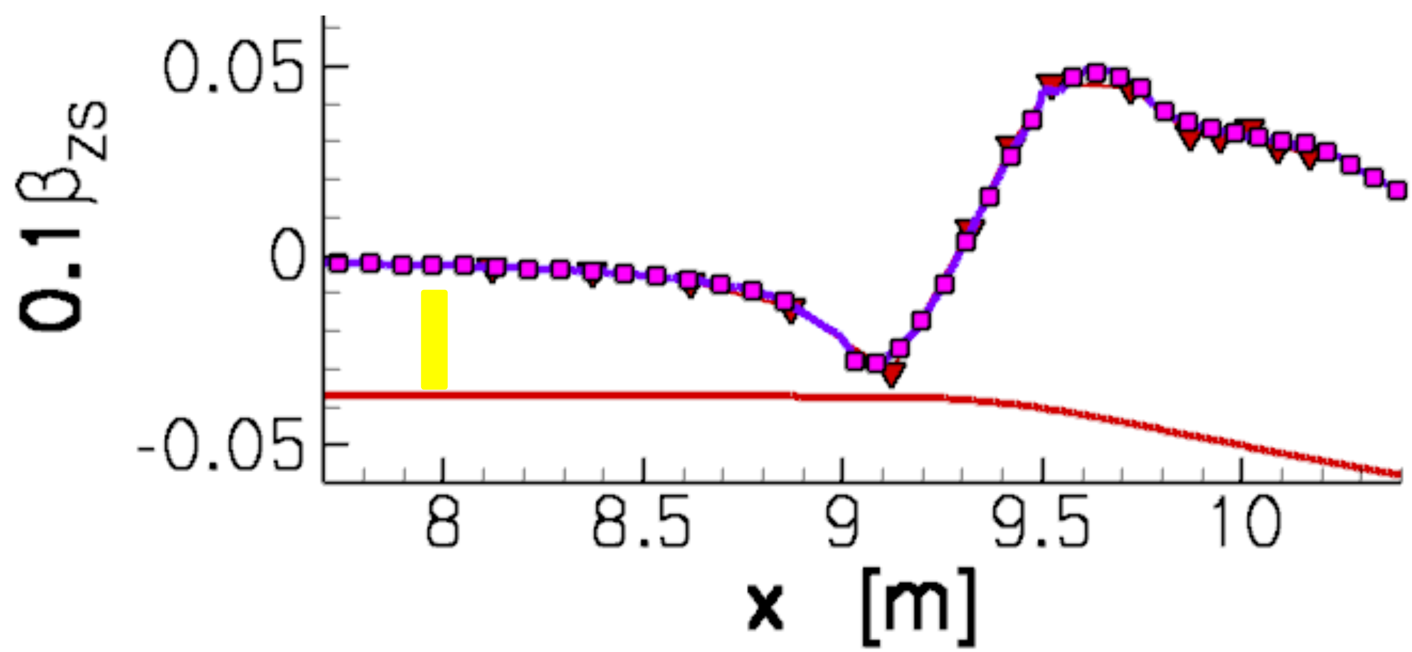
Conclusion.

- The log-law does no longer describe the overlap layer $300 < y^+ < 0.2\ \delta_{99}^+$
- A generalized half-power law (or modified log-law) gives a good description

- Idea of a composite wall-law by Brown & Joubert (1969) is supported
  - Log-law-fit region is thin $60 < y^+ < 130$
    - 1/slope K_i decreasing with increasing Δp_x^+ as devised by Nickels (2004)
  - Modified log-law region for $300\text{-}400 < y^+ < 0.2 - 0.4\ \delta_{99}^+$
    - 1/slope K_o decreasing with increasing Δp_x^+
      - But mean inertial terms need to be accounted for K_o

