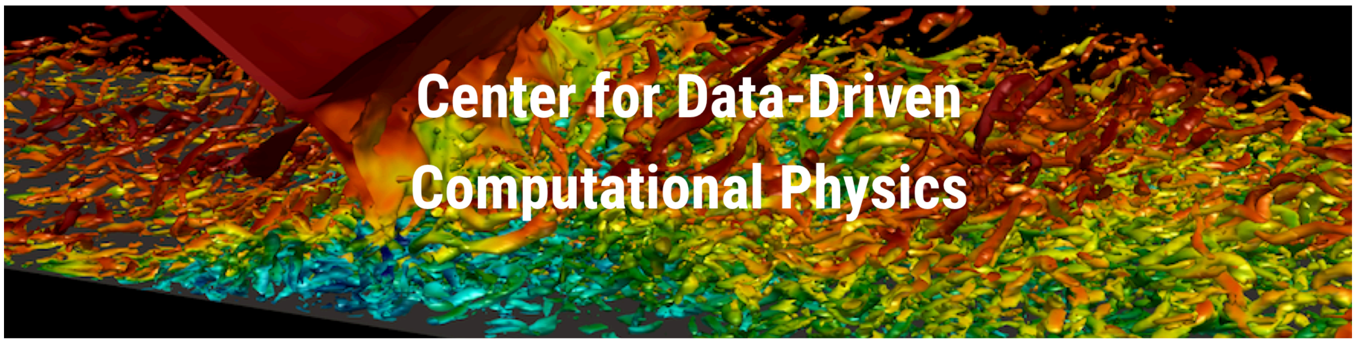
Data-enabled turbulence modeling : Progress and Challenges

Karthik Duraisamy



AEROSPACE ENGINEERING
UNIVERSITY of MICHIGAN

Thanks to: A. Singh, E. Parish, S. Medida, B. Tracey, etc..

NASA

DARPA

NSF

What do we expect from data-driven modeling?

Let's start with a brief history of a success story: Machine Translation

New York Herald Tribune: 8 January 1954

It's all done by machine : Words go in in Russian, English sentence comes out

A huge electronic "brain" with a 250-word vocabulary translated mouth-filling Russian sentences yesterday into simple English in less than ten seconds.

Once the Russian words were fed to the machine no human mind intervened. In demonstrating this feat for the first time scientists of the IBM and Georgetown, said they hoped that within a few years such machines would be freely translating all languages.

At the demonstration, an I.B.M. mathematician, fed into the machine, filling a room as big as a tennis court, a series of cards carrying the Russian sentence: "Myezhdunarodnoye ponyimaniye yavlyatetsya vazhim faktorom vryesnyenyiyi polytyichyeskyix voprosov."

The machine blinked its lights, was quiet for a moment as if thinking and within nine seconds the automatic typewriter clacked and out came: "International understanding constitutes an important factor in decision of political questions."

New York Herald Tribune: 8 January 1954 (Contd..)

Even though the machine can translate such complex sentences, it is limited by its vocabulary, and by its “knowledge” of grammar, according to Dr.Dostert.

Dr.Dostert said that it will not be too long – possibly three to five years – when automatic text-reading machines will feed in Russian sentences automatically into the machines without punched-card intervention.

Then, Dr.Dostert said, complete libraries of Western technical works could be made available to non-industrial nations. “At present,” he added, “we are at the ‘Kitty Hawk’ state.”

What happened after that?

Cls-communications.com

1947-1954

The pioneers:
First public demonstration of computer translation at Georgetown University during the Cold War: Russian into English.

1954-1966

The decade of optimism:
Research was inspired by contemporary developments in linguistics, particularly in models of formal grammar. There were some operational systems, but the quality of output was disappointing.

1966-1980

ALPAC and its aftermath:
Alpac published a report on MT concluding that years of research haven't produced useful results. The outcome was a halt in federal funding for machine translation. However, research went on in Canada, France and Germany.

1980s

The 1980s:
This era witnessed the emergence of a wide variety of MT system types. The dominant strategy was that of ‘indirect’ translation via intermediary representations.

Early 1990s

The 1990s:
A group from IBM published the results of experiments on a system based purely on statistical methods. At the same time, Japanese groups began to use methods based on a body of translation examples.

late 1990s - 2000s

Recent years:
The use of MT and translation aids by large corporations has grown rapidly. Research is ongoing in both statistical and rule-based machine translation.

The big leap : Google Translate

Before september 2016 : phrase-based translation. - mapping roughly equivalent words and phrases without an understanding of linguistic structures can only produce crude results.

After september 2016 : Zero-shot translation: Learned how to make educated guesses about the content, tone, and meaning of phrases based on the context of other words and phrases around them. Google Translate **invented its own language** to help it translate more effectively.

A neural computing system designed to translate content from one human language into another developed its own internal language to make the task more efficient.

Gil Fewster, Medium.com

Human Translation Vs Machine Translation: Competition

- An English-language article from Fox Business
- A selection from Thomas Friedman's *Thank You For Being Late,* also in English.
- Part of a Korean-language opinion column from writer Kim Seo-ryung.
- A selection of the Korean novel *Mothers and Daughters,* by Kang Kyeong-ae

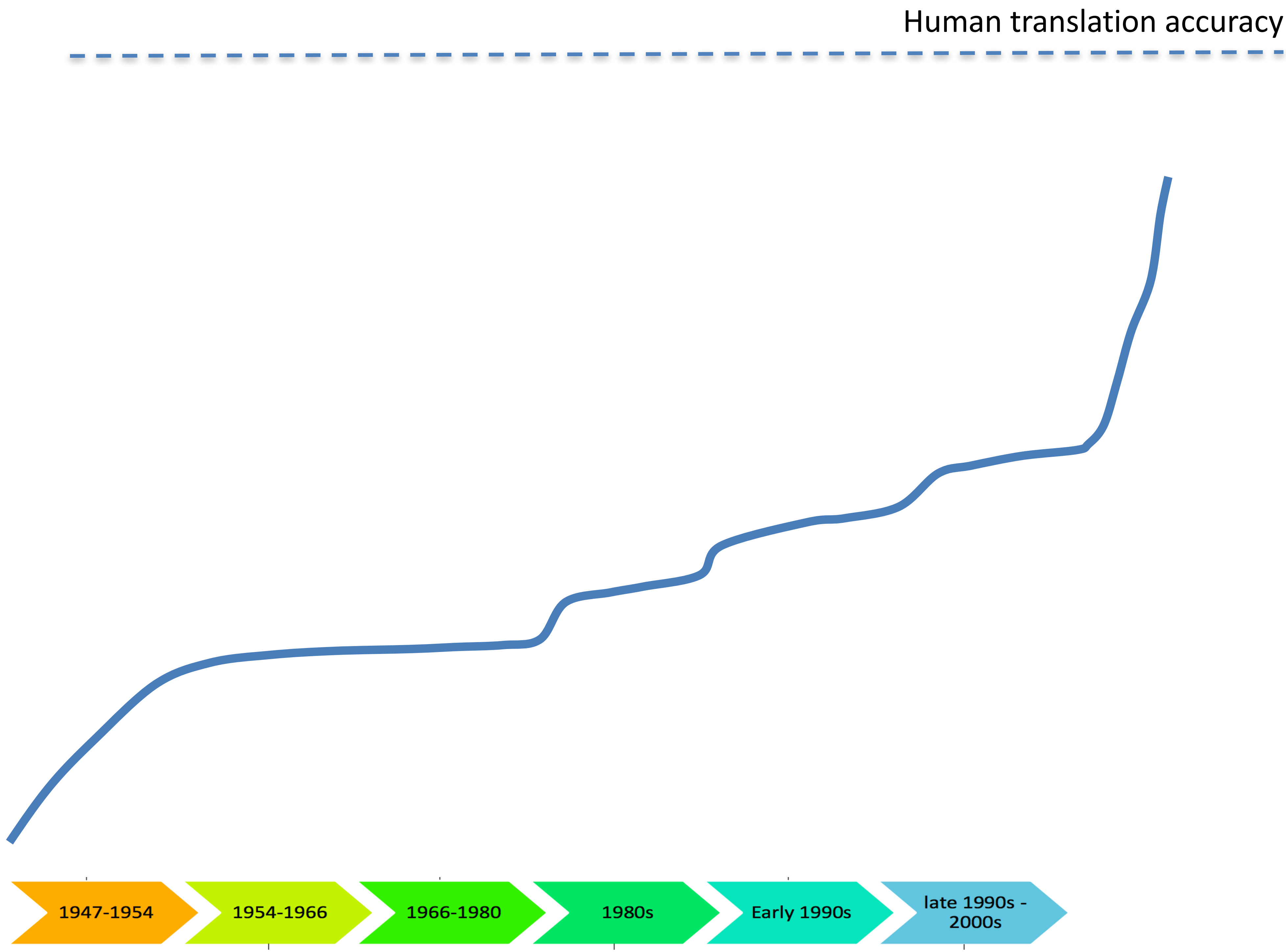
Google's translations scored a 28 out of 60 possible points ; Systran scored 17 out of 60.

The humans, on the other hand, scored much better, with a high score of 49 out of 60.

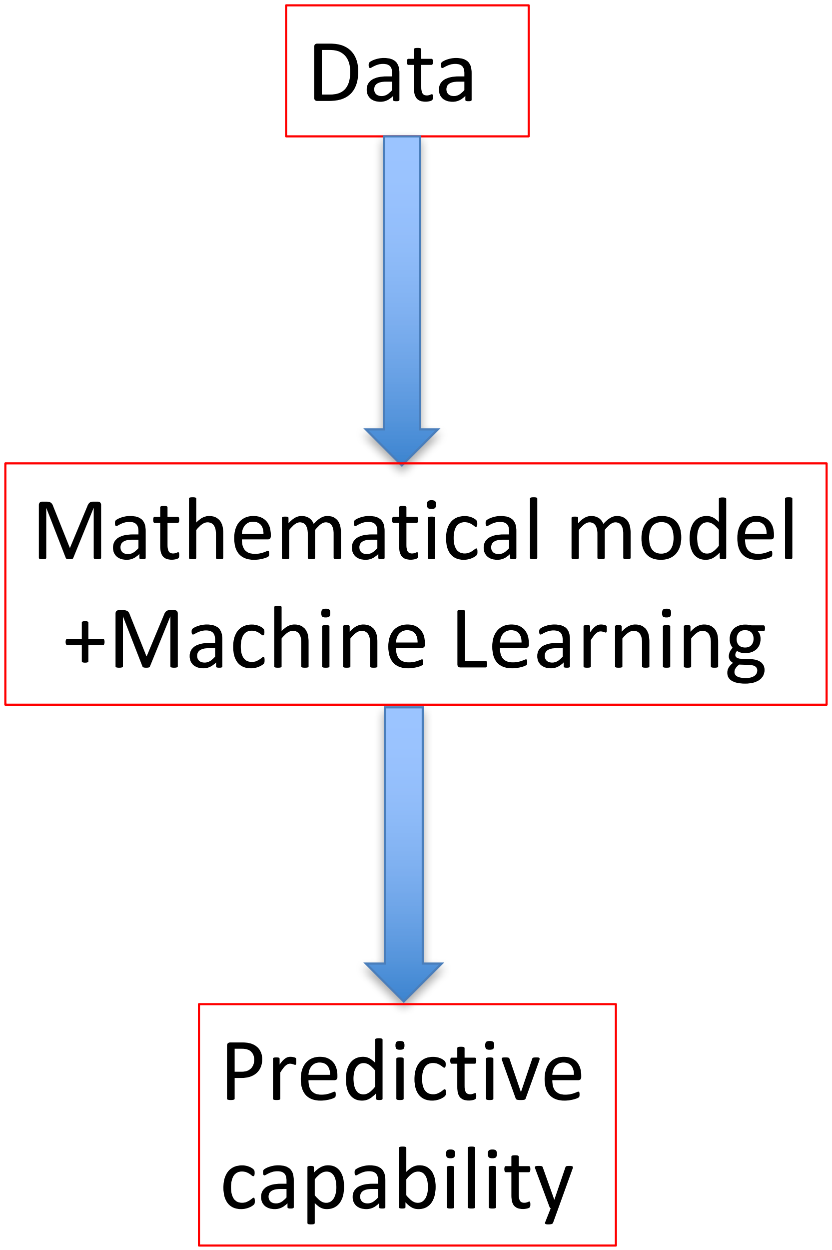
"the machines were much faster. They delivered their translations in just a few minutes, while the human translators took almost an hour"

"No matter how fast the translation programs are, many will doubt they can perfectly translate subtle expressions of emotion in literature. We hope the event shows the relative strengths and weaknesses of AI translation programs and human experts."

Alison Kroulek, K-international.com


Human translation accuracy
1947-1954
1954-1966
1966-1980
1980s
Early 1990s
late 1990s -
2000s

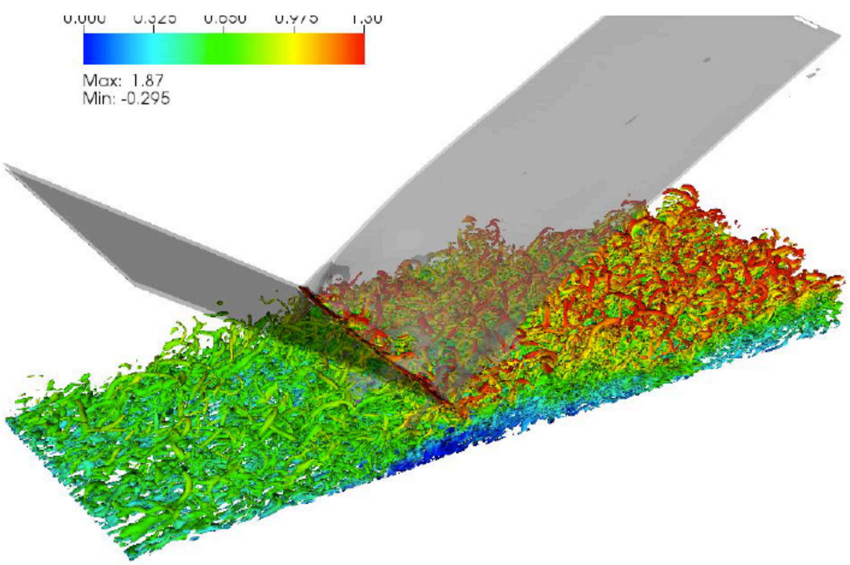
Other examples : Face recognition

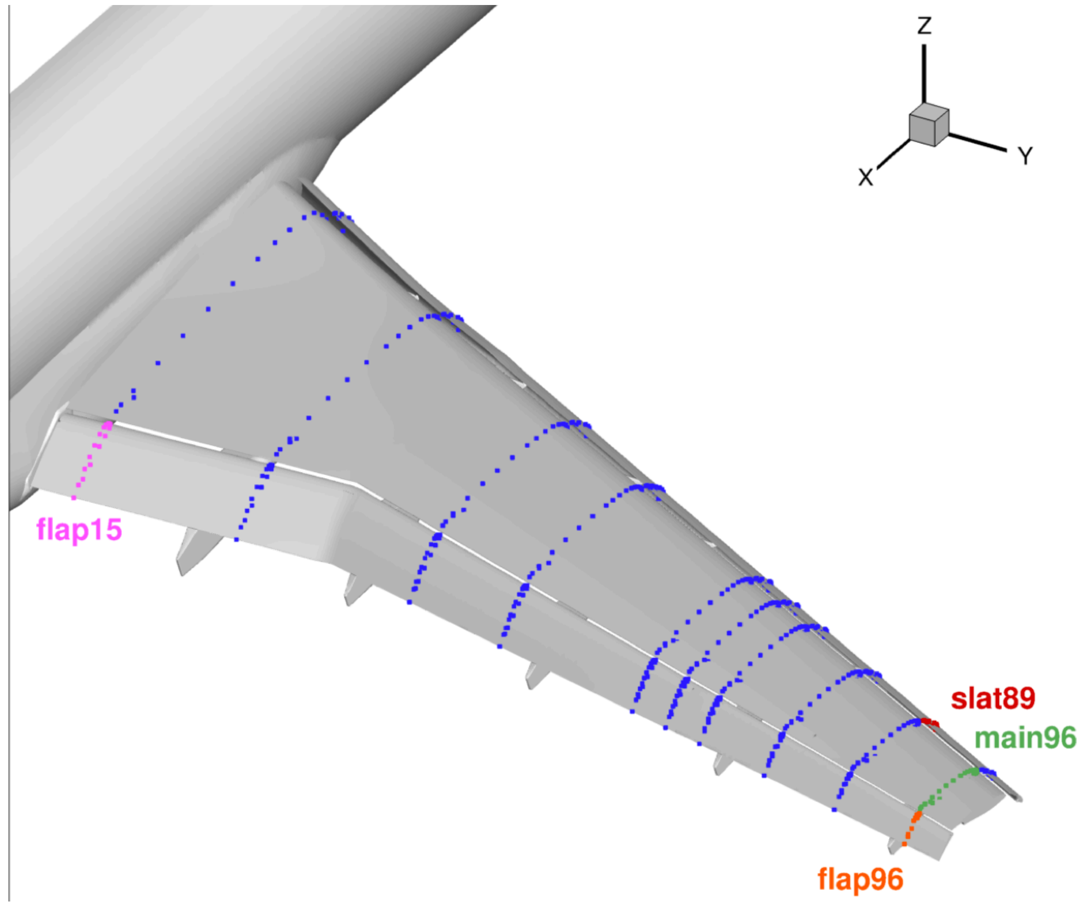


- No physical law ;
- Data is directly useful for model;
- Large amounts of relevant data.

