One-point PDF closure model applied to attached and separated flows

Michael Stoellinger
University of Wyoming, Department of Mechanical Engineering

Advances in Turbulence Modeling
11-13 July 2017, University of Michigan

UNIVERSITY OF WYOMING

Overview

1. Background on PDF modeling
2. Solution method
3. Preliminary results for a plane jet
4. Conclusions and Outlook

Basic idea of PDF modeling

RANS modeling

- Start with exact NS-equations
- Introduce Reynolds averaging operator $\langle \cdot \rangle$
- Derive moment equations $\langle U_i \rangle, \langle u_i u_j \rangle, \ldots$
- Introduce models to "close" at a given moment level $\frac{D\langle u_i u_j \rangle}{Dt} = F\left(\langle U_i \rangle, \langle u_i u_j \rangle, \varepsilon\right)$

PDF modeling

- Define one-point velocity probability density function $f_e(\boldsymbol{V}; \boldsymbol{x}, t)$
- Start from exact NS-equations and derive exact (but unclosed) evolution equation for the PDF $\frac{Df_e}{Dt} = RHS$
- Introduce a closure model for the PDF equation and a time-scale model
- **All moments are closed** i.e. $\langle u_i u_j u_k \rangle$ is also closed
- Solving the closed PDF equation (7-dimensional) is not trivial!

Motivation for PDF modeling approach

- In turbulent reacting flows the chemical source terms and turbulent fluxes are treated exactly if $f(\boldsymbol{V}, \boldsymbol{\psi}; \boldsymbol{x}, t)$ is considered
- The triple correlations $\langle u_i u_j u_k \rangle$ are closed -> possibly improved results in flows where turbulent transport is important (i.e. swirling jets)
- If we can optimize the closure model, we have directly obtained the best possible RANS model!
- New physical insights could be gained by considering Lagrangian statistics
- Could be taken to the two-point level -> Joint velocity – gradient PDF

Main disadvantage: Transported PDF solution is typically 5-20 times more expensive!

Basic idea of PDF modeling

Incompressible NS – equations (constant properties) $\frac{\partial U_i}{\partial x_i} = 0$

$$\frac{DU_i}{Dt} = \frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = A_i = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \Delta^2 U_i$$

Exact PDF evolution equation (derivation see e.g. Pope, "Turbulent Flows")

$$\frac{\partial f_e}{\partial t} + V_i \frac{\partial f_e}{\partial x_i} = -\frac{\partial}{\partial V_i}[f_e \langle A_i | \boldsymbol{V} \rangle] = -\frac{\partial}{\partial V_i}\left[f_e \left(\underbrace{-\frac{1}{\rho}\frac{\partial \langle p \rangle}{\partial x_i} + \nu \Delta^2 \langle U_i \rangle}_{M_i} \right)\right] - \frac{\partial}{\partial V_i}\left[f_e \left\langle -\frac{1}{\rho}\frac{\partial p'}{\partial x_i} \middle| \boldsymbol{V} \right\rangle + f_e \langle \nu \Delta^2 u_i | \boldsymbol{V} \rangle \right]$$

Modeled PDF evolution equation

$$\frac{\partial f}{\partial t} + V_i \frac{\partial f}{\partial x_i} = -\frac{\partial}{\partial V_i}[f M_i] - \frac{\partial}{\partial V_i}\left[f G_{ij} \left(V_j - \langle U_j \rangle\right)\right] + \frac{1}{2} C_0 \varepsilon \frac{\partial^2 f}{\partial V_i \partial V_i}$$

Basic idea of PDF modeling

Modeled PDF evolution equation

$$\frac{\partial f}{\partial t} + V_i \frac{\partial f}{\partial x_i} = -\frac{\partial}{\partial V_i}[f M_i] - \frac{\partial}{\partial V_i}\left[f G_{ij}\left(V_j - \langle U_j \rangle\right)\right] + \frac{1}{2} C_0 \varepsilon \frac{\partial^2 f}{\partial V_i \partial V_i}$$

Can we “measure” the tensor G_{ij}?