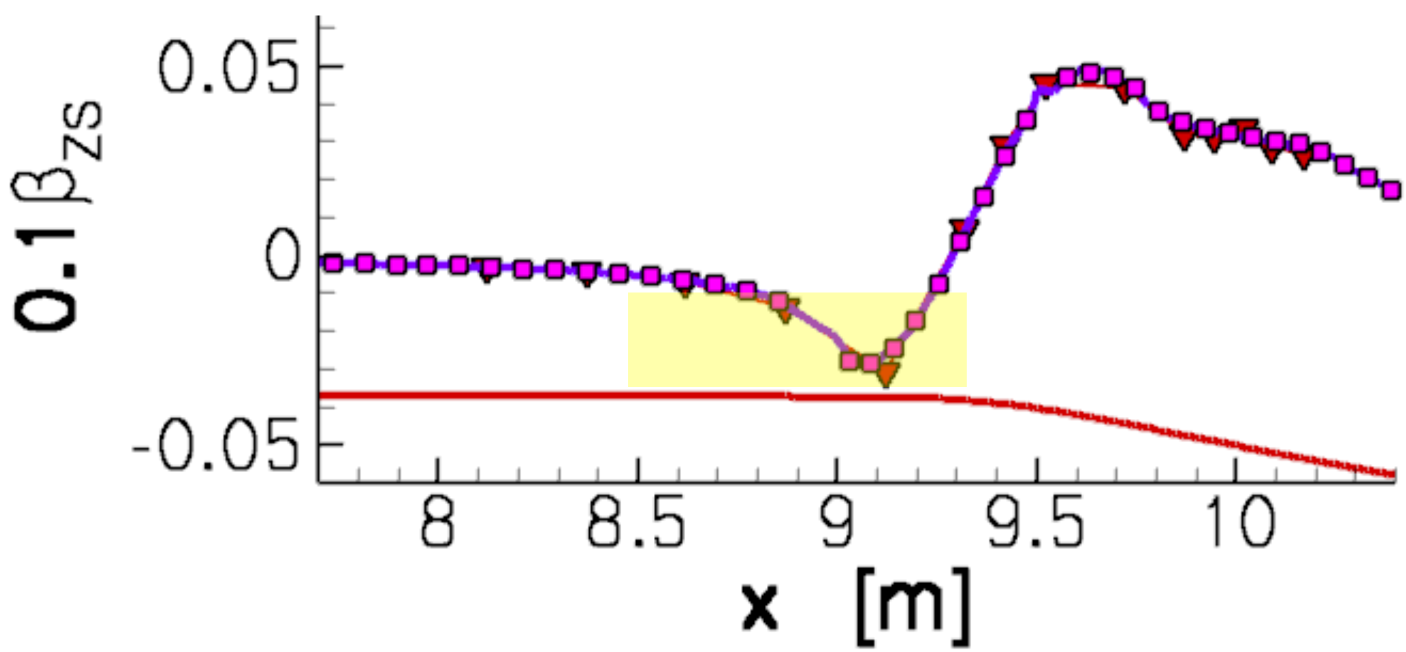
Characterization of the flow

ZPG
log-law



Characterization of the flow

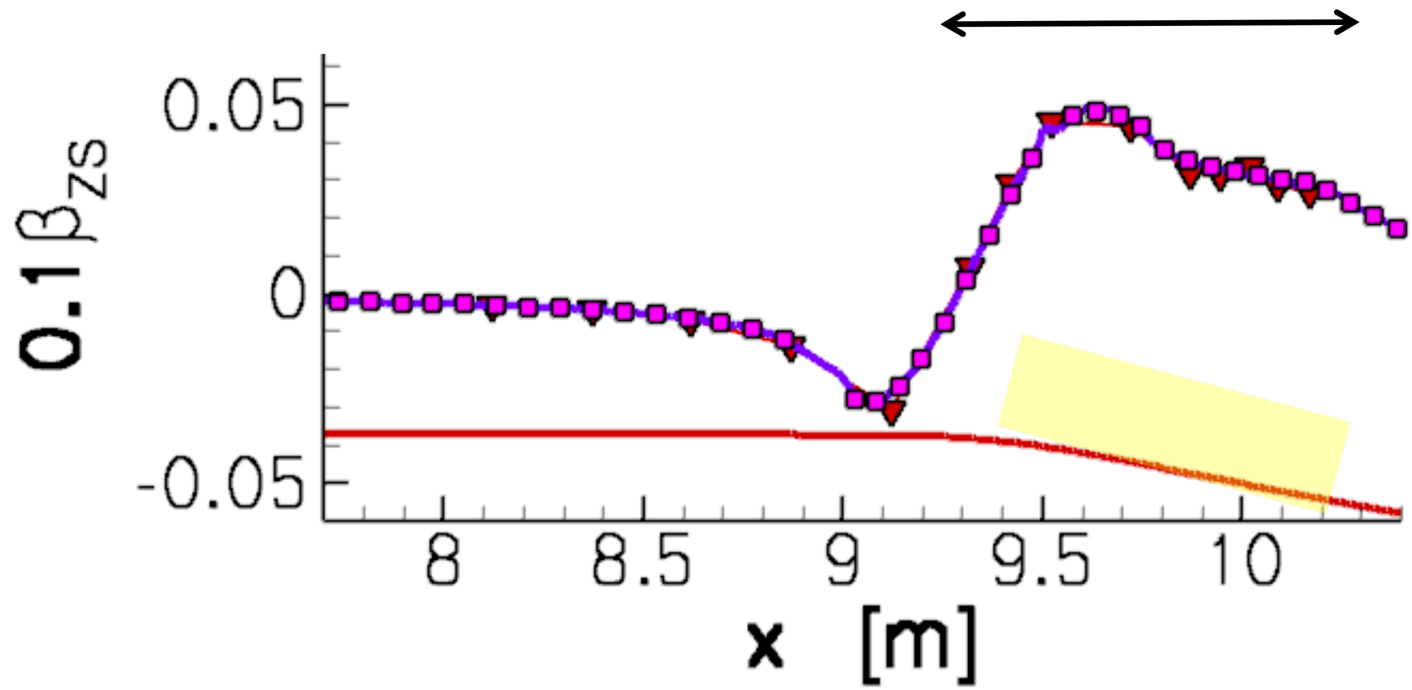