Data deluge…

- DNS and LES have been produced in quantity
- Experimental PIV and MRV high-res data sets

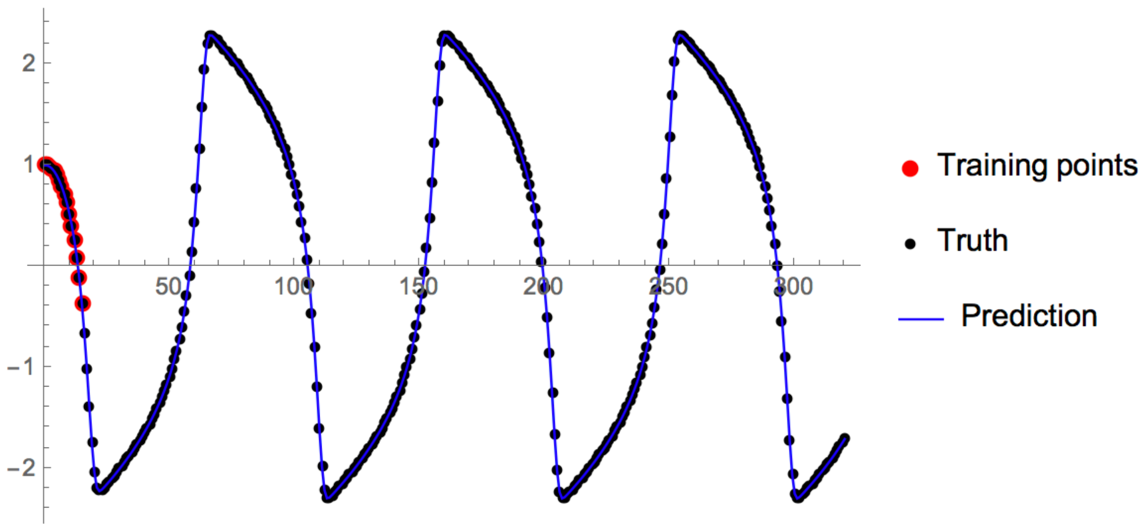


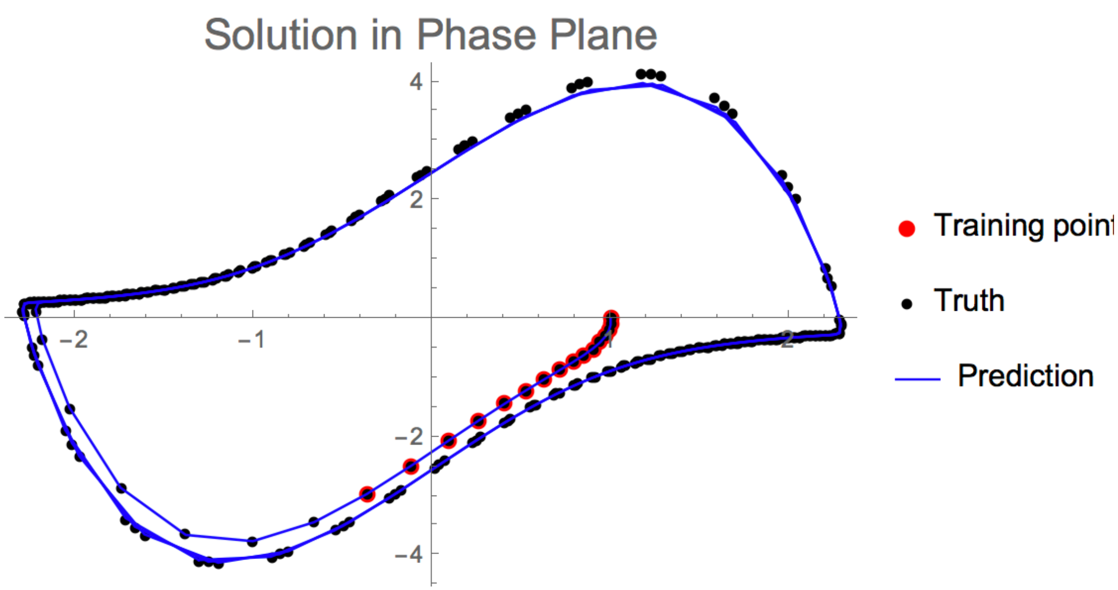


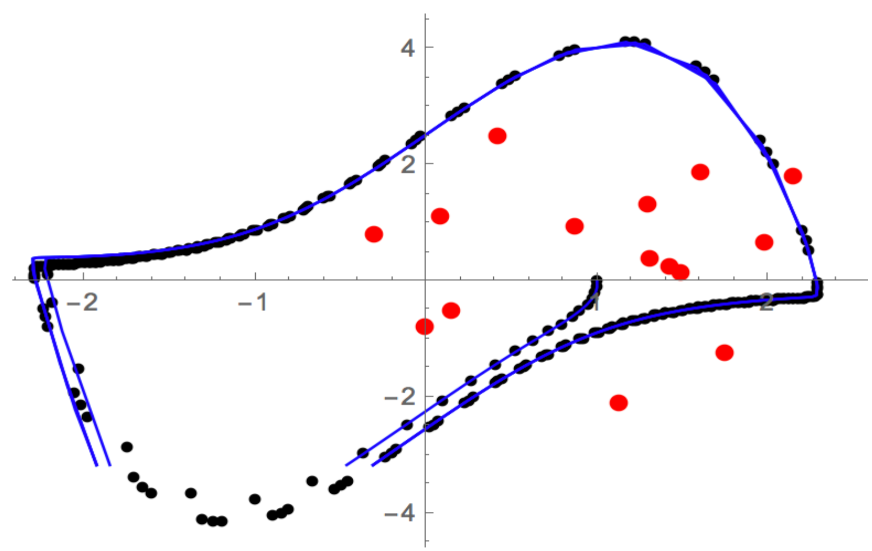
- Data sets have not had a substantial impact on closure modeling

Discovering equations from data

$$\begin{pmatrix} x_1^{n+1} \\ x_2^{n+1} \end{pmatrix} = \begin{pmatrix} x_1^n + \Delta t x_2^n \\ x_2^n + \Delta t(\mu(1 - x_1^{2^n})x_2^n - x_1^n) \end{pmatrix}$$





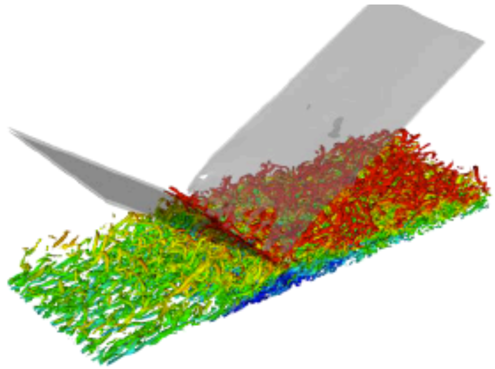


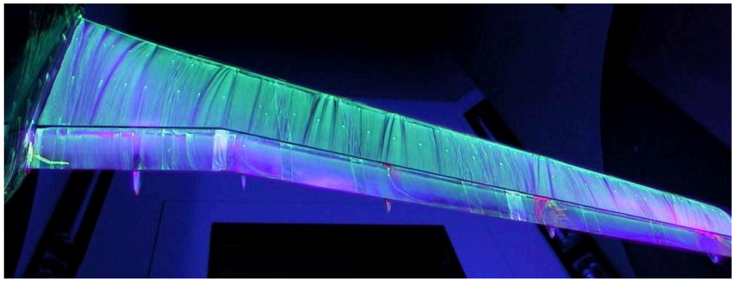
Outline

- What can we learn from other fields ?
- Turbulence modeling , data & tools
- Progress in data-driven turbulence modeling
- Innovative ways of using data
- Perspectives

Can we replicate this type of success in turbulence modeling?

Data





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Physical model + Physics constraints + Machine Learning

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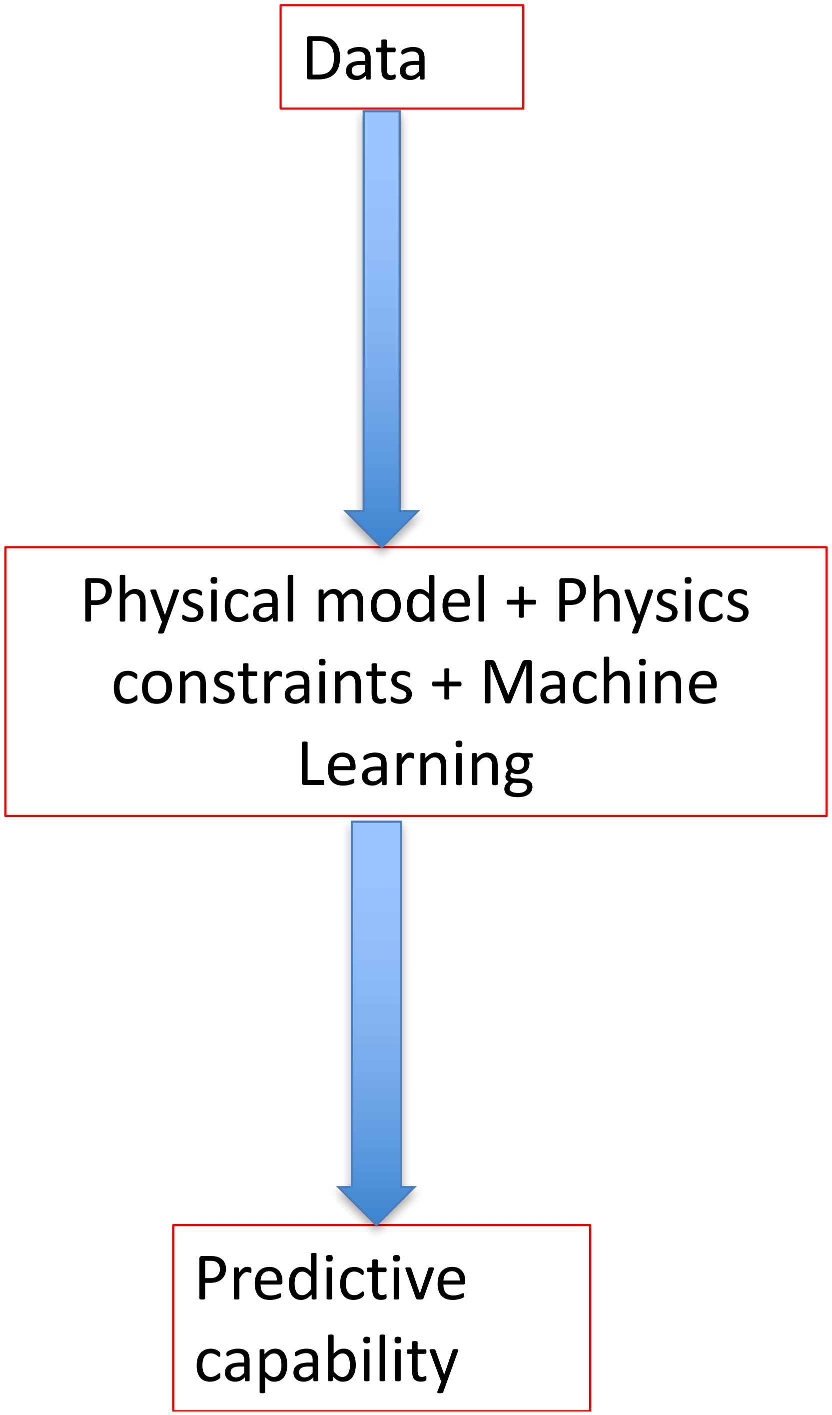
Predictive capability

$$\frac{\partial \bar{u}}{\partial t} + \mathcal{R}(\bar{u}) = \mathbf{M}(\bar{u}, \bar{v})$$

Challenges



- Data contains real quantities; Model contains “modeled” quantities (loss of consistency is severe in turbulence models)
  ➔ k and in the model are not the k and eps in DNS
- Data will be only loosely connected to model (and not objective)
  ➔ How to improve a turbulence model if we only have pressure measurements (or images)?
- Data will be noisy and of variable quality,
- Inherent uncertainty

Turbulence models

$$\frac{\partial \overline{u_i' u_j'}}{\partial t} = C_{ij} + P_{ij} + V_{ij} + T_{ij} + \Pi_{ij} + D_{ij}$$

$$\frac{\partial \overline{u_i' u_j'}}{\partial t} = C_{ij} + P_{ij} + V_{ij} + \tilde{T}_{ij} + \delta_T + \tilde{\Pi}_{ij} + \delta_\Pi + \tilde{D}_{ij} + \delta_D$$

- One - seven transport eqns, and up to 30 adjustable constants.
- Modeling rests on large amounts of intuition and luck, in spite of starting with a "rigorous" approach
- Theories abound for parts of model, but not for output
- Model constants calibrated on very limited data
- Greater sophistication in RANS models, with mixed degree of success
  - ➔ More constants to fit , still use canonical problems

Turbulence modeling discrepancies

$$\frac{\partial \overline{u'_i u'_j}}{\partial t} = C_{ij} + P_{ij} + V_{ij} + T_{ij} + \Pi_{ij} + D_{ij}$$

$$\frac{\partial \overline{u'_i u'_j}}{\partial t} = C_{ij} + P_{ij} + V_{ij} + \tilde{T}_{ij} + \delta_T + \tilde{\Pi}_{ij} + \delta_\Pi + \tilde{D}_{ij} + \delta_D$$

- Balance between the terms matters most (and not accuracy of individual terms)
  ➔ Still respect invariance, symmetries, etc.
- Many "seemingly physical" quantities are just operational variables
  ➔ Use of *apriori* analysis is of limited utility
- There is no beautiful turbulence model waiting to be discovered
  ➔ Look for optimal model, conditional on data & constraints?

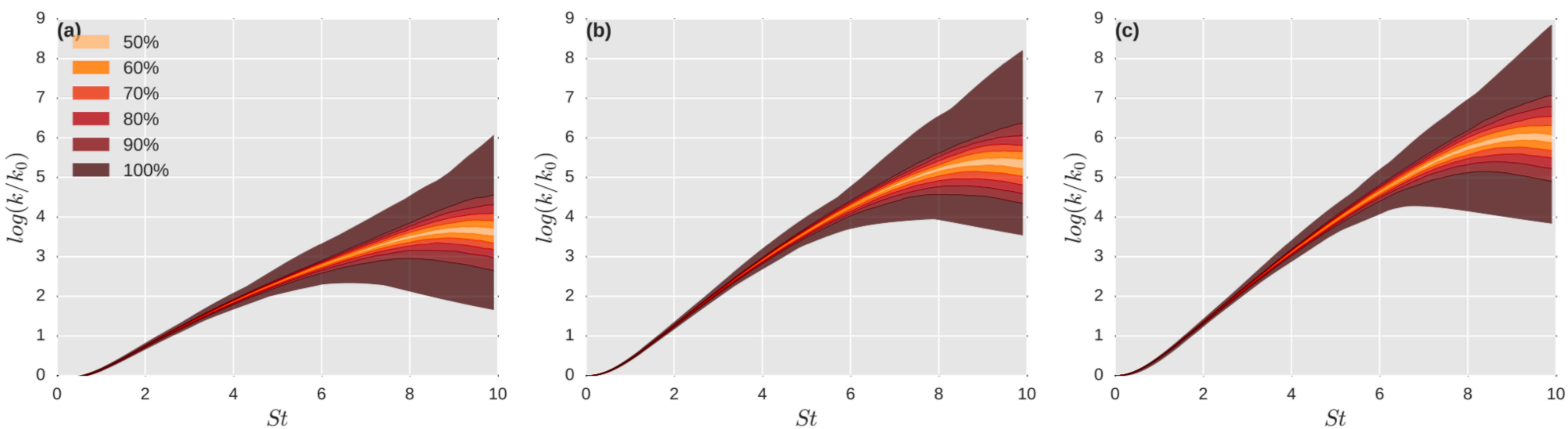
Turbulence models – inherent uncertainty

$$\frac{\partial u}{\partial t} + \mathcal{R}(u) = 0$$

$$\bar{u} = \mathcal{P}u$$

Same macrostate, different microstate – irreducible uncertainty

15



$$\frac{Sk_0}{\epsilon_0} = 3 \qquad \frac{Sk_0}{\epsilon_0} = 15 \qquad \frac{Sk_0}{\epsilon_0} = 27$$

Inference

$$\frac{\partial \bar{u}}{\partial t} + \mathcal{R}(\bar{u}) = \mathbf{M}(\bar{u}) \rightarrow \text{Forward problem} \rightarrow \bar{u}$$

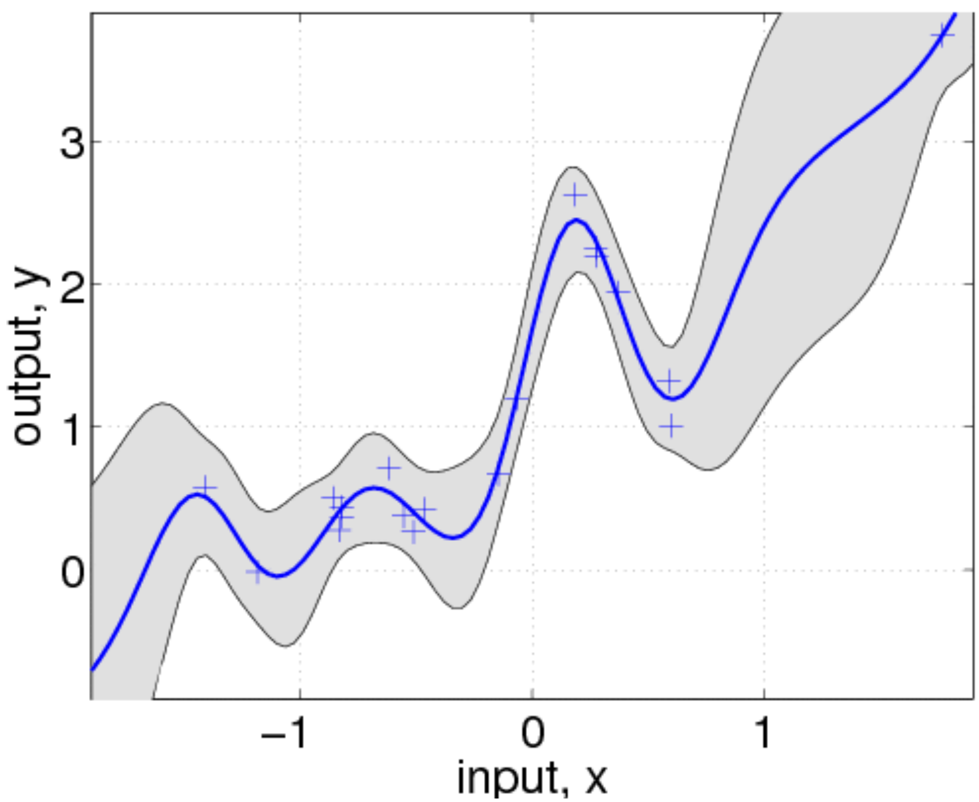
$$\mathbf{M}(\bar{u}); \bar{u} \leftarrow \text{Inverse problem} \leftarrow \bar{u}_{data}; \mathcal{R}(\cdot)$$

Machine Learning

Supervised Learning: Given a set of labeled data $\{x_i, y_i\}$, learn the mapping y(x)

Unsupervised Learning: Given data, discover patterns and groupings

Typically cast in a probabilistic framework ➔ Deep connections with statistical mechanics



GPML

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Turbulence models

$$\frac{\partial u}{\partial t} + \mathcal{R}(u) = 0 \quad \text{NSE}$$

$$\bar{u} = \mathcal{P}(u)$$

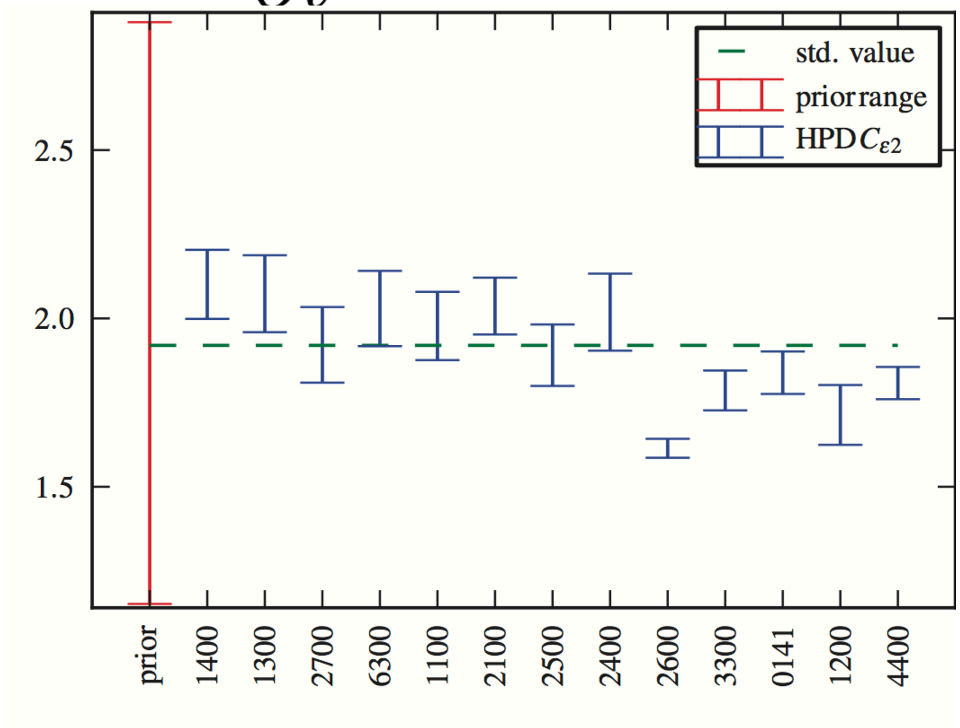
$$\frac{\partial \bar{u}}{\partial t} + \mathcal{R}(\bar{u}) = \mathcal{R}(\bar{u}) - \bar{\mathcal{R}}(u)$$