1) Pope, JFM 2014
   - With moderate assumptions one finds $G_{ij} \sim \langle a_i u_k \rangle \langle u_k u_i \rangle^{-1}$
   - The fluctuating velocity-acceleration correlation $\langle a_i u_k \rangle$ arises from large scale turbulent motions -> could be measured or taken from well resolved LES
2) Work done by Stefan Heinz & Grigory Sarnitsky at UWYO
   - Find G_{ij} from Lagrangian DNS data using

$$\lim_{\tau \to 0} \frac{1}{\tau} \left\langle \left[U_i'(t+\tau) - U_i'(t)\right] U_j'(t) \,\middle|\, \mathbf{X}(t) = \mathbf{x} \right\rangle = \widetilde{G}_{ik}(\mathbf{x}, t) \, \langle u_k' u_j' \rangle (\mathbf{x}, t)$$

Basic idea of PDF modeling

Basic PDF closure model: Simplified Langevin model (SLM)

$$\frac{\partial f}{\partial t} + V_i \frac{\partial f}{\partial x_i} = -\frac{\partial}{\partial V_i}[f M_i] - \frac{\partial}{\partial V_i}[f G_{ij}(V_j - \langle U_j \rangle)] + \frac{1}{2} C_0 \varepsilon \frac{\partial^2 f}{\partial V_i \partial V_i}$$

$$G_{ij} = -\left(\frac{1}{2} + \frac{3}{4} C_0\right) \frac{\varepsilon}{k} \delta_{ij}$$

⇨ Corresponds to Rotta's return to isotropy model in RSM!

Numerical Solution

- PDF equation is 7-dimensional
- Important constraints need to be satisfied numerically

$f(\boldsymbol{V}; \boldsymbol{x}, t) > 0$, and $\int f_e(\boldsymbol{V}; \boldsymbol{x}, t)\, d\boldsymbol{V} = 1$

Best solved with a Monte Carlo method!

Basic idea of PDF modeling

"Particle" Monte Carlo method

SLM is a stochastic model for fluid particles which are at $\boldsymbol{X}^*(t)$

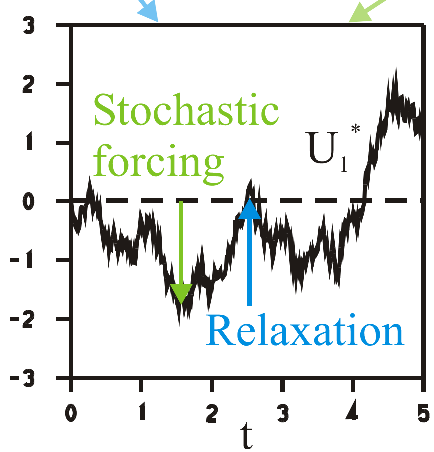
$$dX_i^* = U_i^* \, \mathrm{dt}$$

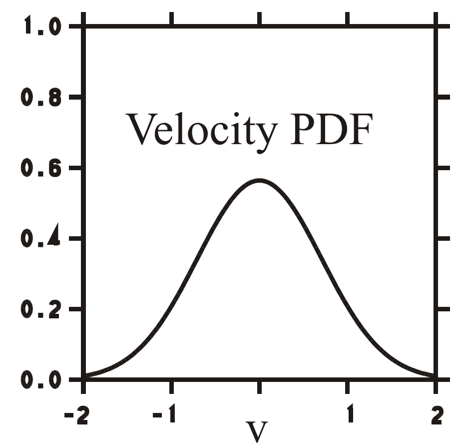
$$dU_i^* = \mathrm{M_i dt} - \left(\frac{1}{2} + \frac{3}{4}C_0\right)\frac{\varepsilon}{k}\left(U_i^* - \langle U_i \rangle\right) + (C_0\varepsilon)^{1/2} dW_i$$

Mean flow → Mean velocity equation exactly satisfied

Relaxation

Stochastic force





Statistics (i.e. PDF) of stochastic process evolve according to the PDF equation!