Favourable dp/ds
L_{FPG} ~ 5 δ_{ref}



Characterization of the flow

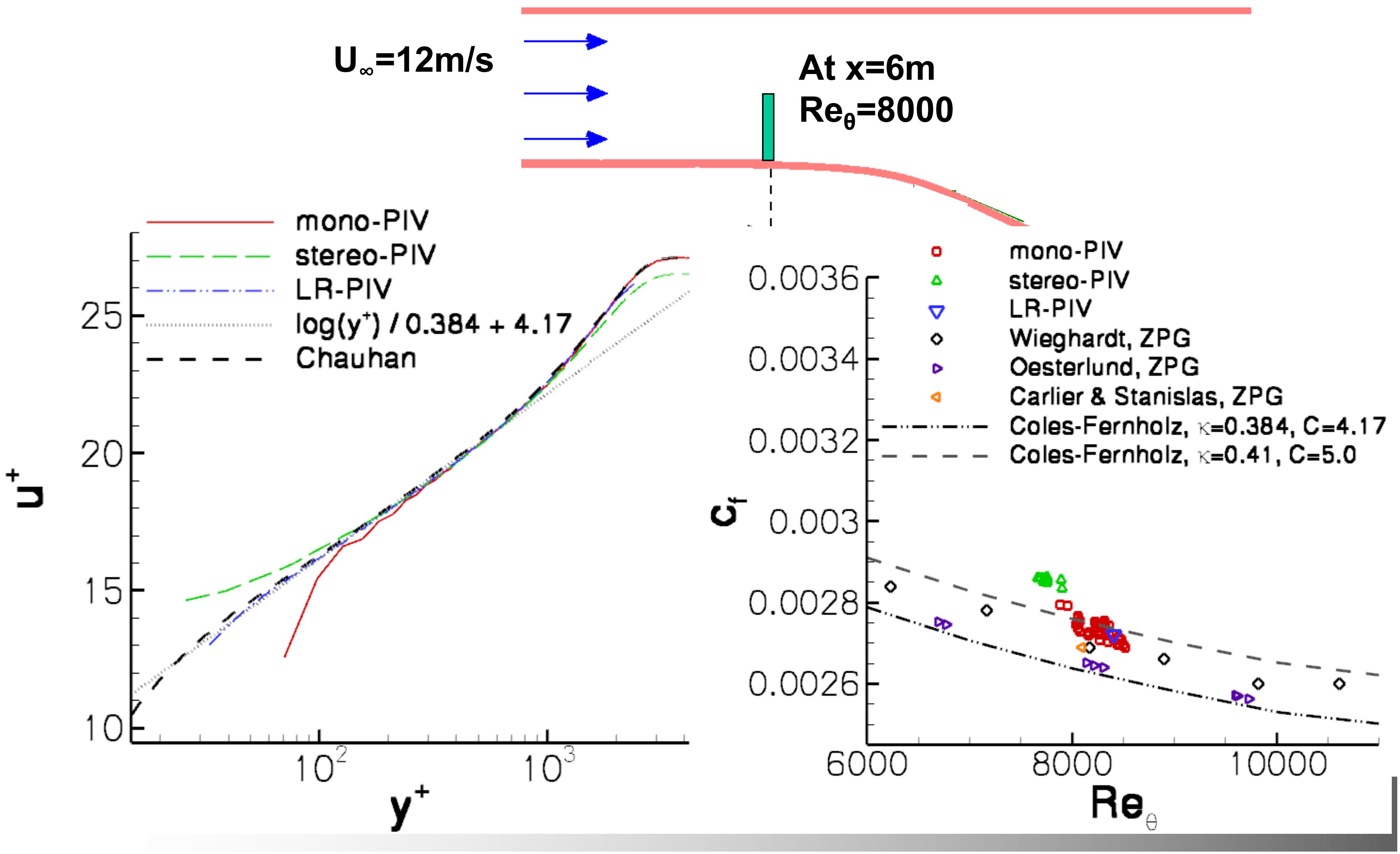
Focus region:

Adverse dp/ds
L_{APG} ~ 5 δ_{ref}

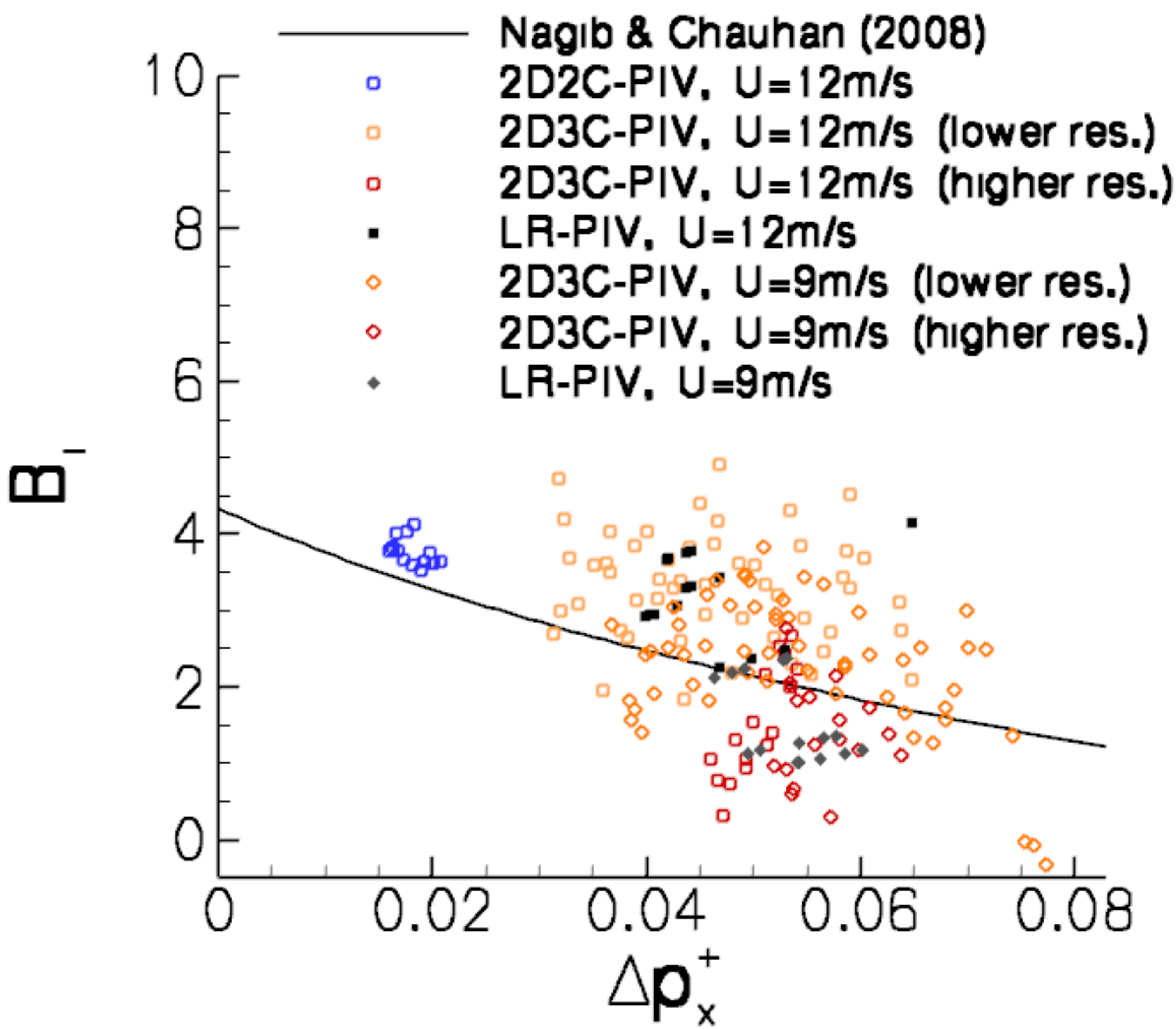


Universal log-law in zero-pressure gradient region