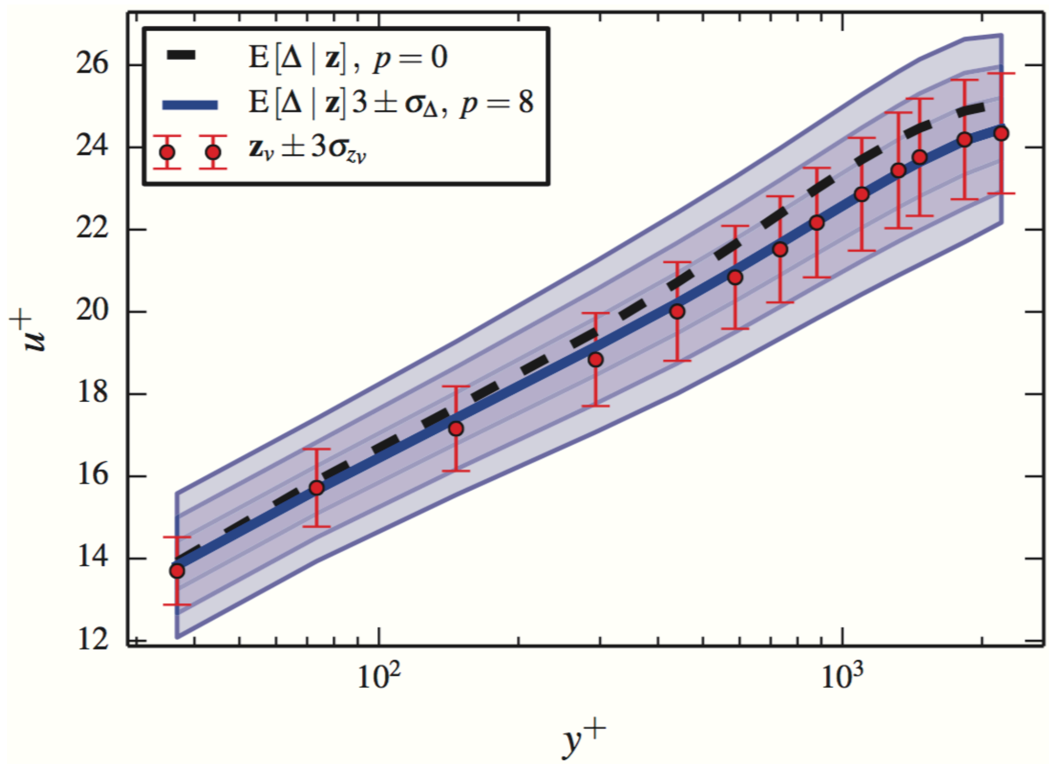
$$\frac{\partial \bar{u}}{\partial t} + \mathcal{R}(\bar{u}) = \mathcal{M}(\bar{u}, u) \quad \text{RANS}$$

$$\frac{\partial \bar{u}}{\partial t} + \mathcal{R}(\bar{u}) = \mathbf{M}(\bar{u}, \bar{v}) \quad \text{RANS + model}$$

Parameter inference / calibration

$$\frac{\partial \bar{u}}{\partial t} + \mathcal{R}(\bar{u}) = \mathbf{M}(\bar{u}, \bar{v}, \alpha) \quad ; \quad \min_{\alpha} ||\bar{u}_{data} - \bar{u}||$$





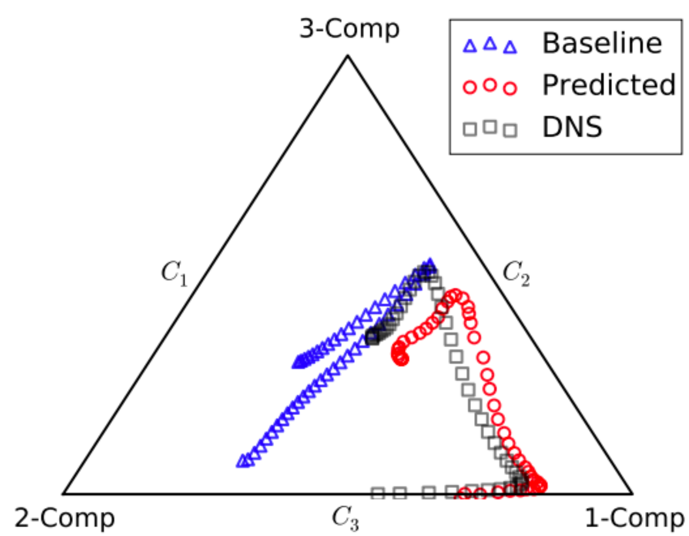
Cheung, et al. (2011): Bayesian uncertainty analysis with applications to turbulence modeling
W. N. Edeling (2014): Bayesian estimates of parameter variability in the k–ε turbulence model
Ray, et al. (2014): Bayesian calibration of a k–ε turbulence model for predictive jet-in-crossflow simulations, 2015.

Structural discrepancy using machine learning

Tracey, Duraisamy, Alonso (2013) : Reynolds stress anisotropy eigenvalues
Xiao et al (2016) : Reynolds stress anisotropy eigenvalues & eigenvectors
Ling et al (2016) : Algebraic RSM functions
Weatheritt (2016) : Evolutionary algorithms to extract Non-linear stress strain

$$\frac{\partial \bar{u}}{\partial t} + \mathcal{R}(\bar{u}) = \mathbf{M}(\bar{u}, \bar{v}, \delta(\bar{u}, \bar{v})) \quad ; \quad \delta(\bar{u}, \bar{v}) \equiv \delta(\bar{u}_{data}, \bar{v}_{data})$$

$$\mathbf{R}_p = 2k \left[\frac{\mathbf{I}}{3} + \mathbf{V}(\Lambda + \delta_\Lambda)\mathbf{V}^T \right]$$



Wang et al., CTR Summer program 2016.

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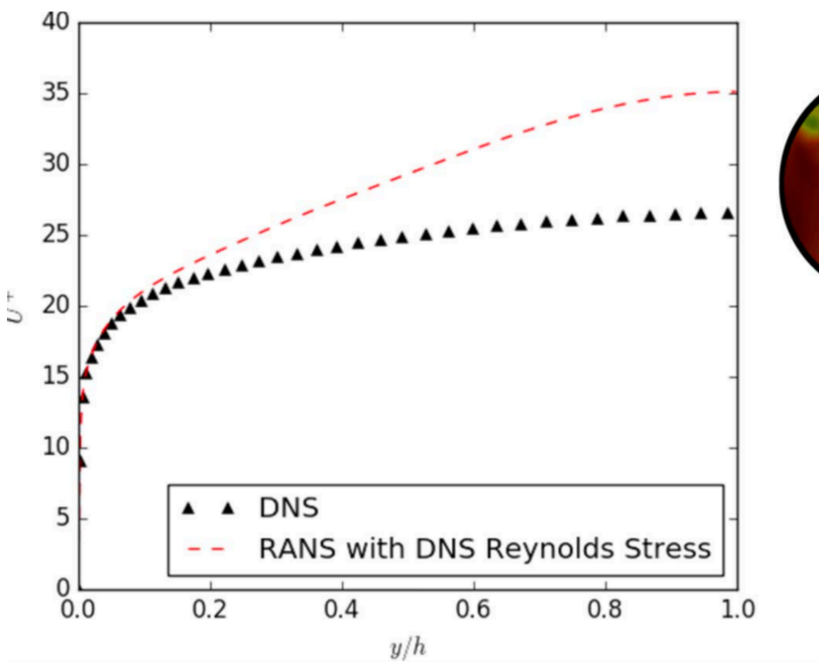
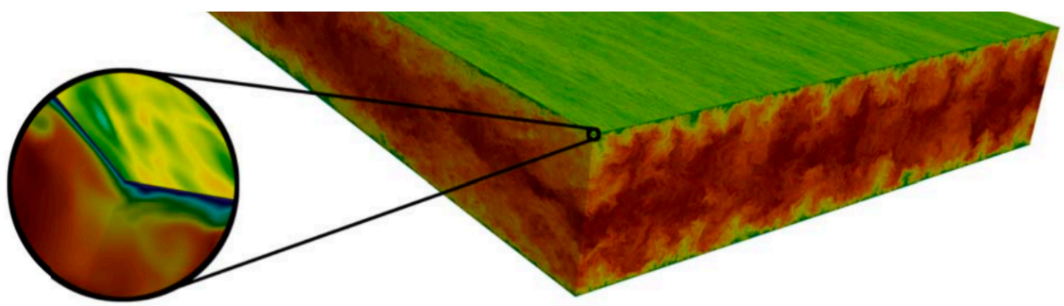


Poroseva, Colmenares F., and Murman Phys. Fluids **28**, 115102 (2016)



Data from UT Austin channel DNS website, Re 5200

$$\frac{d}{dy}\langle u'v' \rangle = \nu \frac{d^2 U}{dy^2} + f$$

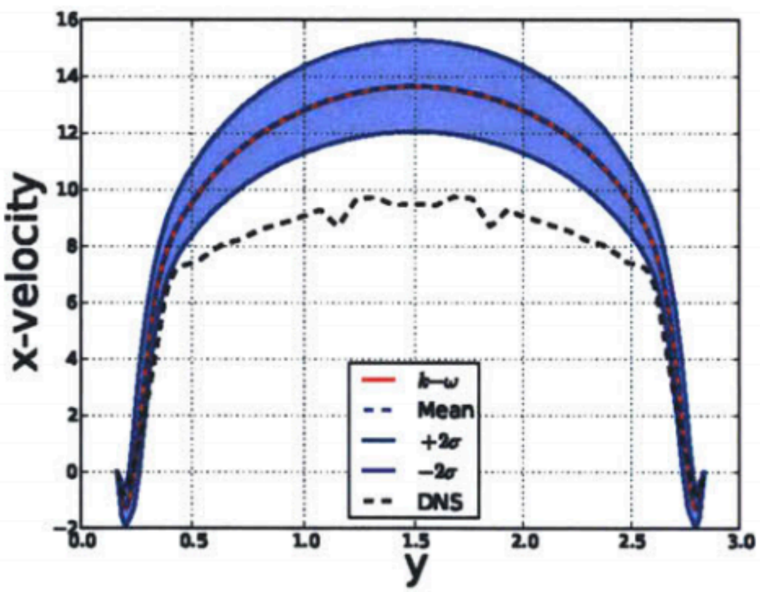
Picture courtesy of Qiqi wang

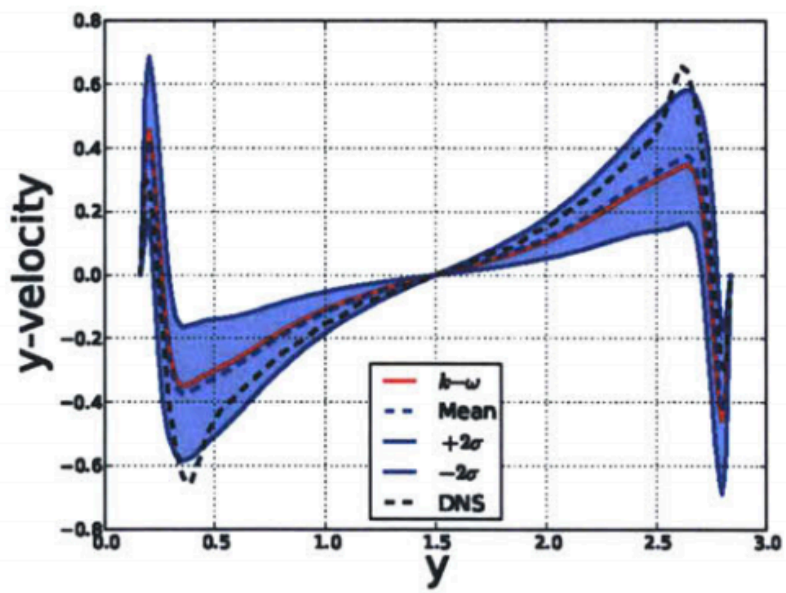
Inversion and spatial mapping of discrepancy in eddy viscosity

Dow and Wang (2012)

$$\frac{\partial \bar{u}}{\partial t} + \mathcal{R}(\bar{u}) = \mathbf{M}(\bar{u}, \bar{v}, \delta(\vec{x})) \quad ; \quad \min_{\delta(\vec{x})} ||\bar{u}_{data} - \bar{u}||$$

$$\frac{\partial \bar{u}}{\partial t} + \mathcal{R}(\bar{u}) = \mathbf{M}(\bar{u}, \bar{v}, \delta(\vec{x}))$$

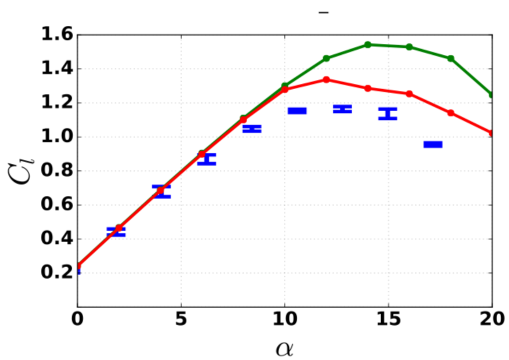




Field inversion + machine learning

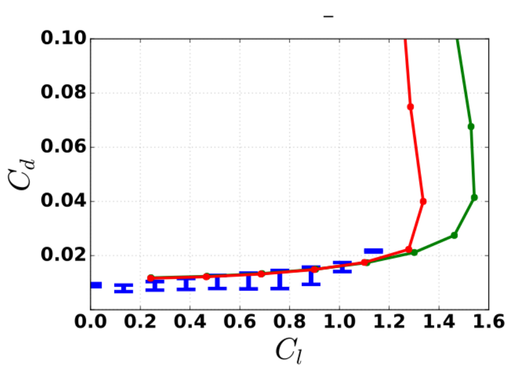
Duraisamy et al. (2014) : Inversion and machine learning
Tracey, Duraisamy, Alonso (2015) : Machine learning + embedding
Duraisamy (2016) : Inference + machine learning + embedding

$$\frac{\partial \bar{u}}{\partial t} + \mathcal{R}(\bar{u}) = \mathbf{M}(\bar{u}, \bar{v}, \delta(\vec{x})) \quad ; \quad \min_{\delta(\vec{x})} ||\bar{u}_{data} - \bar{u}||$$

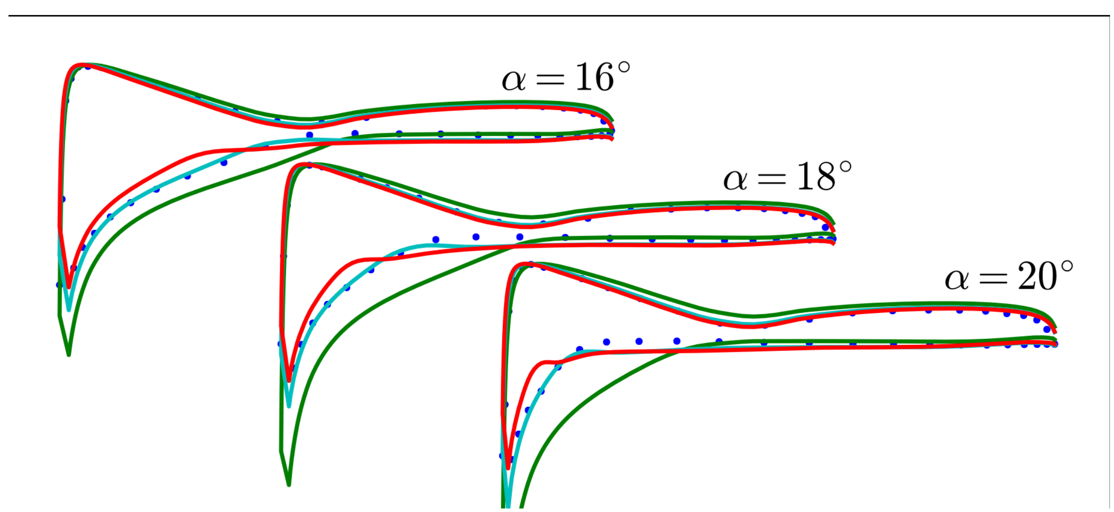


(a) $Re = 1 \times 10^6$

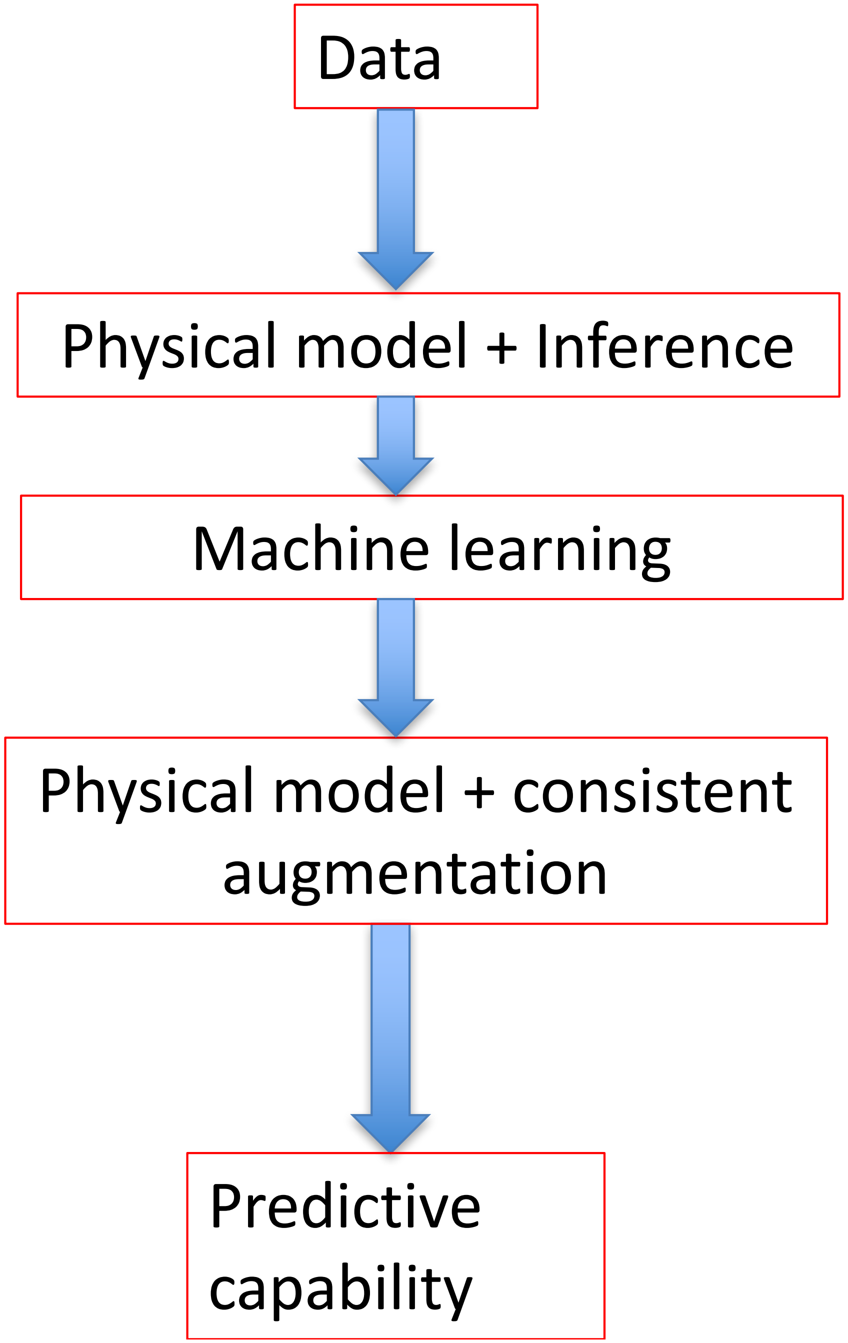
$$\frac{\partial \bar{u}}{\partial t} + \mathcal{R}(\bar{u}) = \mathbf{M}(\bar{u}, \bar{v}, \delta(\bar{u}, \bar{v}))$$



(d) $Re = 1 \times 10^6$



Field Inversion + Machine learning to Augment Physics-based, Consistent Models



- Data contains real quantities; Model contains “modeled” quantities (loss of consistency is bad in turbulence models)