Hybrid FV – Monte Carlo solution method

Statistical error of Monte Carlo Method decreases as $\frac{1}{\sqrt{N_p}}$

-> To reduce the effects of the statistical and bias errors we use a hybrid FV – Monte Carlo method to solve for the joint velocity – turbulence frequency PDF

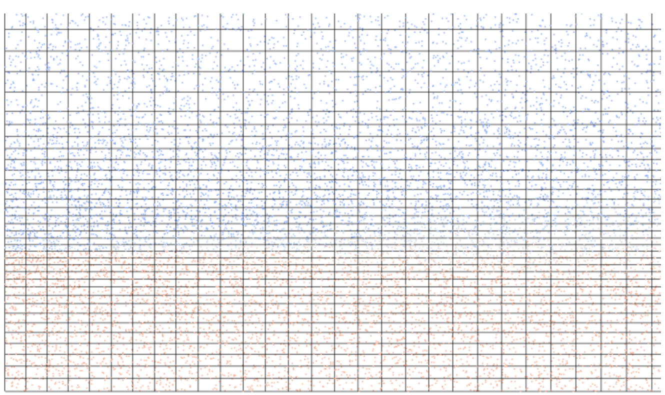
- Consider a FV mesh with Nc = 50-100 particles per cell
- Solve mean velocity $\langle U_i \rangle$ and pressure equations with FV method and Reynolds stress taken from particles (particle in cell average + time averaging)
- Solve particle equation with a simple Euler-Maruyama method

$$u_i^*(t+\Delta t) = u_i^*(t) + \frac{\partial \langle u_i u_j \rangle}{\partial x_j}\Delta t - u_j^*(t)\frac{\partial \langle U_i \rangle}{\partial x_j} - \left(\frac{1}{2} + \frac{3}{4}C_0\right)\Omega u_j^*(t) + (C_0 \mathrm{k}\Omega\Delta t)^{1/2}\xi_i$$

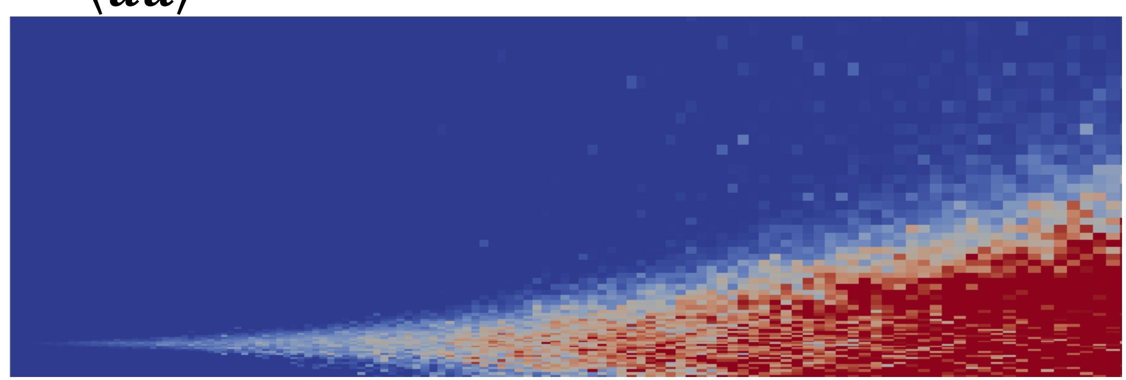
$$\omega^*(t+\Delta t) = \omega^*(t) - \mathrm{C}_{\omega,3}(\omega^* - \langle\omega\rangle)\Omega\Delta t - S_\omega\Omega\omega^*\Delta t + \left(2C_{\omega,3}C_{\omega,4}\langle\omega\rangle\omega^*\Omega\Delta t\right)^{1/2}\xi'$$