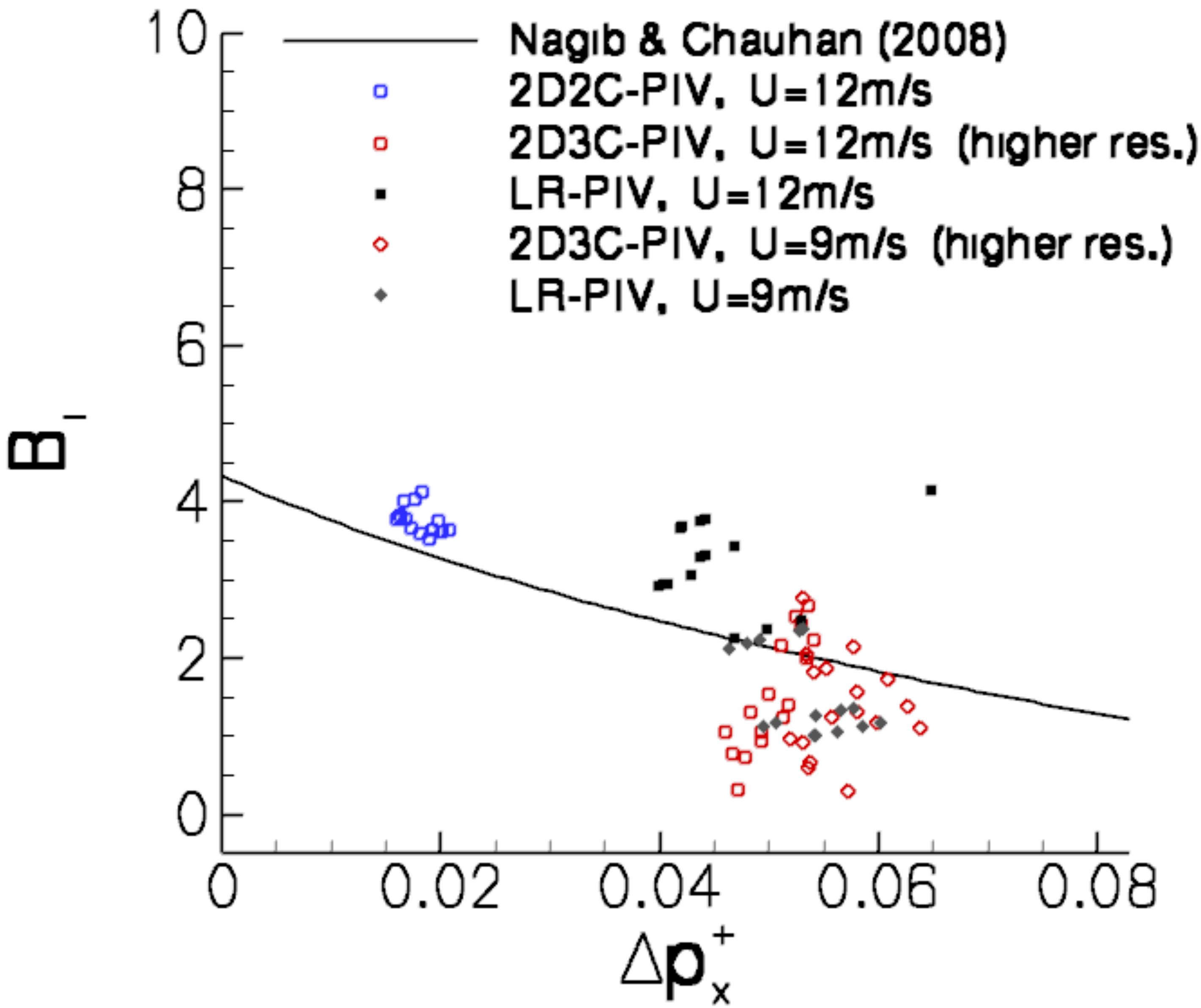
- Universal log-law at the inlet of the adverse pressure gradient section