  ➔ Inference connects real quantities to modeled ones

- Data will be only loosely connected to model (and not objective)

  ➔ Inference connects secondary, non-objective data to model quantites

- Data will be noisy and of variable quality, inherent uncertainty

  ➔ Probabilistic casting of inference and learning

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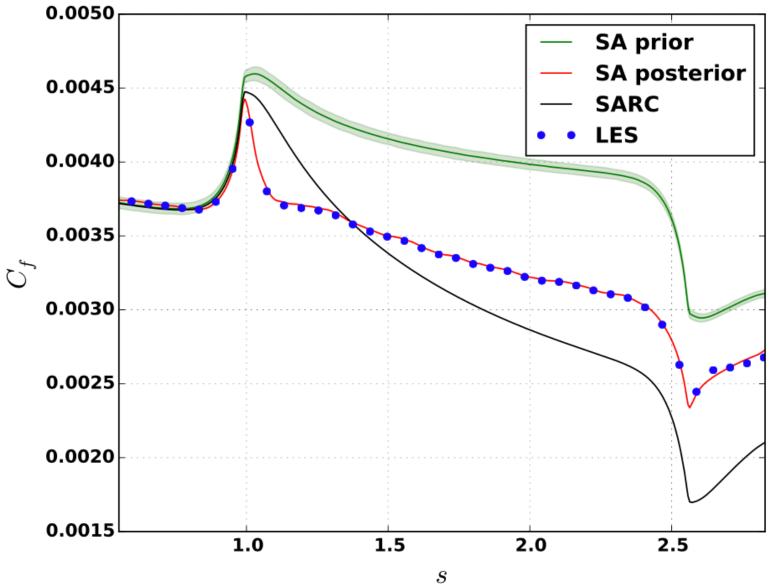
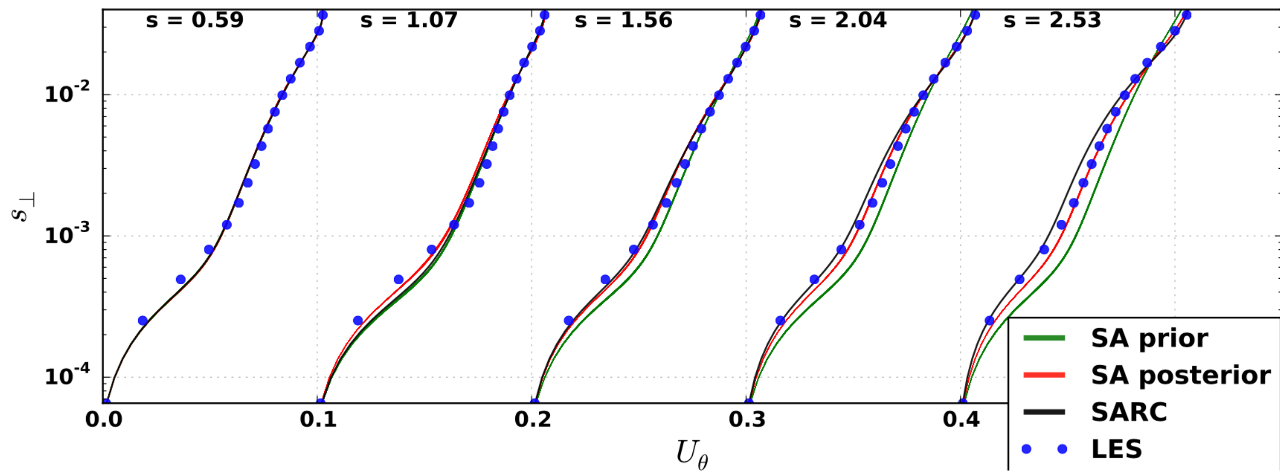
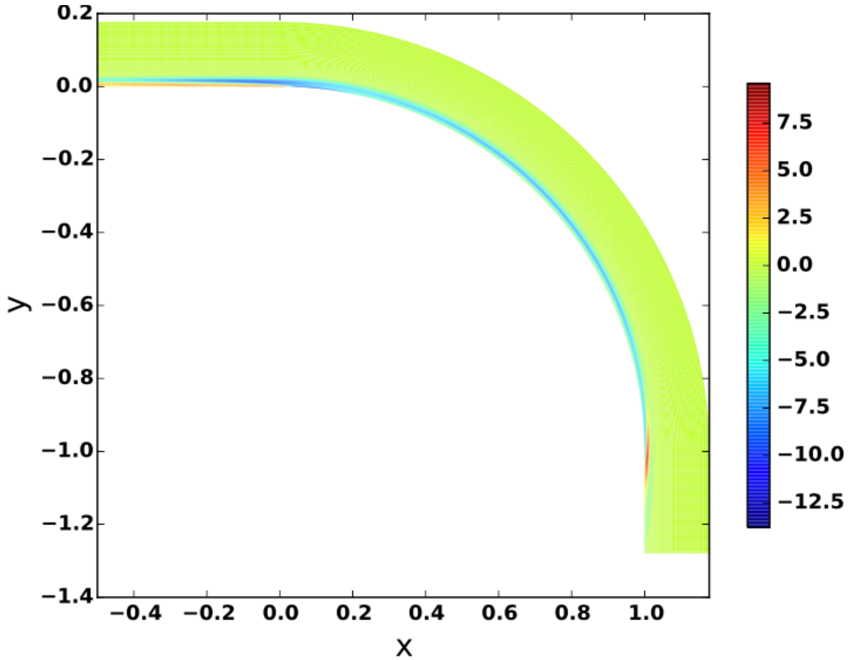
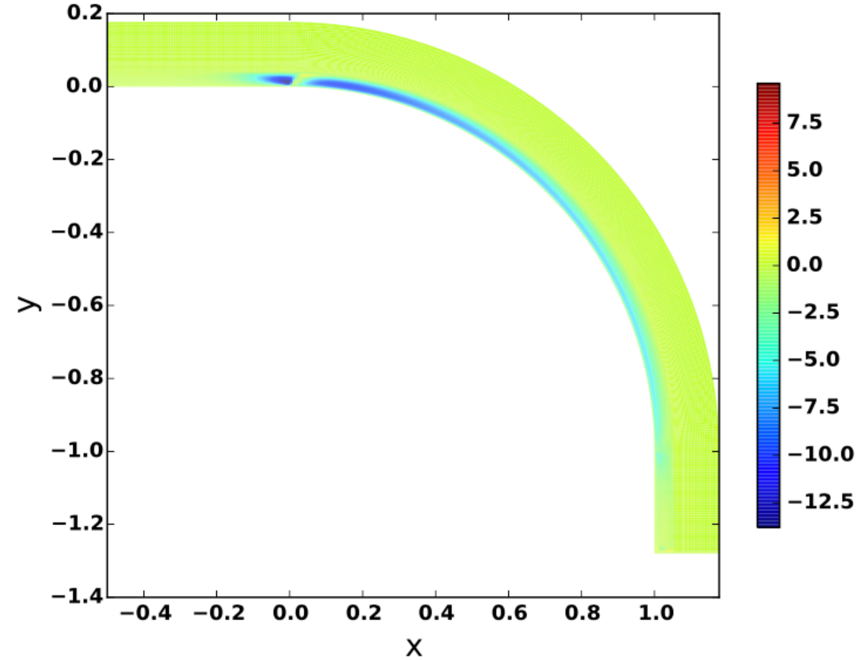
Quantitative information on model inadequacy









Modeler knows what is wrong, quantitatively

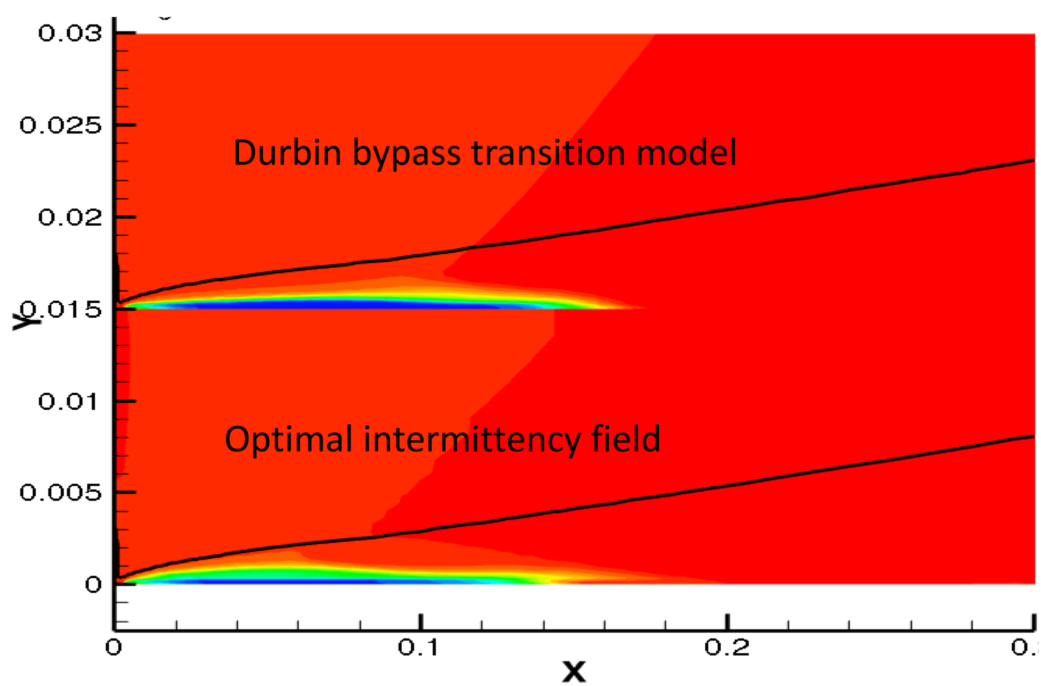
What does a sub-model require?

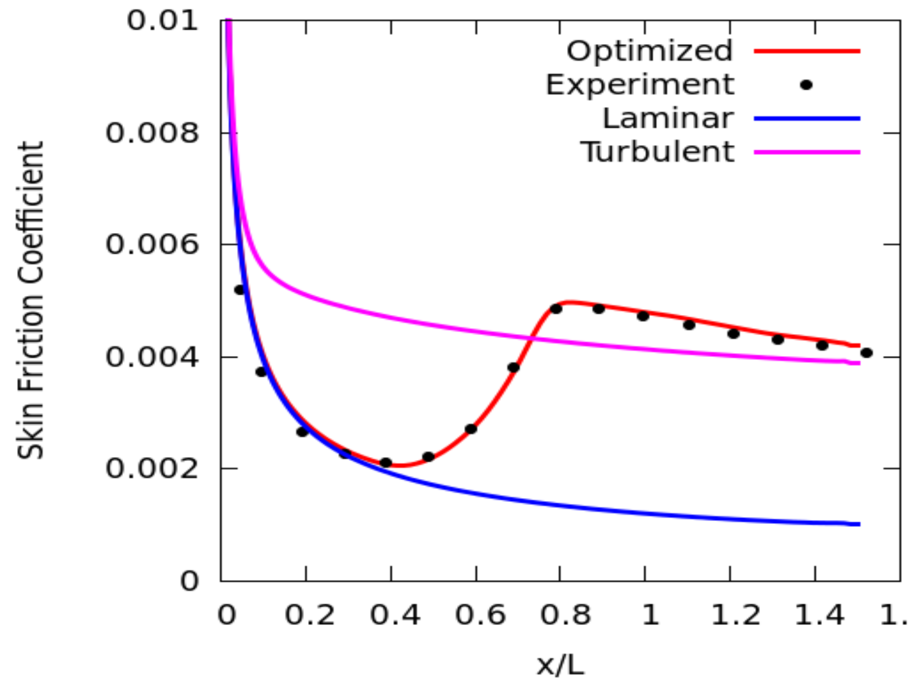
$$\frac{Dk}{Dt} = 2\nu_T|S|^2\gamma - C_\mu k\omega + \partial_j\left[\left(\nu + \frac{\nu_T}{\sigma_k}\right)\partial_j k\right]$$

$$\frac{D\omega}{Dt} = 2C_{\omega 1}|S|^2 - C_{\omega 2}\omega^2 + \partial_j\left[\left(\nu + \frac{\nu_T}{\sigma_\omega}\right)\partial_j \omega\right]$$

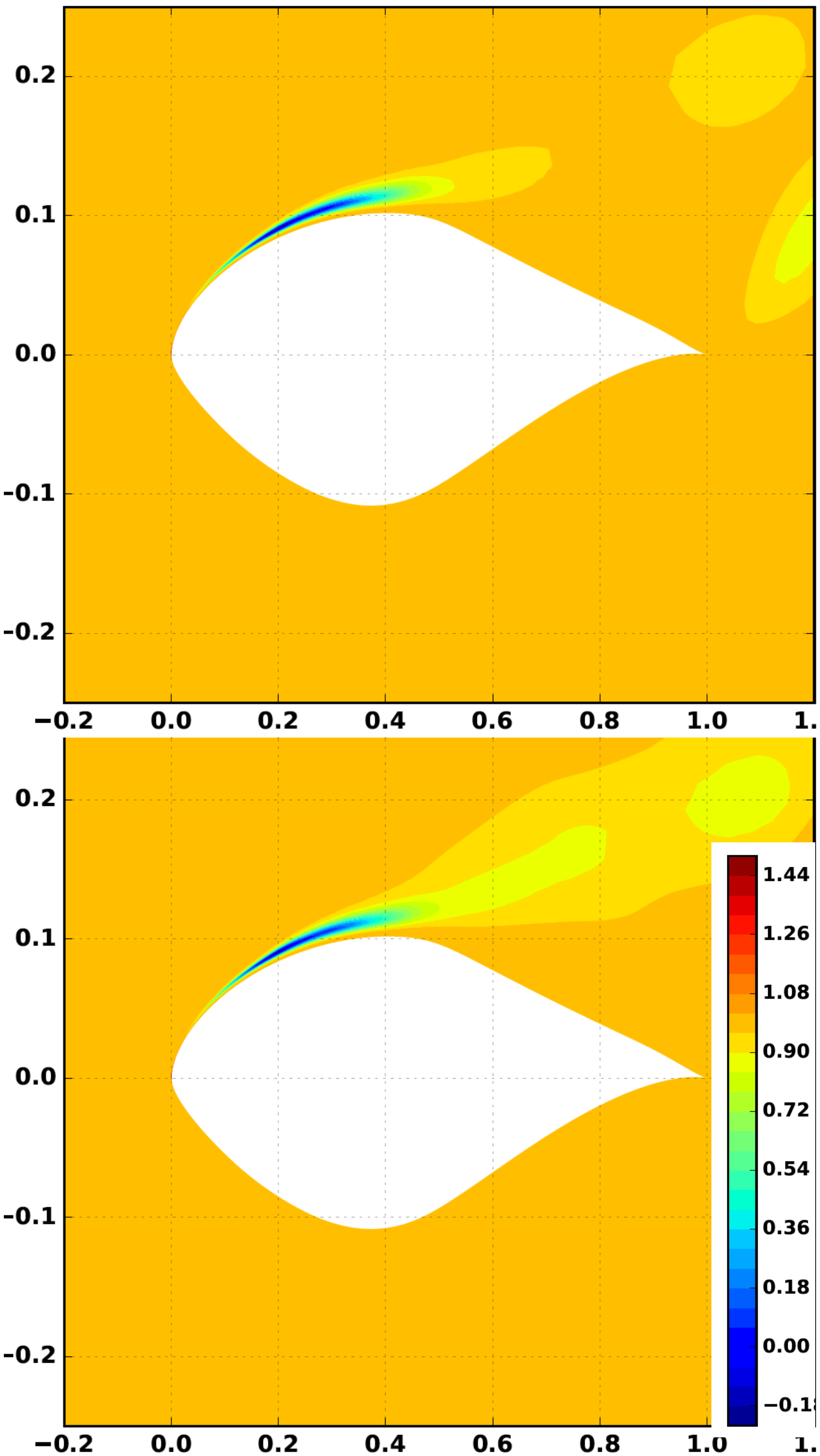
$$\frac{D\gamma}{Dt} = \partial_j\left[\left(\frac{\nu}{\sigma_l} + \frac{\nu_T}{\sigma_\gamma}\right)\partial_j \gamma\right] + P_\gamma - E_\gamma$$

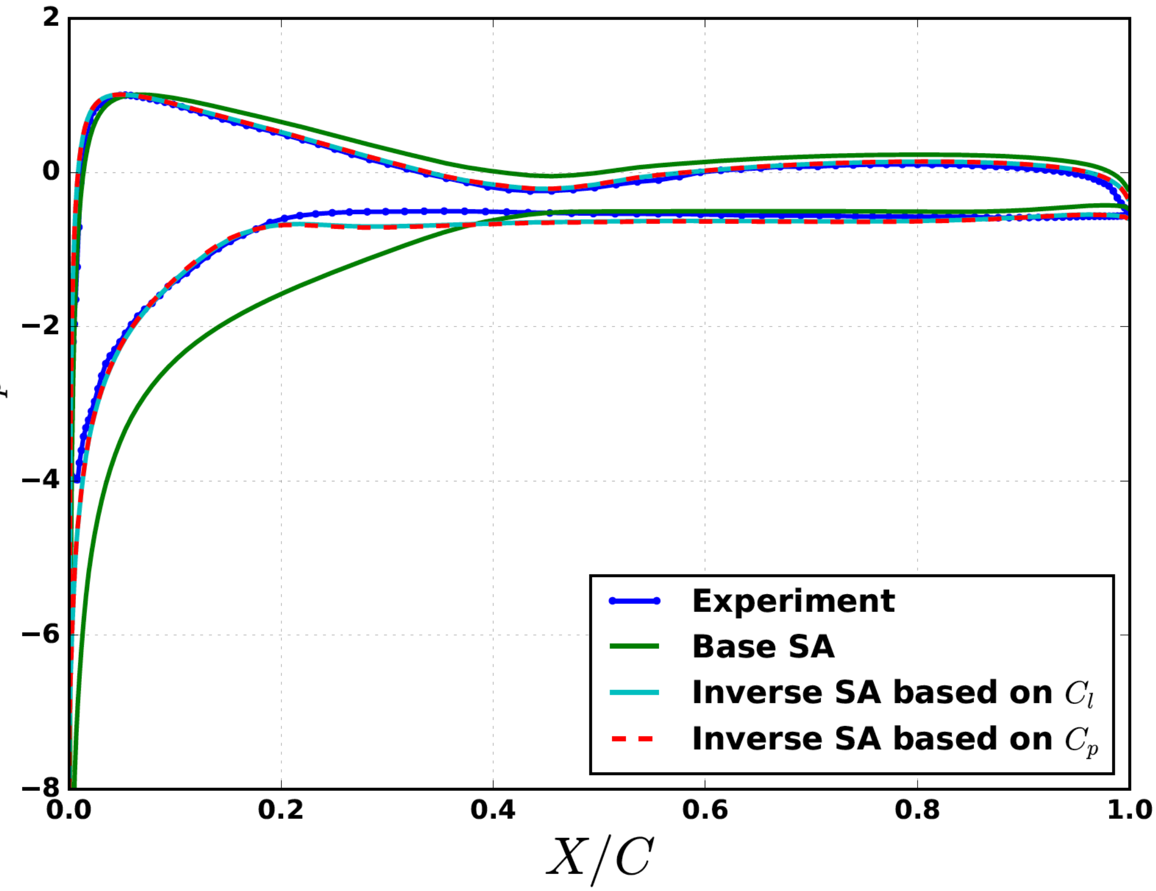
Optimal intermittency function
vs. model prediction





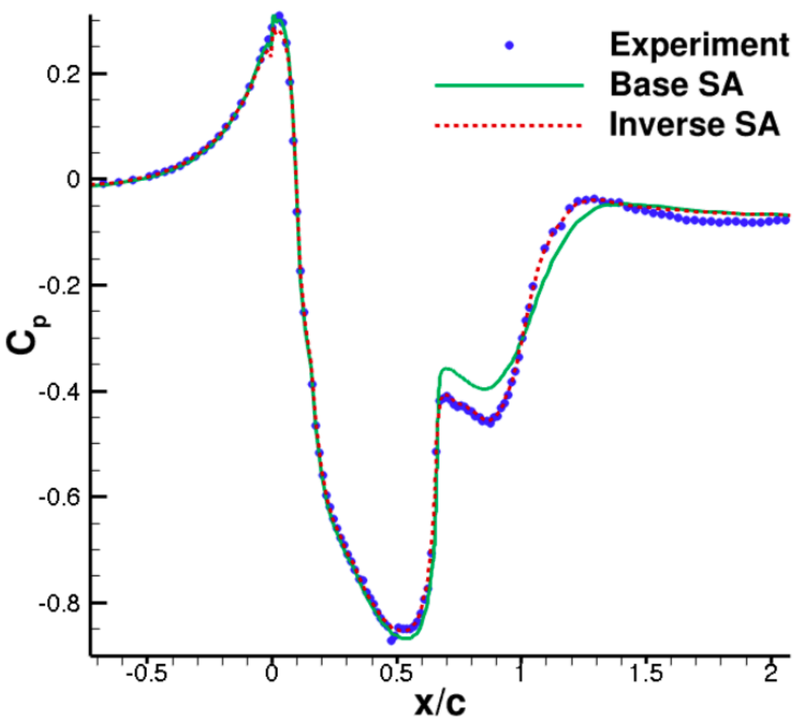
Improving the value of data!