$$X_i^*(t+\Delta t) = X_i^*(t) + \left(\langle U_i \rangle + u_i^*(t+\Delta t)\right)\mathrm{dt}$$

$$\Omega = C_\Omega \langle \omega^* | \omega^* > \langle\omega\rangle \rangle$$

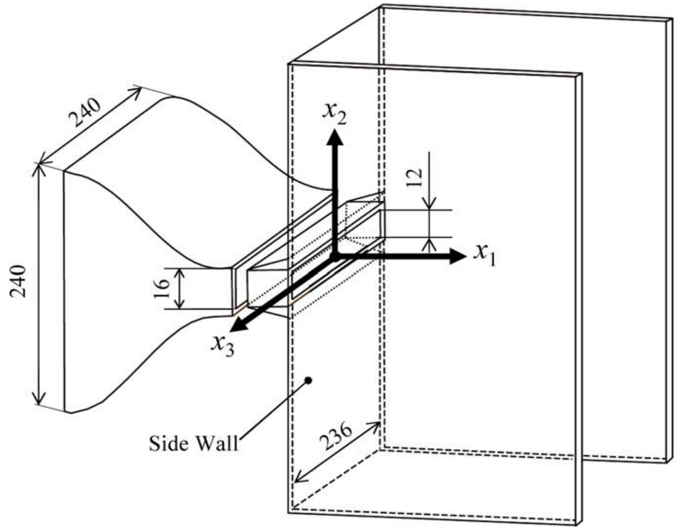


$\langle \boldsymbol{uu} \rangle$



Preliminary results: Plane jet at Re = 22,000

“Simultaneous measurement of all three velocity components and pressure in a plane jet”
Terashima et al, Meas. Sci. Technol. **25** (2014)



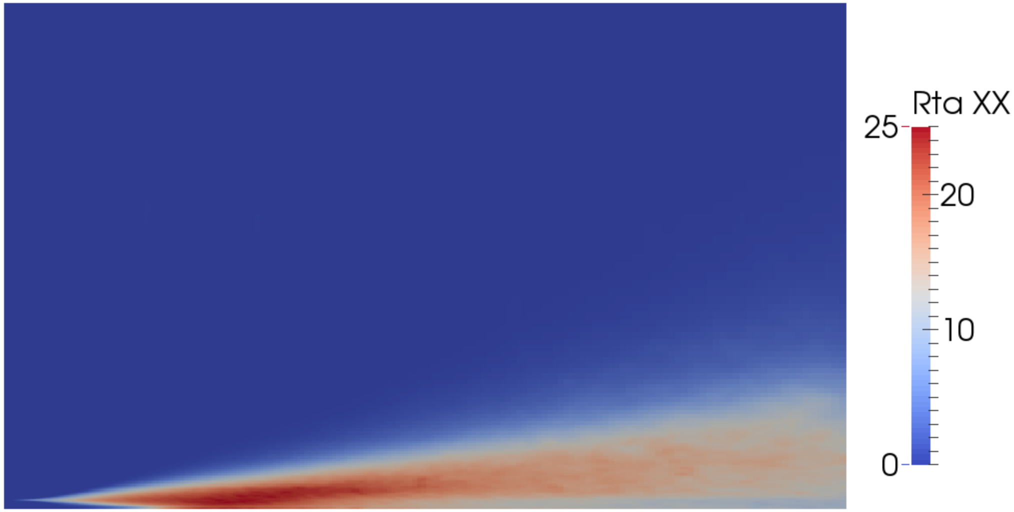
- Measured velocity and joint pressure-velocity statistics (one-point)
- Performed LES to “Validate” the measurements

Simulations

- Hybrid FV- Monte Carlo solver implemented in OpenFOAM (based on pisoFOAM)
- Implementation suitable for unstructured grids and fully parallelized
- 50d x 20d domain with 240x110 grid points
- Np = 100 particles per cell, time averaging over 200 time steps
- For comparison we use a Rotta’s RSM model with Daily-Harlow turbulent diffusion and standard ε-equation coefficients (RottaDH)
- PDF solution is based on a “frozen” mean velocity field from RottaDH solution