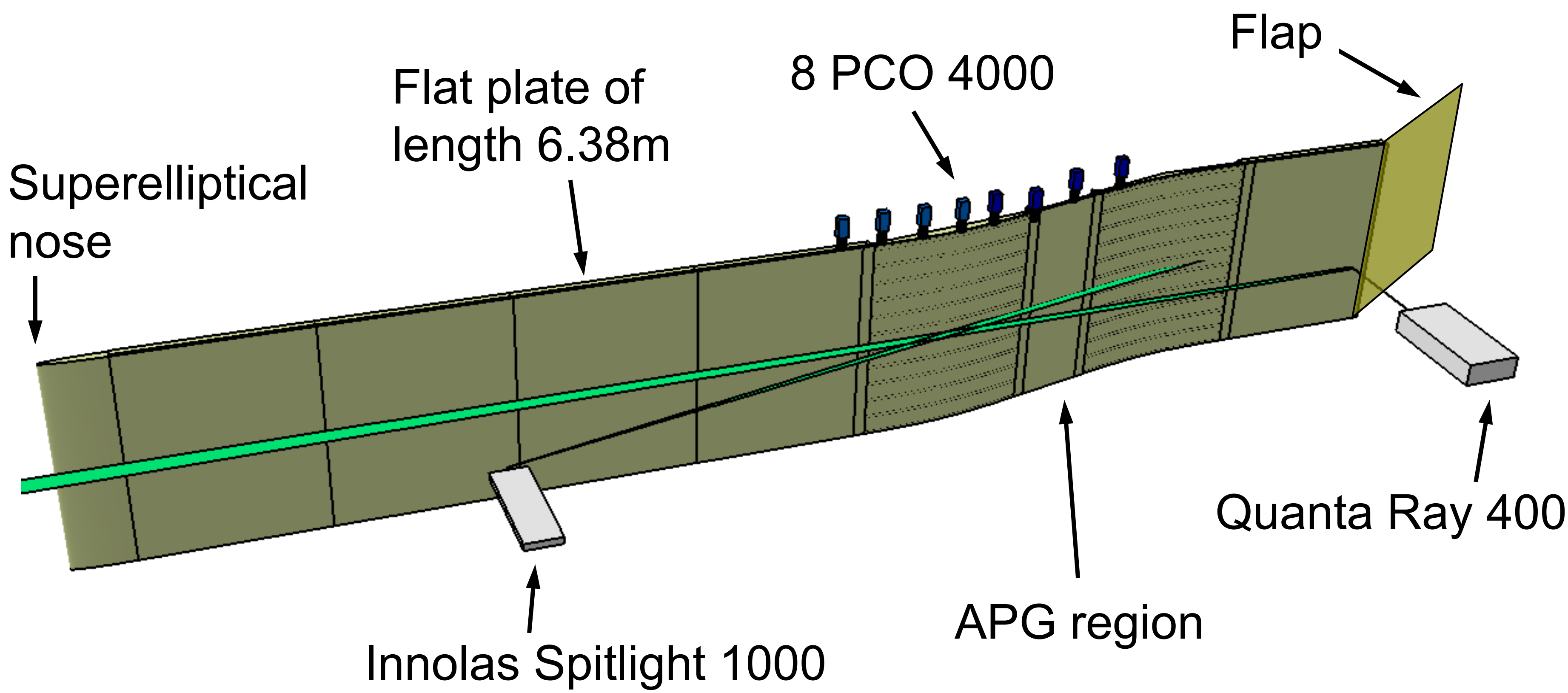
Systematic trend/quantitative formula for variation of B_i