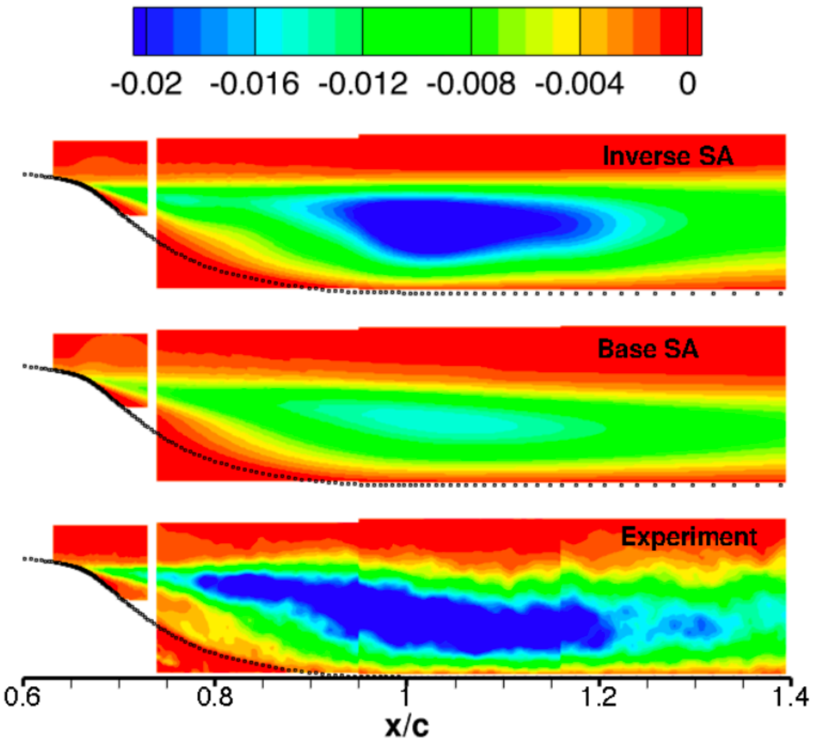




Ability to work on sparse amount of data is critical

Sanity check : Are we getting answers for the right reasons?

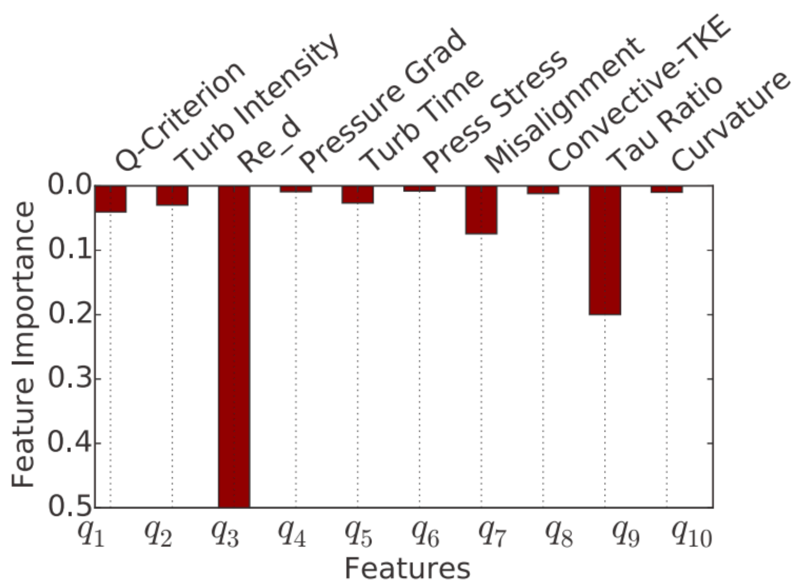




Use pressure data for inference, check if Reynolds stresses are correct

What are the right features?

Feature (q_β)	Description	Raw feature ($\hat{q}_\beta$)	Normalization factor (q^*_β)
q_1	Ratio of excess rotation rate to strain rate (Q criterion)	$\frac{1}{2}(\|\boldsymbol{\Omega}\|^2 - \|\mathbf{S}\|^2)$	$\|\mathbf{S}\|^2$
q_2	Turbulence intensity	k	$\frac{1}{2}U_i U_i$
q_3	Wall-distance based Reynolds number	$\min\left(\frac{\sqrt{k}d}{50\nu}, 2\right)$	not applicable[a]
q_4	Pressure gradient along streamline	$U_k \frac{\partial P}{\partial x_k}$	$\sqrt{\frac{\partial P}{\partial x_j}\frac{\partial P}{\partial x_j} U_i U_i}$
q_5	Ratio of turbulent time scale to mean strain time scale	$\frac{k}{\varepsilon}$	$\frac{1}{\|\mathbf{S}\|}$
q_6	Cratio of pressure normal stresses to shear stresses	$\sqrt{\frac{\partial P}{\partial x_i}\frac{\partial P}{\partial x_i}}$	$\frac{1}{2}\rho \frac{\partial U_k^2}{\partial x_k}$
q_7	Nonorthogonality between velocity and its gradient [28]	$\lvert U_i U_j \frac{\partial U_i}{\partial x_j}\rvert$	$\sqrt{U_l U_l \, U_i \frac{\partial U_i}{\partial x_j} U_k \frac{\partial U_k}{\partial x_j}}$
q_8	Ratio of convection to production of TKE	$U_i \frac{dk}{dx_i}$	$\lvert \overline{u'_j u'_k} S_{jk} \rvert$
q_9	Ratio of total to normal Reynolds stresses	$\|\overline{u'_i u'_j}\|$	k
q_{10}	Streamline curvature	$\lvert \frac{D\boldsymbol{\Gamma}}{Ds} \rvert$ where $\boldsymbol{\Gamma} \equiv \mathbf{U}/\lvert\mathbf{U}\rvert$, $Ds = \lvert\mathbf{U}\rvert Dt$	$\frac{1}{L_c}$



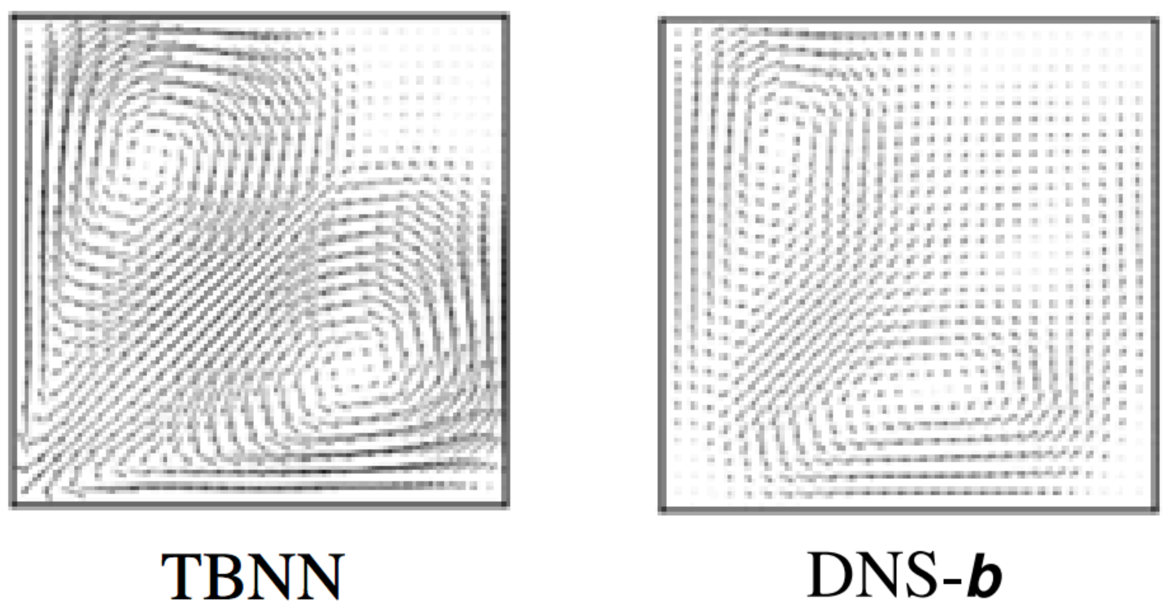
Wang, Wu, Xiao, PRF 2017.

Embedded invariance

$$\frac{\partial \bar{u}}{\partial t} + \mathcal{R}(\bar{u}) = \mathbf{M}(\bar{u}, \bar{v}, \delta(\bar{u}, \bar{v})) \quad ; \quad \delta(\bar{u}, \bar{v}) \equiv \delta(\bar{u}_{data}, \bar{v}_{data}$$

ML algorithm embeds rotational invariance by enforcing that the predicted anisotropy tensor lies on a basis of isotropic tensors.

Ling et al., JFM 2016.



It is really a set of ideas

$$\frac{D\omega}{Dt} = P_\omega - \beta(x) D_\omega + T_\omega$$

Singh & Duraisamy, PoF 2016

Parish & Duraisamy, Aviation 2014

$$\frac{DR_{ij}}{Dt} = C_{ij} + P_{ij} + T_{ij} + \Pi_{ij} + D_{ij} + \beta(x)_{ij} \epsilon_{ij}$$

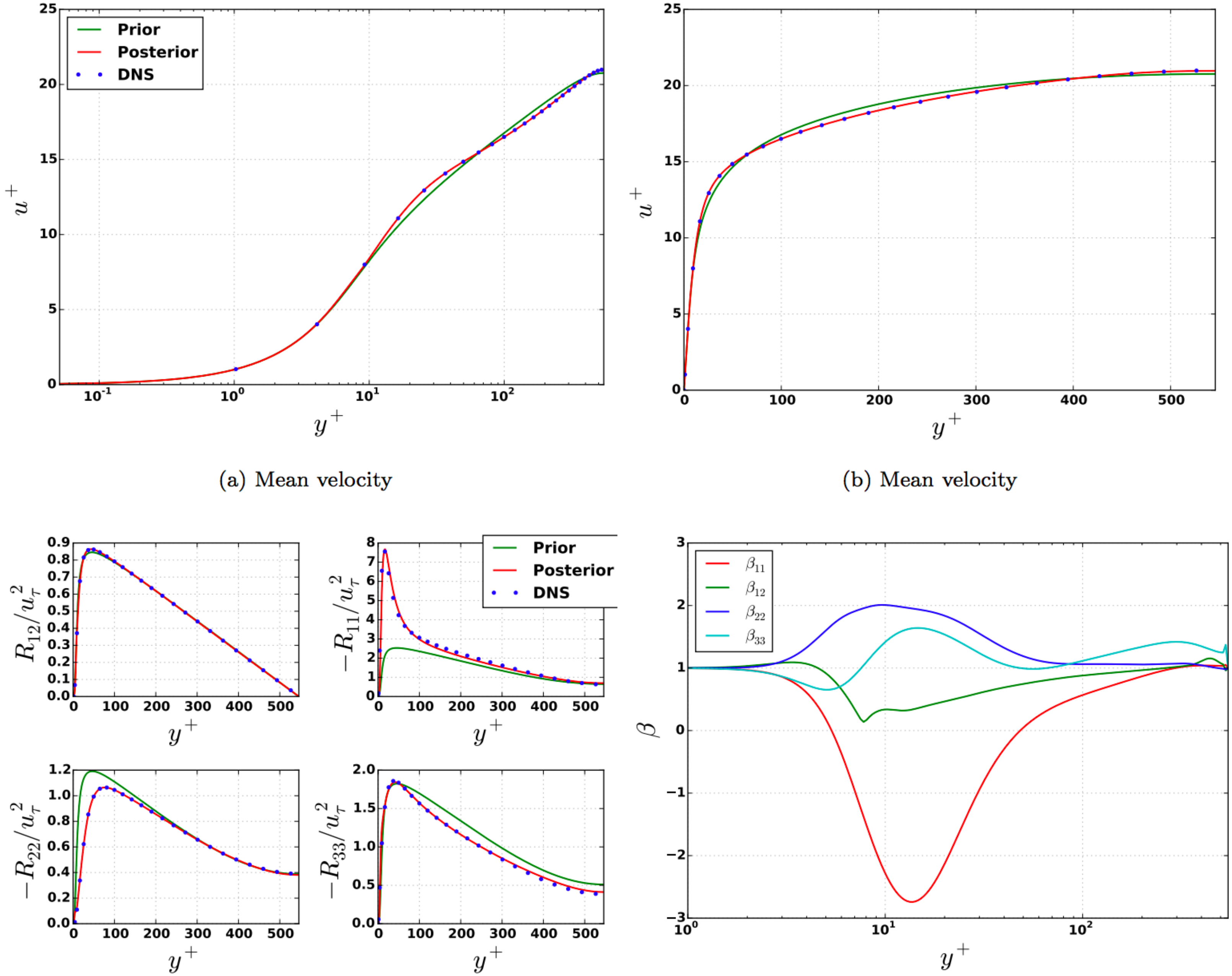
$$\frac{DR_{ij}}{Dt} = \beta(x)_{ij} a_o \omega (R_{ij,eq} - R_{ij})$$

Singh & Duraisamy, Scitech 2016

$$\mathbf{R}_p = 2k \left[\frac{\mathbf{I}}{3} + \mathbf{V} (\Lambda + \vec{\beta}(x)) \mathbf{V}^T \right]$$