Preliminary results: Plane jet at Re = 22,000

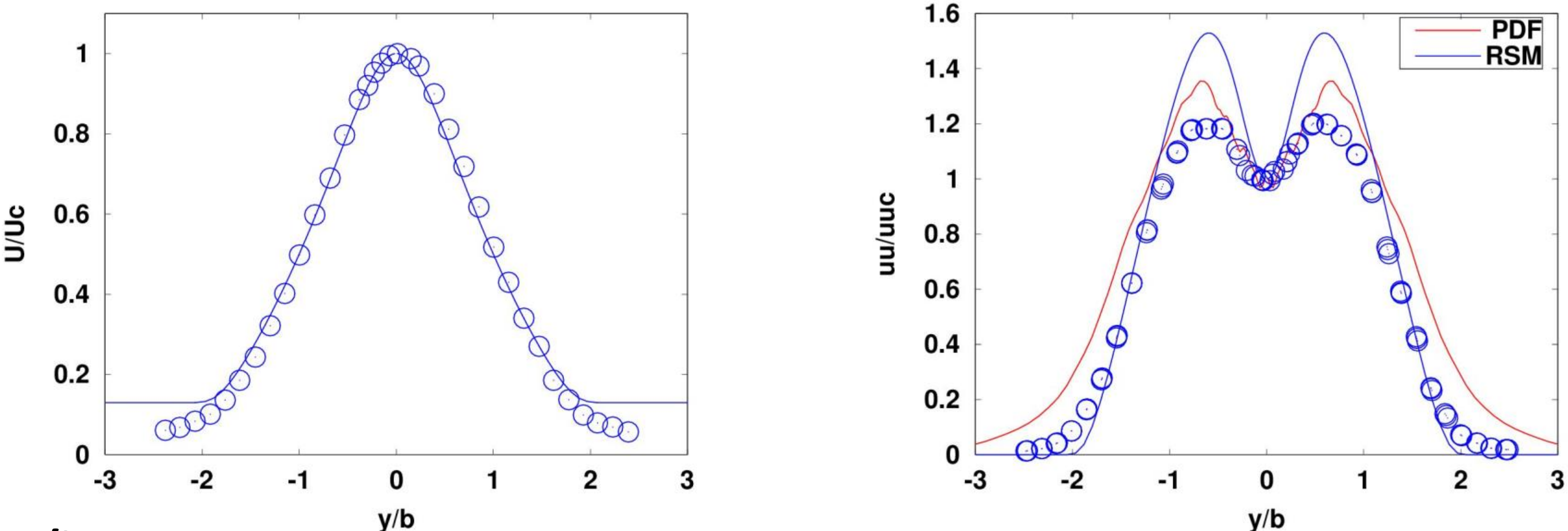
PDF: $\langle uu \rangle$



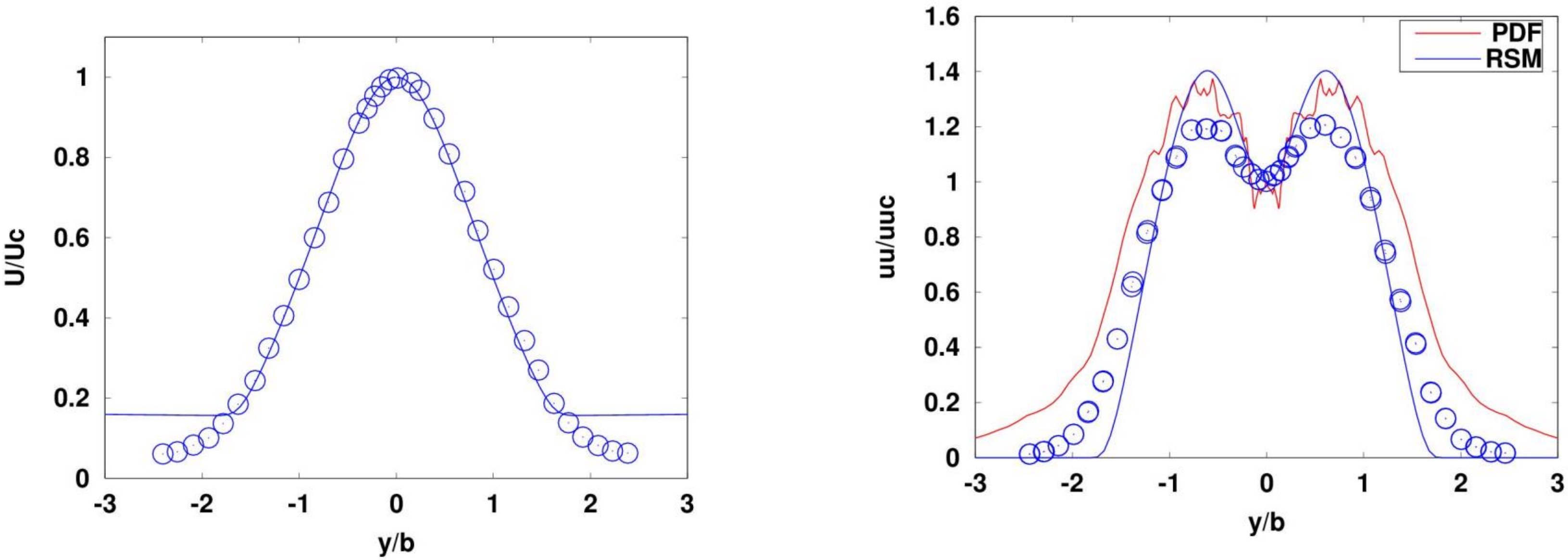
RottaDH RSM: $\langle uu \rangle$

Preliminary results: Plane jet at Re = 22,000

x/h = 20

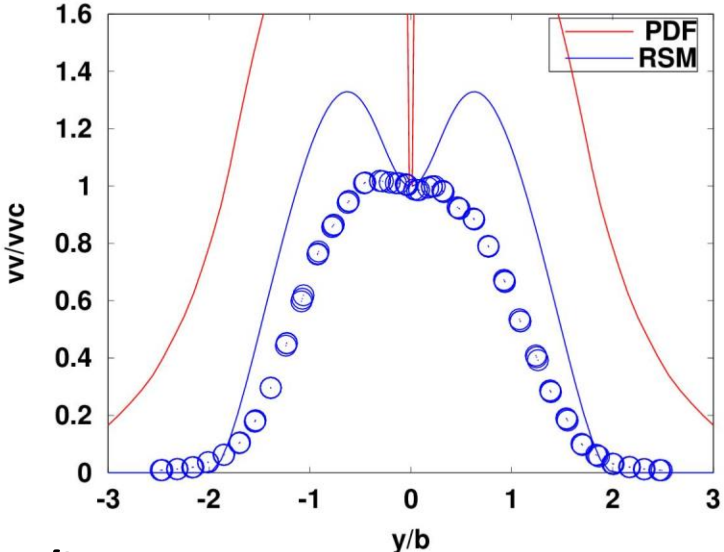


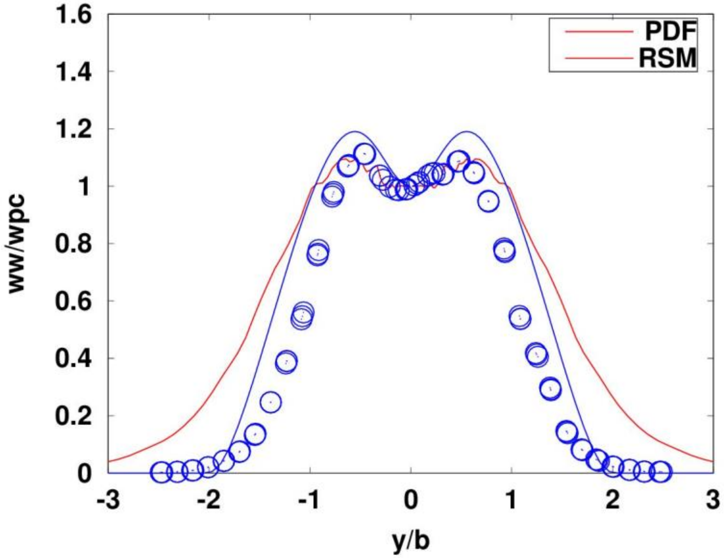