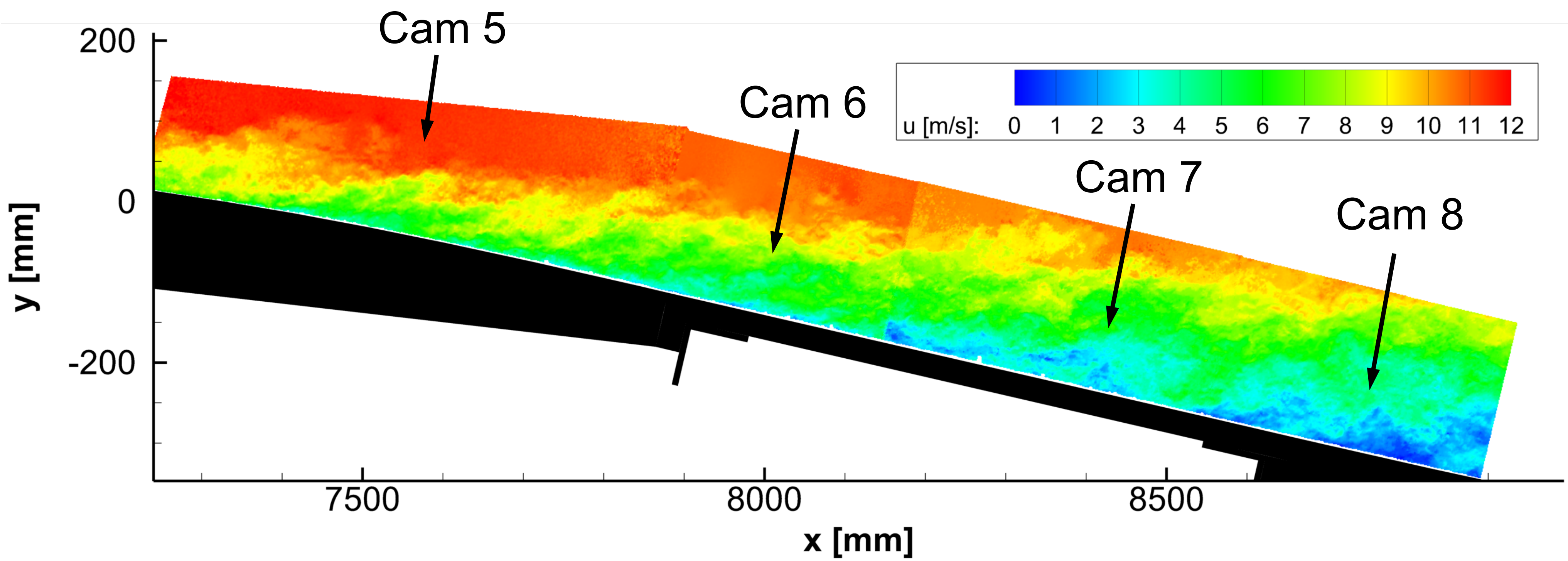
Systematic trend/quantitative formula for variation of B_i



Measurement technique using PIV



Large-scale 2D2C PIV. Instantaneous flow field



DLR

C²A²S²E

Motivation: Digital aerospace products

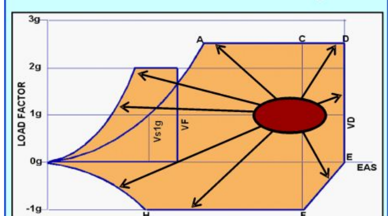
Numerical simulation based future aircraft design

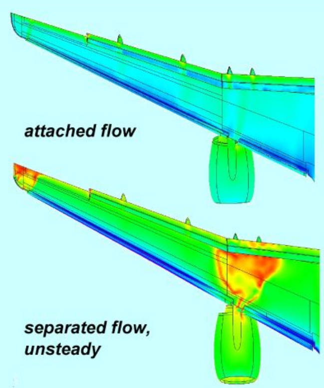
Challenges

- **A/δ_{99} extremely large: large Re and large A**
- **#simulations large for design and optimization**

=> RANS, not LES

Full flight envelope coverage: *CFD mostly done near cruise point*





configurations:

clean

airbrakes deployed

high lift

50	flight points
100	mass cases
10	a/c configurations
5	maneuvers
20	gusts (gradient lengths)
4	control laws

~ 20,000,000 simulations

Engineering experience for **current** configurations and technologies

~ 100,000 simulations

Motivation: Turbulent boundary layers at adverse pressure gradient

During take-off and landing
Significant adverse pressure gradient (APG)

Goals:
⇒ Wall-laws at APG
⇒ Improvement RANS models