Duraisamy, SIAM 2016

Application to RSMs



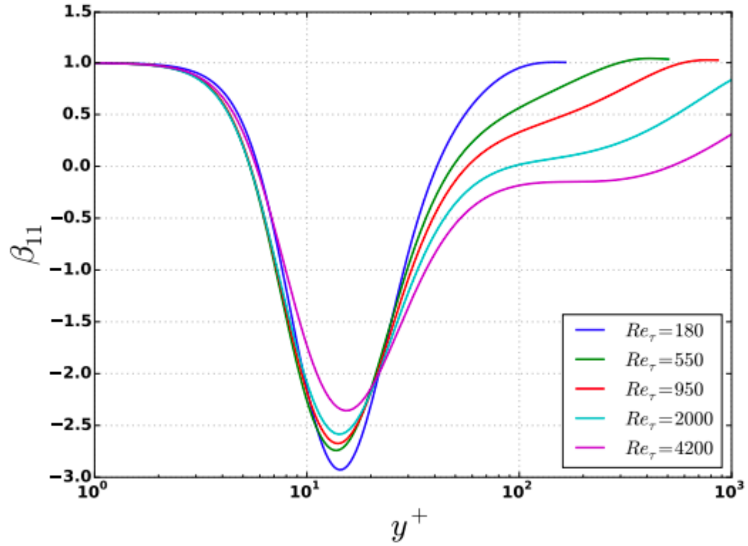
(a) Mean velocity

(b) Mean velocity

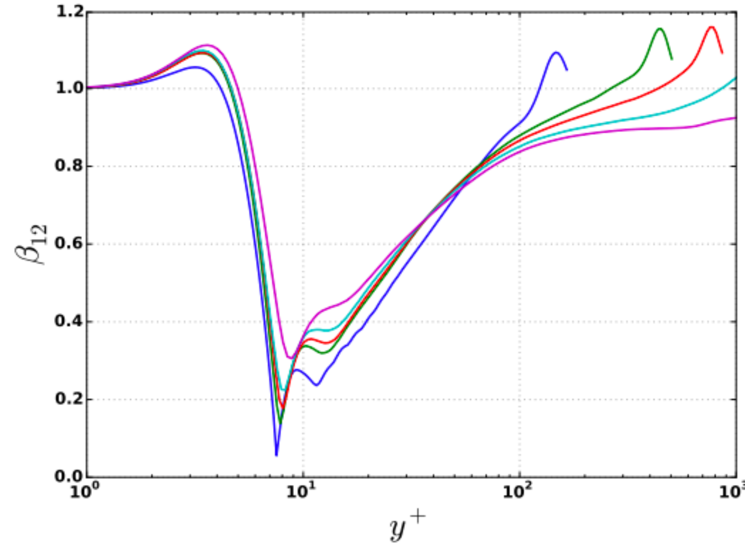
(c) Reynolds Stresses

(d) Multiplier function $\boldsymbol{\beta}_{ij}$

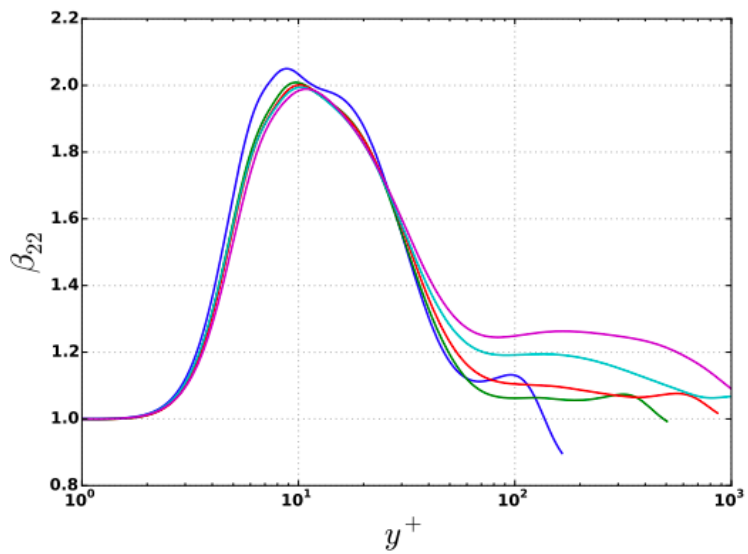
Application to RSMs



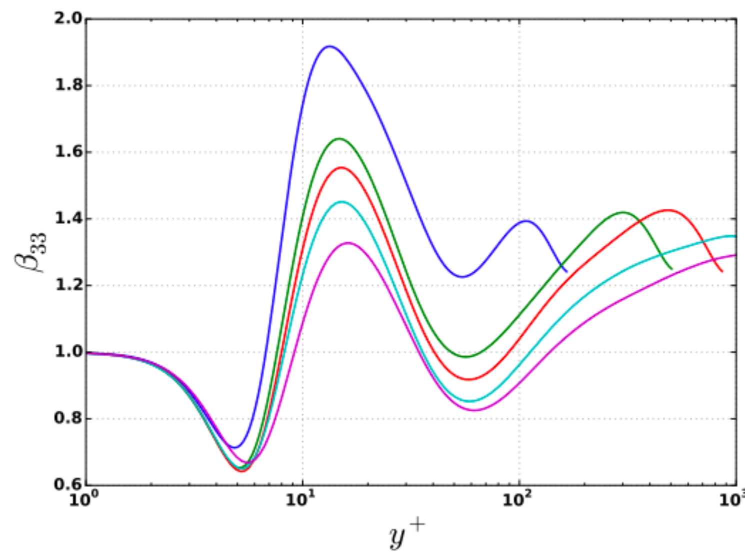
(a) $\boldsymbol{\beta}_{11}$



(b) $\boldsymbol{\beta}_{12}$



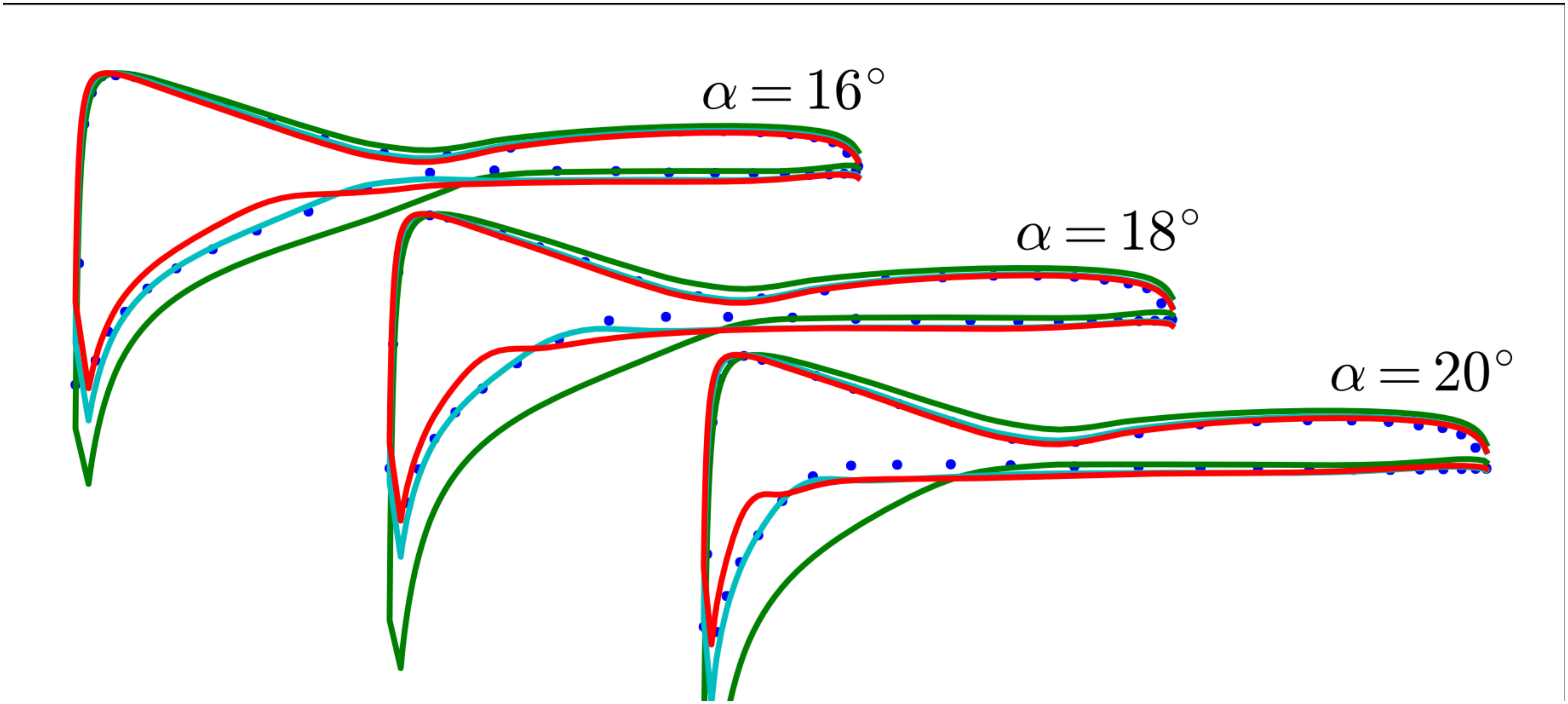
(c) $\boldsymbol{\beta}_{22}$



(d) $\boldsymbol{\beta}_{33}$

True prediction !

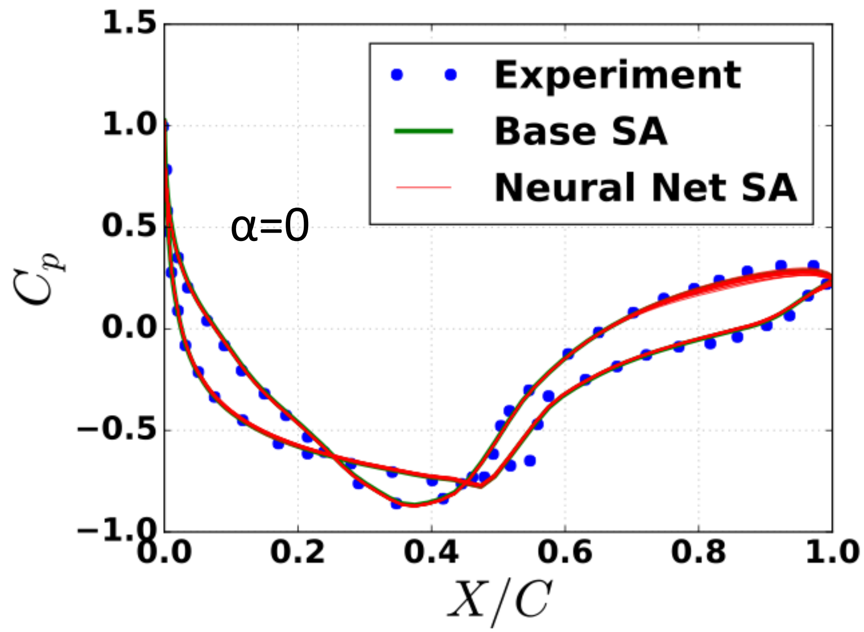
S 809, Re=2 Million

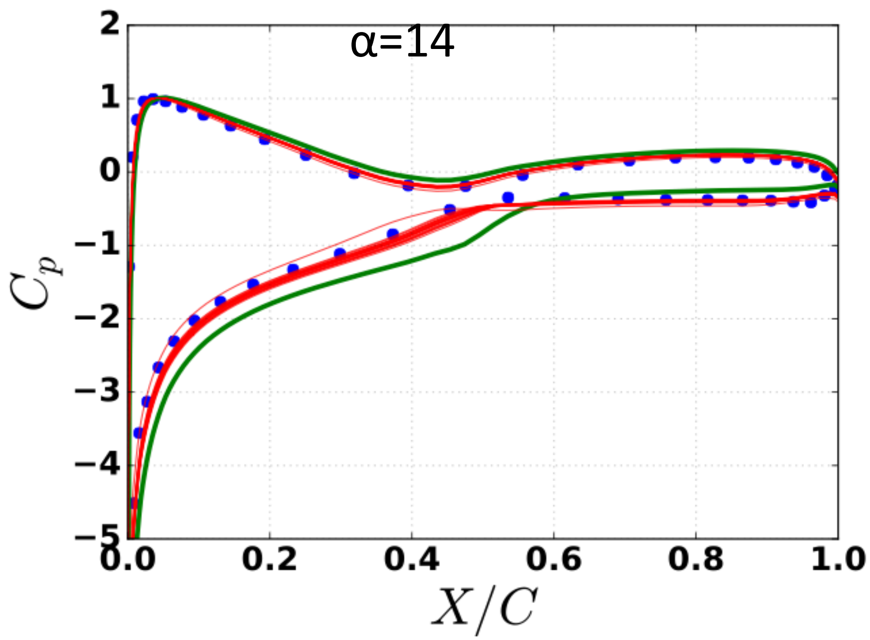


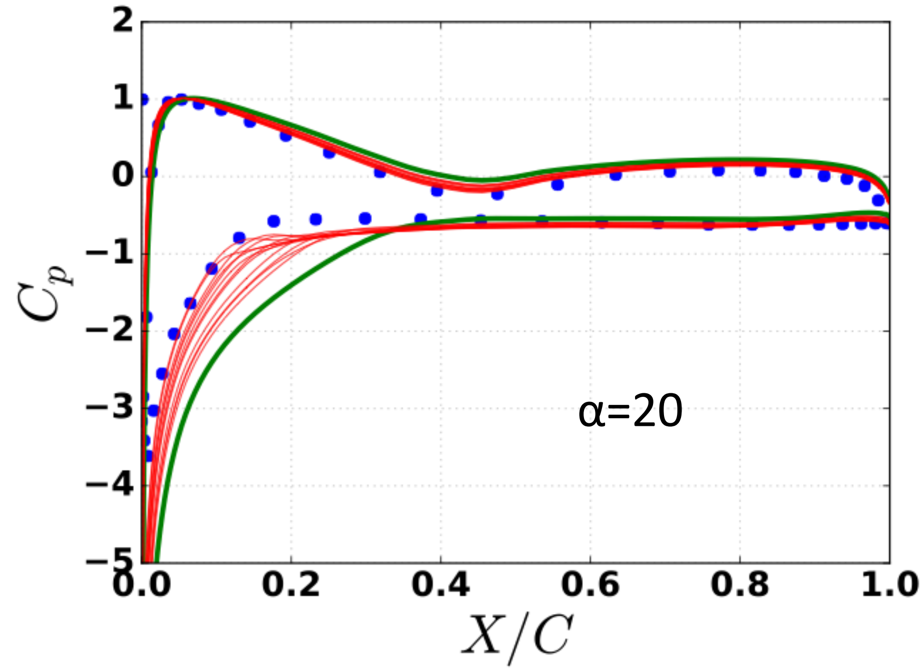
Inference used only CL data, NN-augmented model provides considerable predictive improvements of Cp

Variability

S 809, Re=2 Million

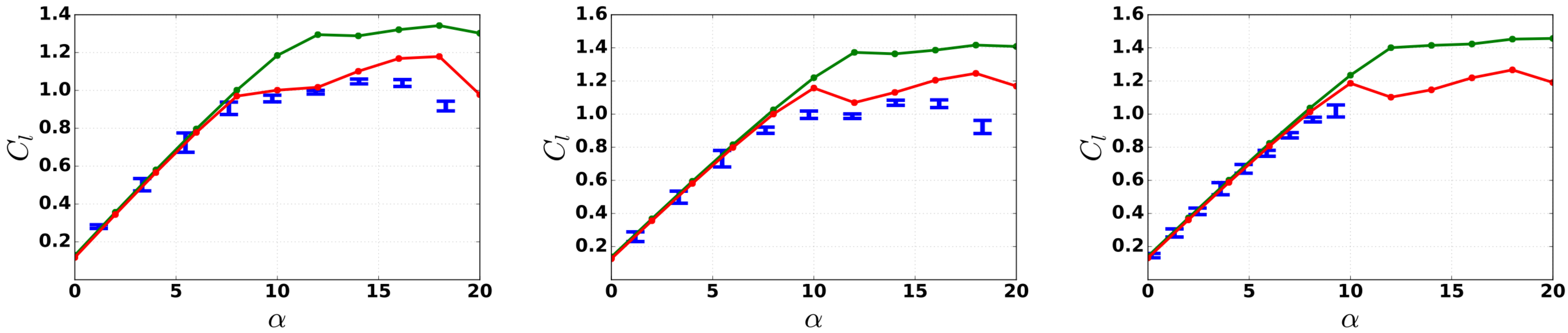




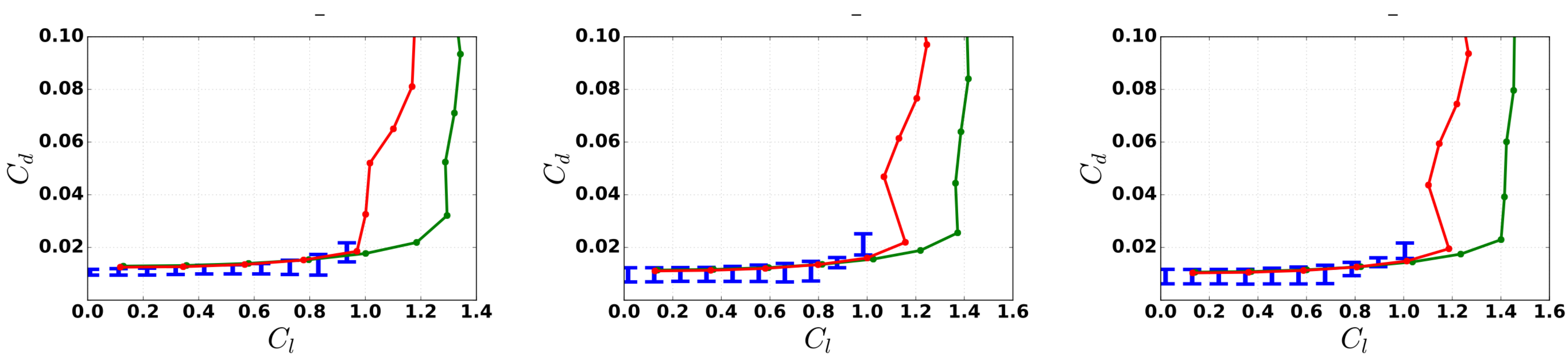


Training from different sets

Portability :
Implementation in AcuSolve



(a) $Re = 1 \times 10^6$ (b) $Re = 2 \times 10^6$ (c) $Re = 3 \times 10^6$



S809 Airfoil : Predictive results in Commercial CFD solver

Growing community for data-driven turbulence modeling

2011: Cheung, Moser, et al (parametric UQ)
2012: Dow & Wang (non-parametric UQ)
2013: Tracey, Duraisamy, Alonso (ML for non-parametric UQ)
2014: Duraisamy et. (Inversion + ML for model improvement)
2015: Ling & Templeton, Weatheritt & Sandberg (apriori ML)
2016: Xiao et al. (ML for model improvement)
2017: Mishra, Iaccarino, Edeling (physics, data-based)

Also, Dwight, Cinella, Arunjatesan et al.,
Companies: Altair, Inc. ; UTRC; xxxx; yyyy

Perspectives 1/2

- “Kitty hawk” state, much work remains
- Be careful about what data we can use for calibration and how we can use it
- Machine learning
  - ➔ Can function as indicator
  - ➔ Is an optional step
  - ➔ Can be fed by theory and asympotics
- If there is an underlying “exact” model, we can discover it
- There is no (and will ever be a) universally accurate model waiting to be discovered
  - ➔ Optimal model, conditional on data and assumptions possible
  - ➔ Avoid tendency to overfit
  - ➔ Small number of sensible features (Galilean invariant)
  - ➔ Absolutely the most sensible thing to do in an industrial setting

(Lots of data for a class of problems, Lots of expertise/knowhow)

Perspectives 2/2

- Modeling has ALWAYS been data-driven & we have always been using machine learning (and inversion too)
- Data-driven approach is not a substitute to turbulence modeling
- Data-driven approach is not a new way of modeling. It is a new tool.
  - Uses (other than prediction):
    - ➔ Model credibility: Can validate/invalidate model structures
    - ➔ Uncertainty quantification: Can obtain modeling error bounds
    - ➔ Robust design
    - ➔ Feature selection
    - ➔ Input for modeler (forget machine learning)

Vision for the future

A continuously augmented curated database / website of inferred corrections that are input to the machine learning process

Users upload/download/process data, generate maps.

Turbulence Modeling Gateway

43 Registered users

Home | Team | Research | Publications | Symposium 2017 | Support

Welcome to the Turbulence Modeling Gateway Server. The goal of our project is to develop new techniques for turbulence modeling. We are exploring a range of techniques including data-driven techniques, advanced structure based modeling and hybrid RANS-LES methods from a predictive modeling as well as an uncertainty quantification context. We treat all these techniques as natural allies in the broad goal of turbulence model improvement.

Currently, the prime focus of our efforts is on the development of the science behind data driven turbulence modeling and demonstrate the utility of large-scale data-driven techniques in turbulence modeling. Our work involves the development of domain-specific learning techniques suited for the representation of turbulence and its modeling, the establishment of a trusted ensemble of data for the creation and validation of new models, and the deployment of these models in complex aerospace problems.
We are grateful to the following agencies for funding:

- NASA : RCA (2011-2014) & LEARN (2014-2017)
- NSF : CDESE (2015-2018)
- DARPA : EQUiPS (2015-2018)
- ONR : Wall Turbulence BRC (2017-2021)

We have several collaborators at the University of Michigan, Stanford University, and Iowa State University. We also consult with Boeing Commerical Airplanes and interact with NASA Langley Research Center.

We will highlight our research on this website, will maintain a wiki and we hope to make this a portal which users can upload/download/process data and turbulence models. You can register using the bar on the right.

Email

Enter email

Password

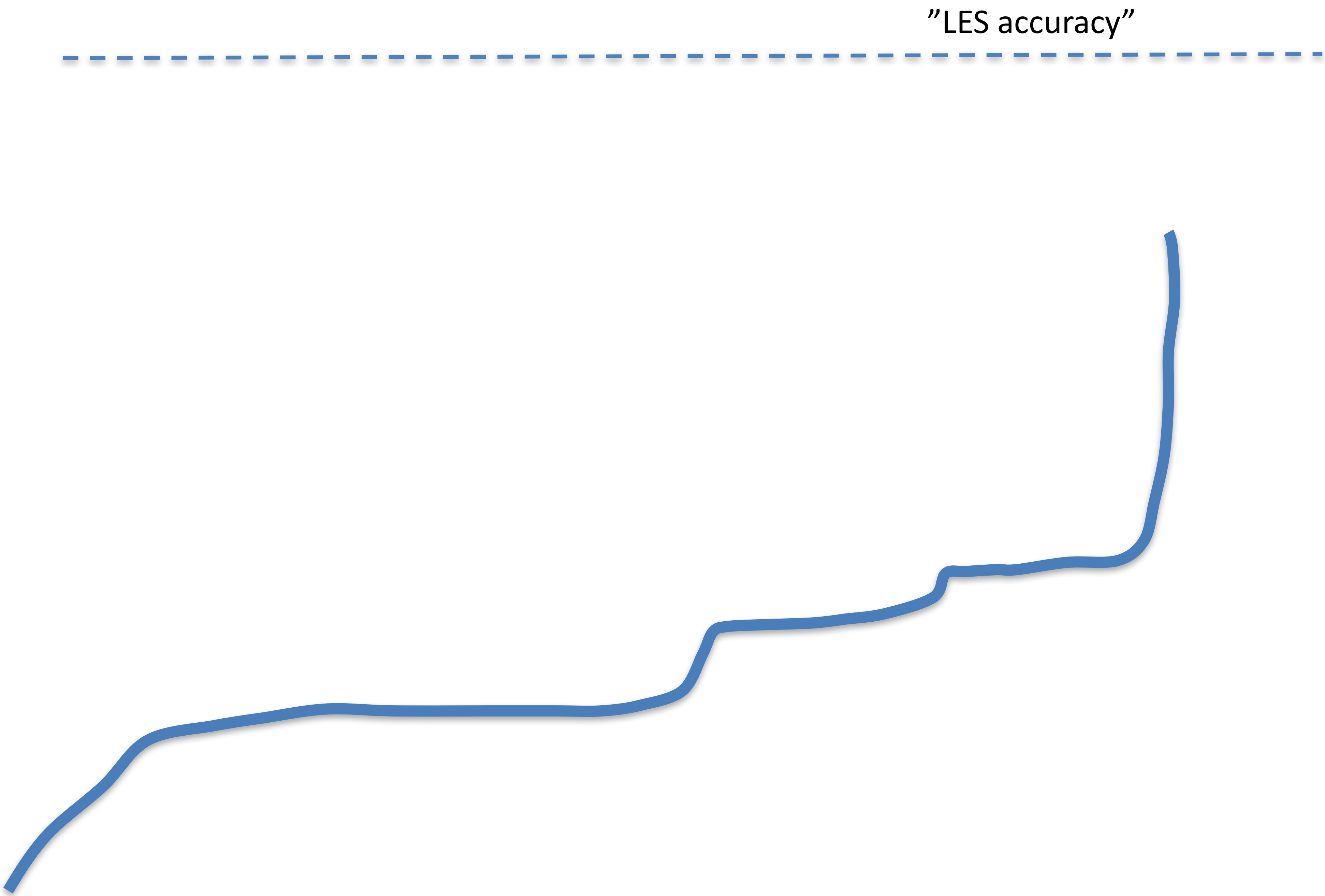
Password

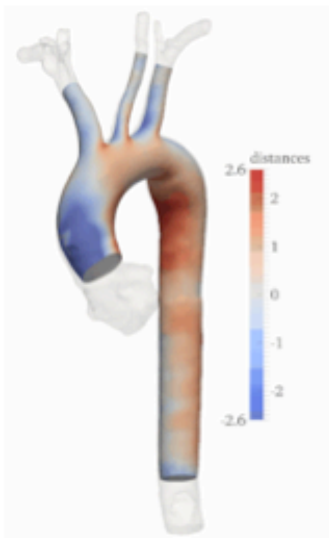
Login

Not a member? Sign up
Forgot password? Click here

Links

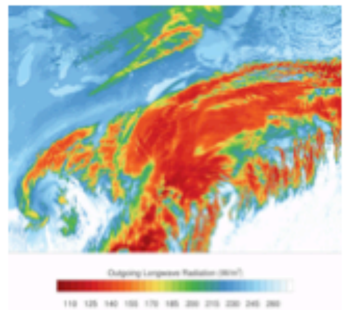
- NASA Langley's Turbulence Modeling Resource page
- Johns Hopkins Turbulence Database
- Universidad Politecnica de Madrid Database