x/h = 40



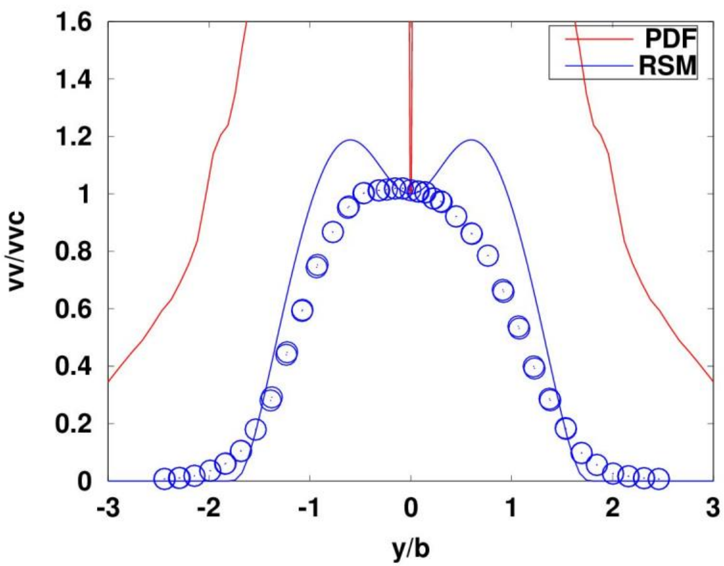
Preliminary results: Plane jet at Re = 22,000

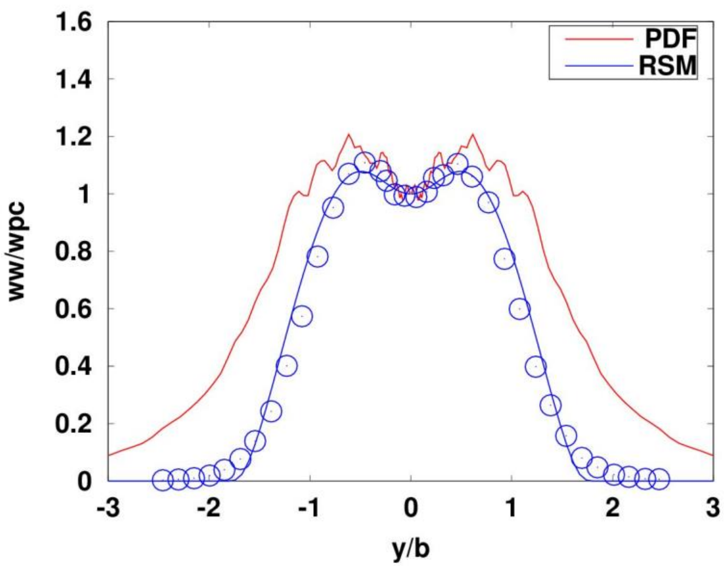
x/h = 20





x/h = 40





Conclusions and Outlook

- PDF closure modeling provides a “complete” moment-closure
- Importance of higher order moments such as the turbulent transport could be evaluated and new models for RSM approaches proposed
- A basic hybrid FV – Monet Carlo solution method has been implemented in OpenFOAM (unstructured grids, fully parallelized with encouraging strong scaling)
- More debugging required
- Analyze k-budget for the plane jet also for different G_{ij} models