”LES accuracy”



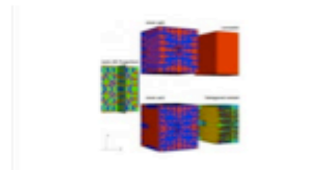
Subject-Specific Blood Flow Modeling

Subject specific blood flow modeling has developed notably over the last decades. From the pioneering academic work in the 1990s to today's landscape in which computer-based simulations to assess the severity of coronary disease have been recently approved by the FDA, the progress



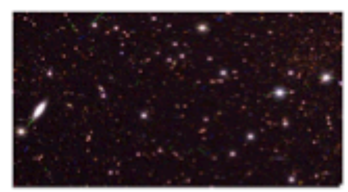
Climate Systems Interaction

The Earth's climate system is composed of multiple interacting components that span spatial scales of 13 orders of magnitude and temporal scales that range from microseconds to centuries. Decades of research and development have produced a global multivariate observing system, global numerical process models, sophisticated model-data fusion algorithms, and ever-increasing computational capacity. While the physics that underlie climate system interactions are now well understood, the key responses and feedbacks in the system are controlled by



COMPUTATIONAL MATERIALS PHYSICS

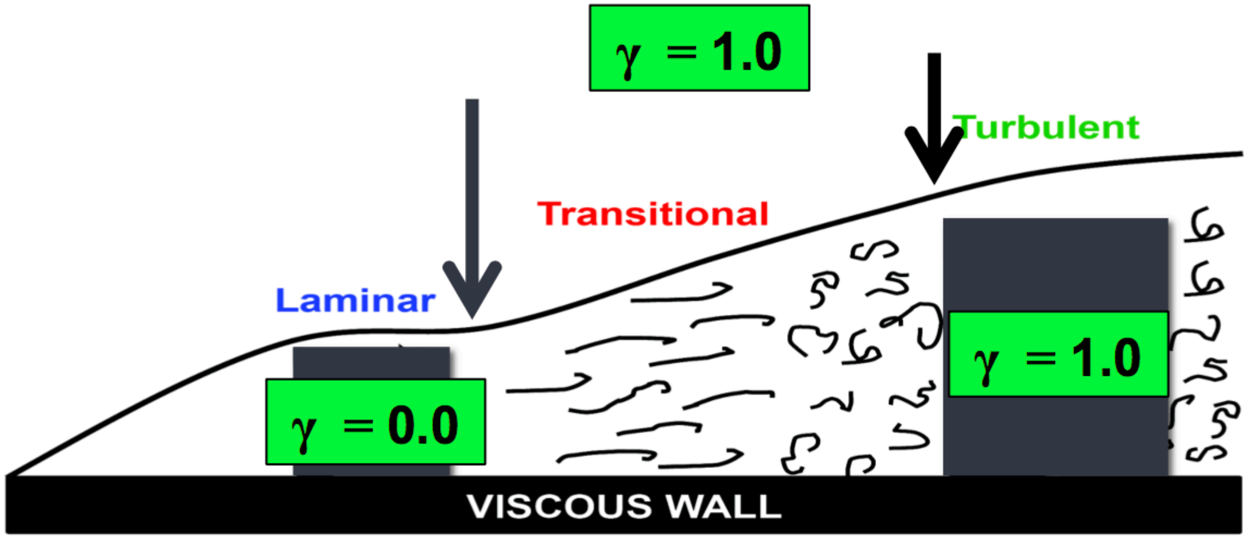
Computational materials physics aims to model the behavior of metallic alloys, polymers, biological and biologically inspired materials, semiconductors, glasses, battery materials and many other categories of substances (materials). The goal is to identify, explain, predict and ultimately to design the properties and responses of these materials.



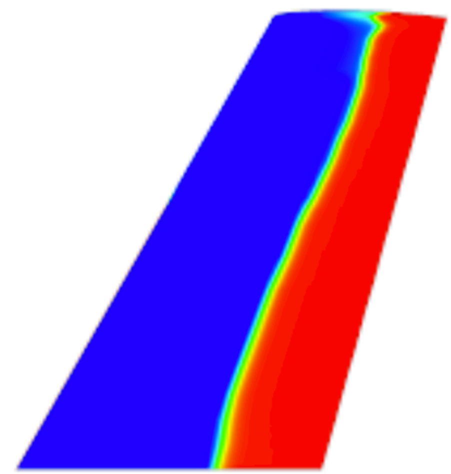
Cosmology

Backups

γ-Re_θ Transition Model



ONERA M-6 WING



Blue: Laminar

Red: Turbulent

$$\gamma \Rightarrow \text{Transition Model} \Rightarrow \frac{D\gamma}{Dt} = P_\gamma + D_\gamma + M_\gamma$$

$$\mu_t \Rightarrow \text{Modified S-A Model} \Rightarrow \frac{D\mu_t}{Dt} = \gamma P_\mu + D_\mu + M_\mu$$

- Predicts natural and bypass transition
- Intermittency $\gamma = 0$ in laminar, 1 in turbulent BL
- Use γ to selectively turn on/off turbulence production
- Solves additional equation for Re_θ and uses experimental correlations

Intermittency

$$\frac{\partial(\rho\gamma)}{\partial t} + \frac{\partial(\rho U_j \gamma)}{\partial x_j} = P_\gamma - E_\gamma + \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_f}\right)\frac{\partial \gamma}{\partial x_j}\right]$$

$$P_{\gamma 1} = F_{\text{length}} c_{a1} \rho S [\gamma F_{\text{onset}}]^{0.5} (1 - c_{e1}\gamma)$$

$$E_\gamma = c_{a2} \rho \Omega \gamma F_{\text{turb}} (c_{e2}\gamma - 1)$$

$$Re_V = \frac{\rho y^2 S}{\mu} \qquad R_T = \frac{\rho k}{\mu \omega}$$

$$F_{\text{sublayer}} = e^{-\left(\frac{R_\omega}{0.4}\right)^2}$$

$$F_{\text{onset1}} = \frac{Re_v}{2.193 \cdot Re_{\theta c}}$$

$$R_\omega = \frac{\rho y^2 \omega}{500\mu}$$

$$F_{\text{length}} = F_{\text{length}}(1 - F_{\text{sublayer}}) + 40.0 \cdot F_{\text{sublayer}}$$

$$F_{\text{onset2}} = \min(\max(F_{\text{onset1}}, F_{\text{onset1}}^4), 2.0)$$

$$F_{\text{onset}} = \max(F_{\text{onset2}} - F_{\text{onset3}}, 0)$$

$$F_{\text{onset3}} = \max\left(1 - \left(\frac{R_T}{2.5}\right)^3, 0\right)$$

$$F_{\text{length}} = \begin{cases} [398.189 \cdot 10^{-1} + (-119.270 \cdot 10^{-4})\tilde{R}e_{\theta t} + (-132.567 \cdot 10^{-6})\tilde{R}e_{\theta t}^2], & \tilde{R}e_{\theta t} < 400 \\ [263.404 + (-123.939 \cdot 10^{-2})\tilde{R}e_{\theta t} + (194.548 \cdot 10^{-5})\tilde{R}e_{\theta t}^2 + (-101.695 \cdot 10^{-8})\tilde{R}e_{\theta t}^3], & 400 \le \tilde{R}e_{\theta t} < 596 \\ [0.5 - (\tilde{R}e_{\theta t} - 596.0) \cdot 3.0 \cdot 10^{-4}], & 596 \le \tilde{R}e_{\theta t} < 1200 \\ [0.3188], & 1200 \le \tilde{R}e_{\theta t} \end{cases}$$

$$Re_{\theta c} = \begin{cases} [\tilde{R}e_{\theta t} - (396.035 \cdot 10^{-2} + (-120.656 \cdot 10^{-4})\tilde{R}e_{\theta t} + (868.230 \cdot 10^{-6})\tilde{R}e_{\theta t}^2 \\ \quad + (-696.506 \cdot 10^{-9})\tilde{R}e_{\theta t}^3 + (174.105 \cdot 10^{-12})\tilde{R}e_{\theta t}^4)], & \tilde{R}e_{\theta t} \le 1870 \\ [\tilde{R}e_{\theta t} - (593.11 + (\tilde{R}e_{\theta t} - 1870.0) \cdot 0.482)], & \tilde{R}e_{\theta t} > 1870 \end{cases}$$

Re_θ

$$\frac{\partial(\rho \tilde{R}e_{\theta t})}{\partial t} + \frac{\partial(\rho U_j \tilde{R}e_{\theta t})}{\partial x_j} = P_{\theta t} + \frac{\partial}{\partial x_j}\left[\sigma_{\theta t}(\mu + \mu_t)\frac{\partial \tilde{R}e_{\theta t}}{\partial x_j}\right]$$

$$P_{\theta t} = c_{\theta t}\frac{\rho}{t}(Re_{\theta t} - \tilde{R}e_{\theta t})(1.0 - F_{\theta t}) \qquad t = \frac{500\mu}{\rho U^2}$$

$$F_{\theta t} = \min\left(\max\left(F_{\text{wake}} \cdot e^{-(\frac{y}{\delta})^4}, 1.0 - \left(\frac{\gamma - 1/c_{e2}}{1.0 - 1/c_{e2}}\right)^2\right), 1.0\right)$$

$$\theta_{\text{BL}} = \frac{\tilde{R}e_{\theta t}\mu}{\rho U}; \qquad \delta_{\text{BL}} = \frac{15}{2}\theta_{\text{BL}}; \qquad \delta = \frac{50\Omega y}{U} \cdot \delta_{\text{BL}}$$

$$Re_\omega = \frac{\rho\omega y^2}{\mu}; \qquad F_{\text{wake}} = e^{-(\frac{Re_\omega}{1E+5})^2} \quad \lambda_\theta = \frac{\rho\theta^2}{\mu}\frac{dU}{ds}$$

$$Re_{\theta t} = \left[1173.51 - 589.428Tu + \frac{0.2196}{Tu^2}\right]F(\lambda_\theta), \quad Tu \le 1.3$$

$$Re_{\theta t} = 331.50[Tu - 0.5658]^{-0.671}F(\lambda_\theta), \qquad Tu > 1.3$$

$$F(\lambda_\theta) = 1 - [-12.986\lambda_\theta - 123.66\lambda_\theta^2 - 405.689\lambda_\theta^3]e^{-[\frac{Tu}{1.5}]^{1.5}}, \quad \lambda_\theta \le 0$$

$$F(\lambda_\theta) = 1 + 0.275[1 - e^{[-35.0\lambda_\theta]}]e^{[\frac{-Tu}{0.5}]}, \qquad \lambda_\theta > 0$$

$$-0.1 \le \lambda_\theta \le 0.1 \qquad Tu \ge 0.027 \qquad Re_{\theta t} \ge 20$$

$$\gamma_{sep} = \min\left(s_1 \max\left[0, \left(\frac{Re_v}{3.235 Re_{\theta c}}\right) - 1\right] F_{reattach}, 2\right) F_{\theta t}$$

$$F_{reattach} = e^{-\left(\frac{R_T}{20}\right)^4}$$

$$\gamma_{eff} = \max(\gamma, \gamma_{sep})$$

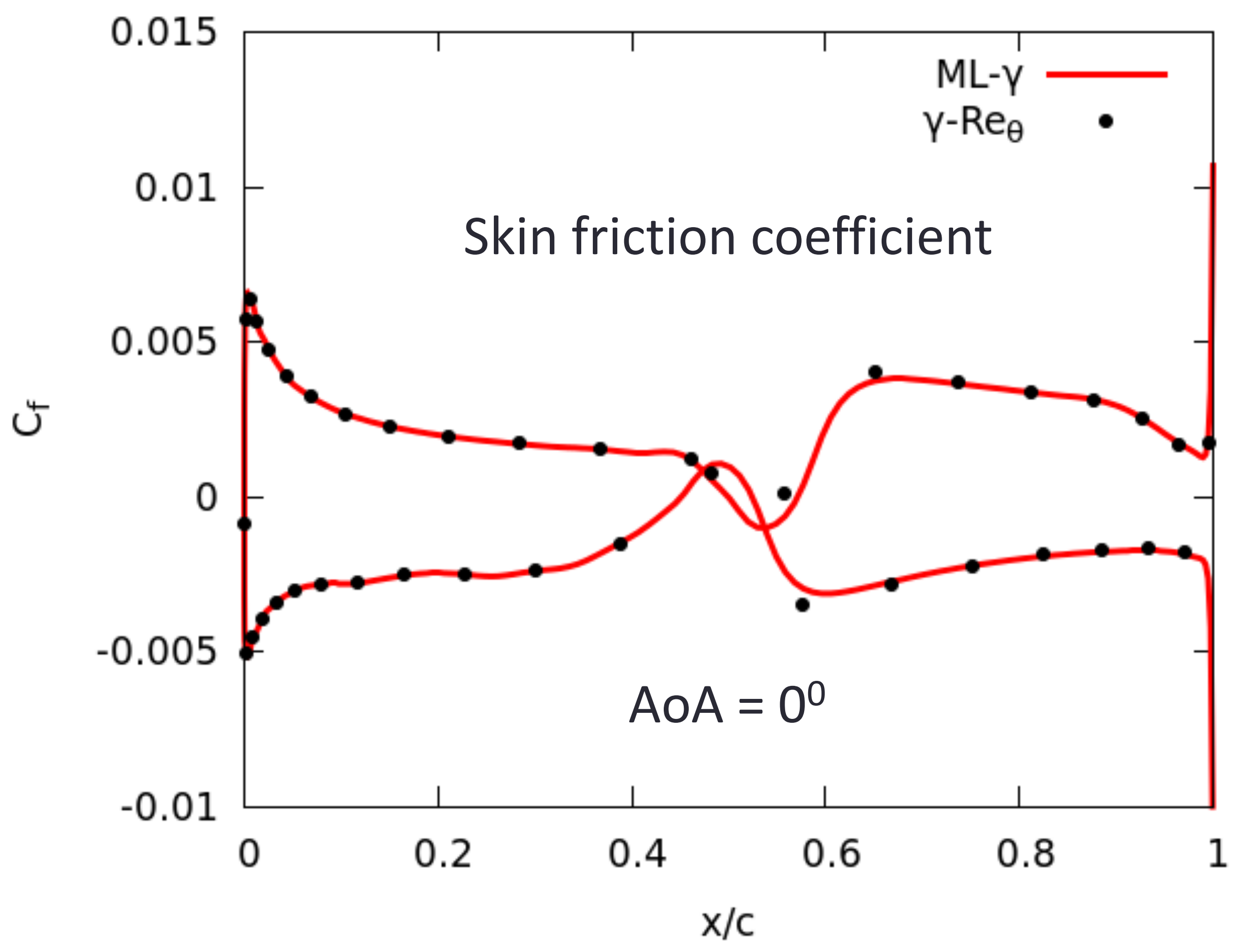
$$c_{e1} = 1.0; \quad c_{a1} = 2.0 \quad c_{e2} = 50; \quad c_{a2} = 0.06; \quad \sigma_f = 1.0$$

$$c_{\theta t} = 0.03; \quad \sigma_{\theta t} = 2.0 \quad s_1 = 2$$

All that effort to compute intermittency source terms and predict natural and bypass transition

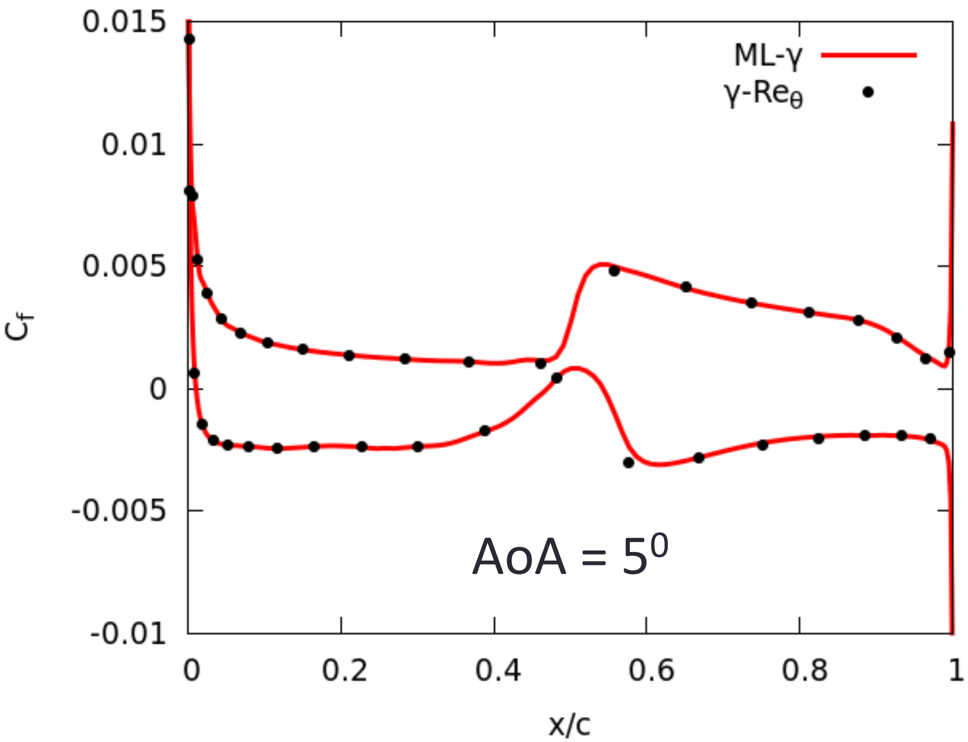
$$\gamma \Rightarrow \text{Transition Model} \Rightarrow \frac{D\gamma}{Dt} = P_\gamma + D_\gamma + M_\gamma$$

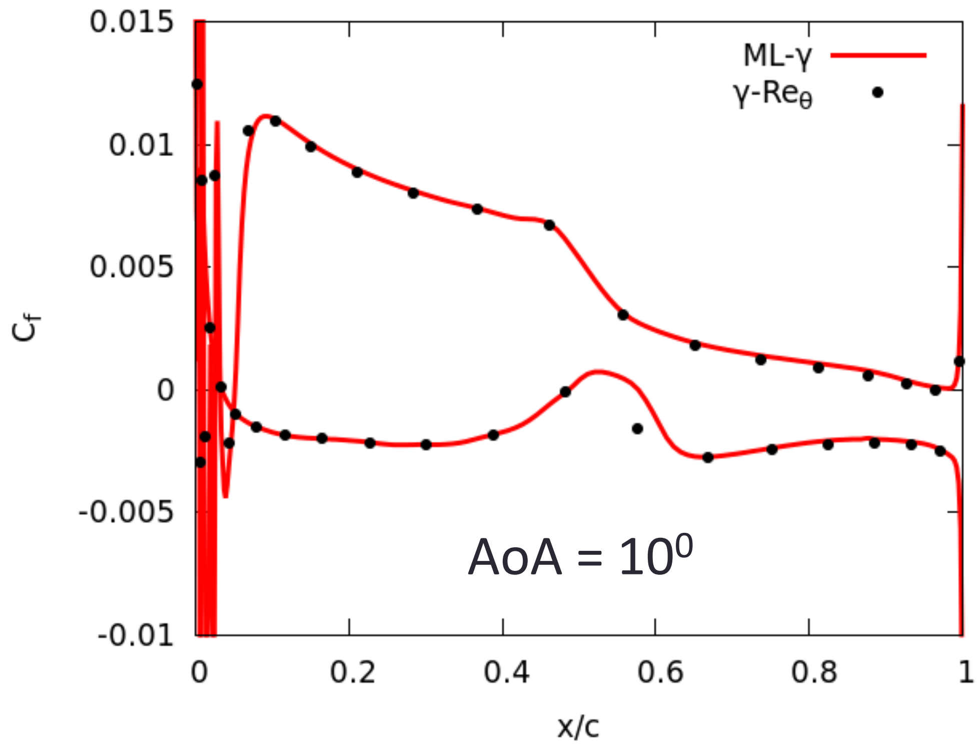
Preliminary Results – S809 Airfoil, Re = 2 x 10^6



- Excellent agreement between ML-γ and γ-Re$_\theta$ models
  - Sanity check – ANN tested on same data as on which it was trained

Preliminary Results – S809 Airfoil, Re = 2 x 10^6





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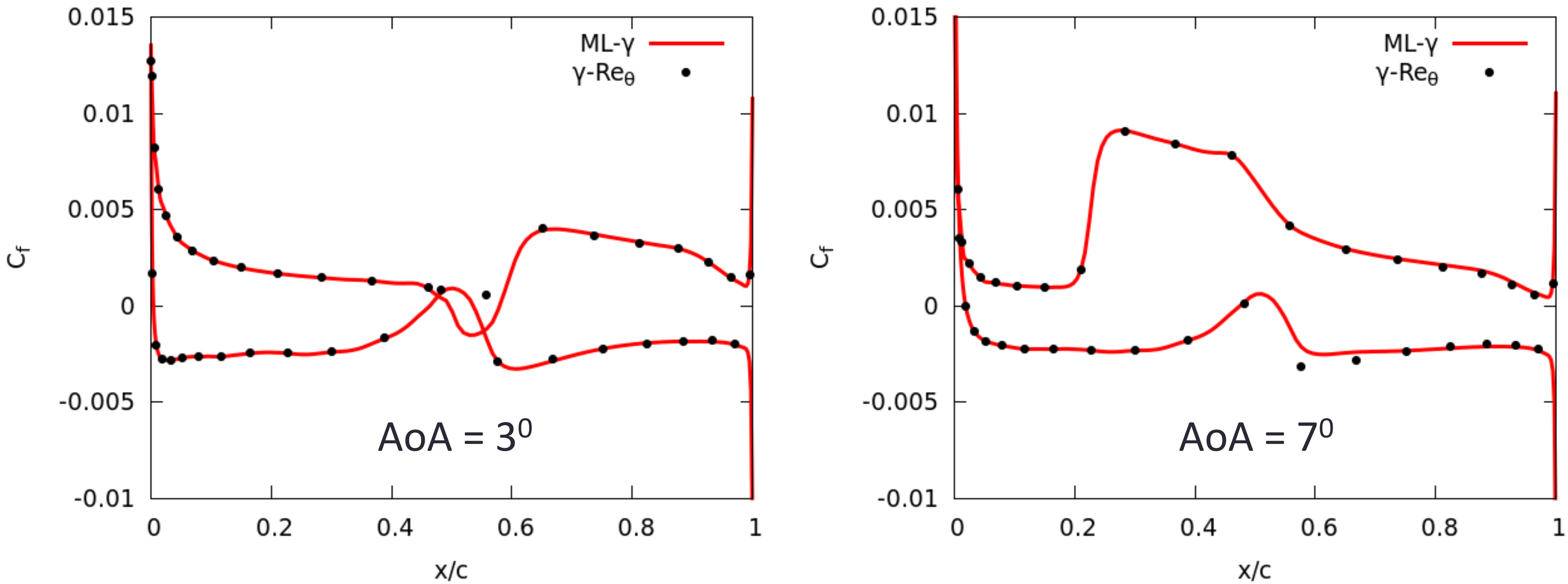
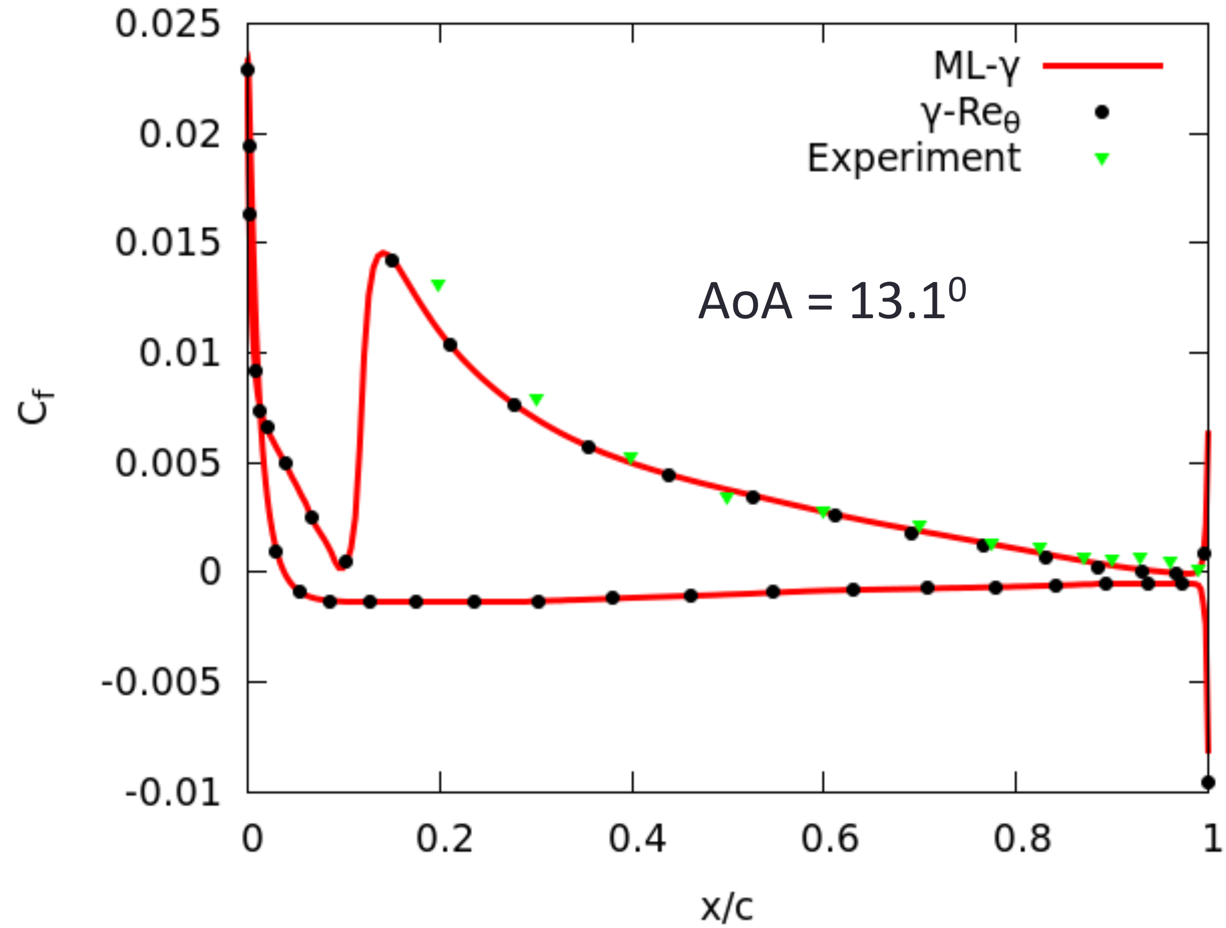


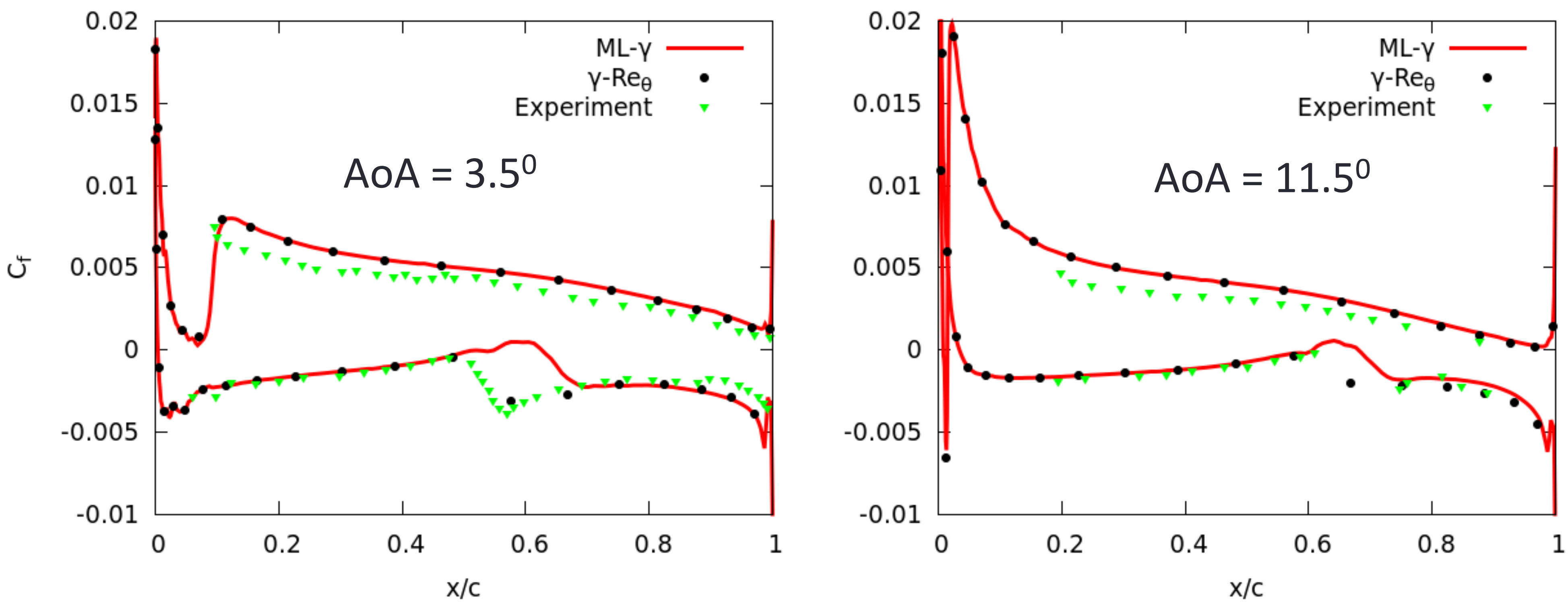
- Excellent agreement between ML-γ and γ-Re$_\theta$ models
  - ANN tested on new data

Aerospatiale-A Airfoil, Re = 2.1 x 10^6



- Excellent agreement between ML-γ and γ-Re$_\theta$ models
  - ANN tested on new data

VA-2 Airfoil, Re = 2.0 x 10^6



- Good agreement between ML-γ and γ-Re_θ models
  - ANN tested on new data

Introducing discrepancies

$$\frac{D\omega}{Dt} = P_\omega - \beta(x) D_\omega + T_\omega$$

Singh & Duraisamy, PoF 2016

Parish & Duraisamy, Aviation 2014

$$\frac{DR_{ij}}{Dt} = C_{ij} + P_{ij} + T_{ij} + \Pi_{ij} + D_{ij} + \beta(x)_{ij} \epsilon_{ij}$$

$$\frac{DR_{ij}}{Dt} = \beta(x)_{ij} a_o \omega (R_{ij,eq} - R_{ij})$$

Singh & Duraisamy, Scitech 2016

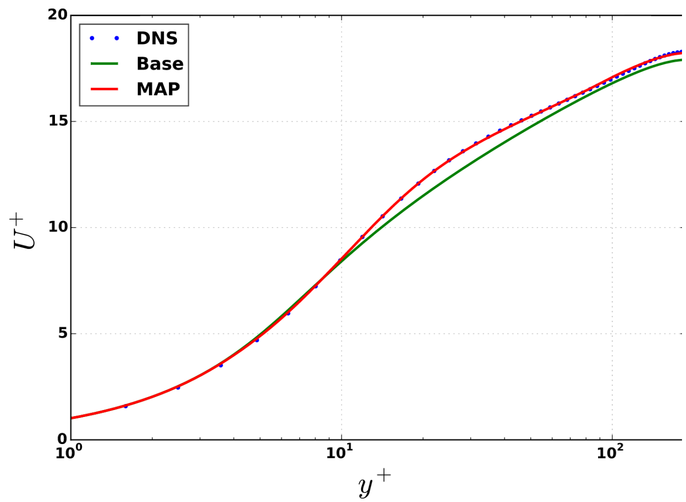
$$\mathbf{R}_p = 2k \left[\frac{\mathbf{I}}{3} + \mathbf{V}(\Lambda + \vec{\beta}(x)) \mathbf{V}^T \right]$$

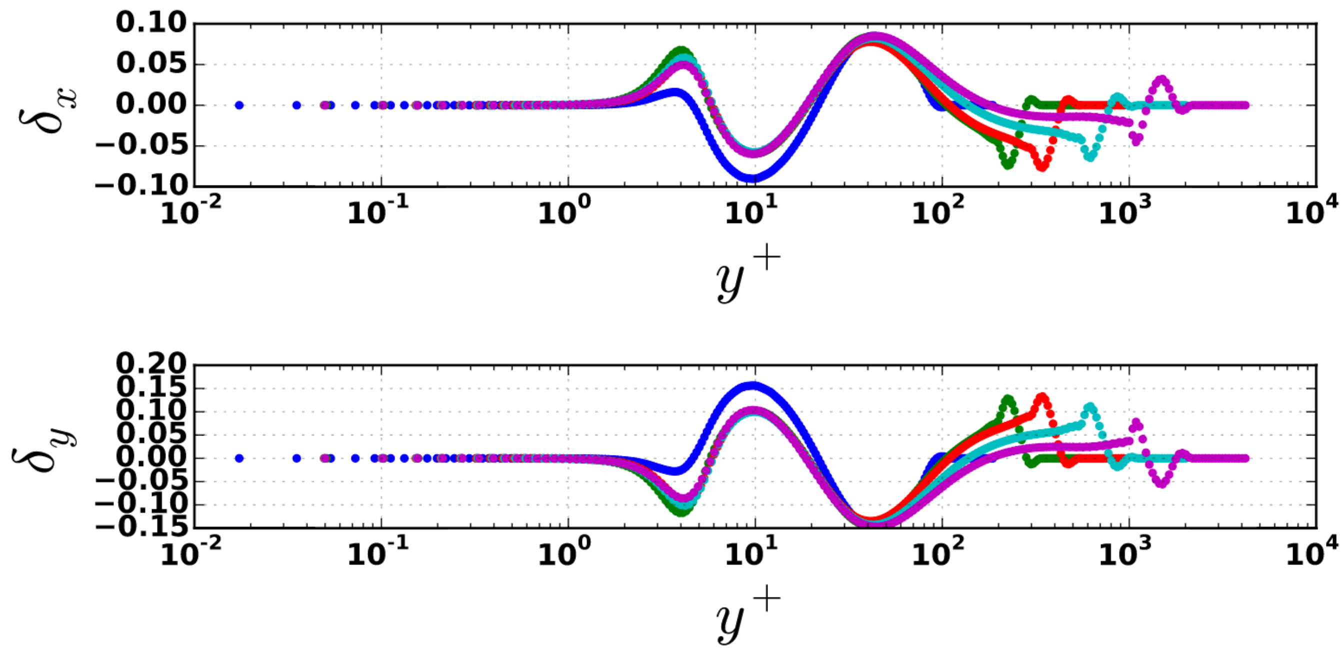
Duraisamy, SIAM 2016

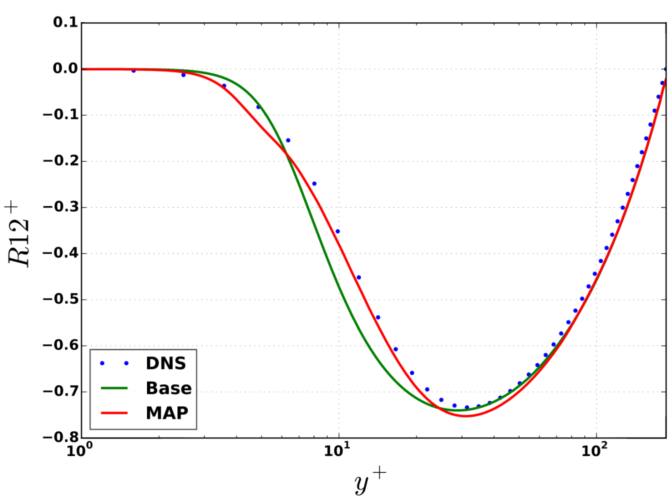
Introducing discrepancies in stress perturbations

$$\mathbf{R}_p = 2k\left[\frac{\mathbf{I}}{3} + \mathbf{V}(\Lambda + \vec{\beta}(x))\mathbf{V}^T\right]$$

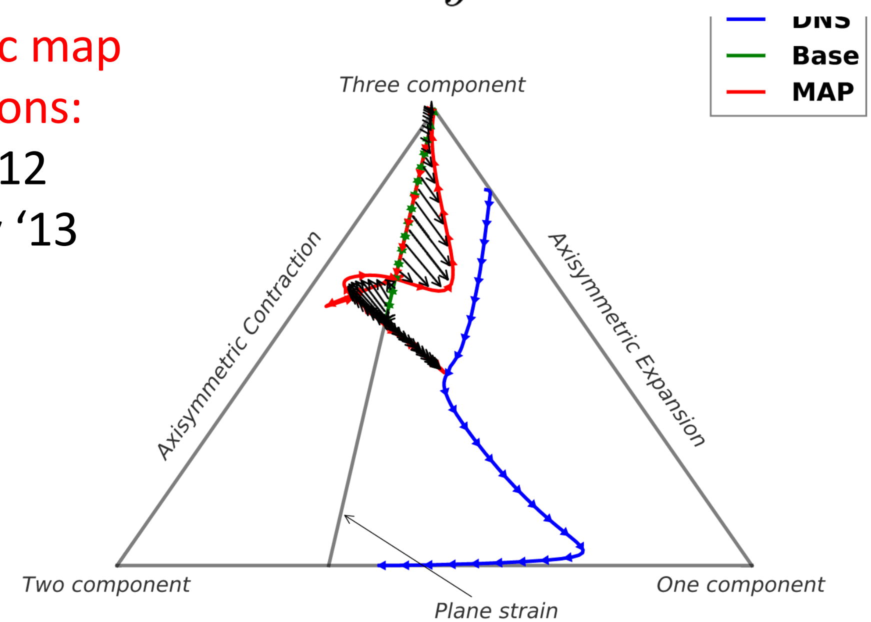
k-ω model



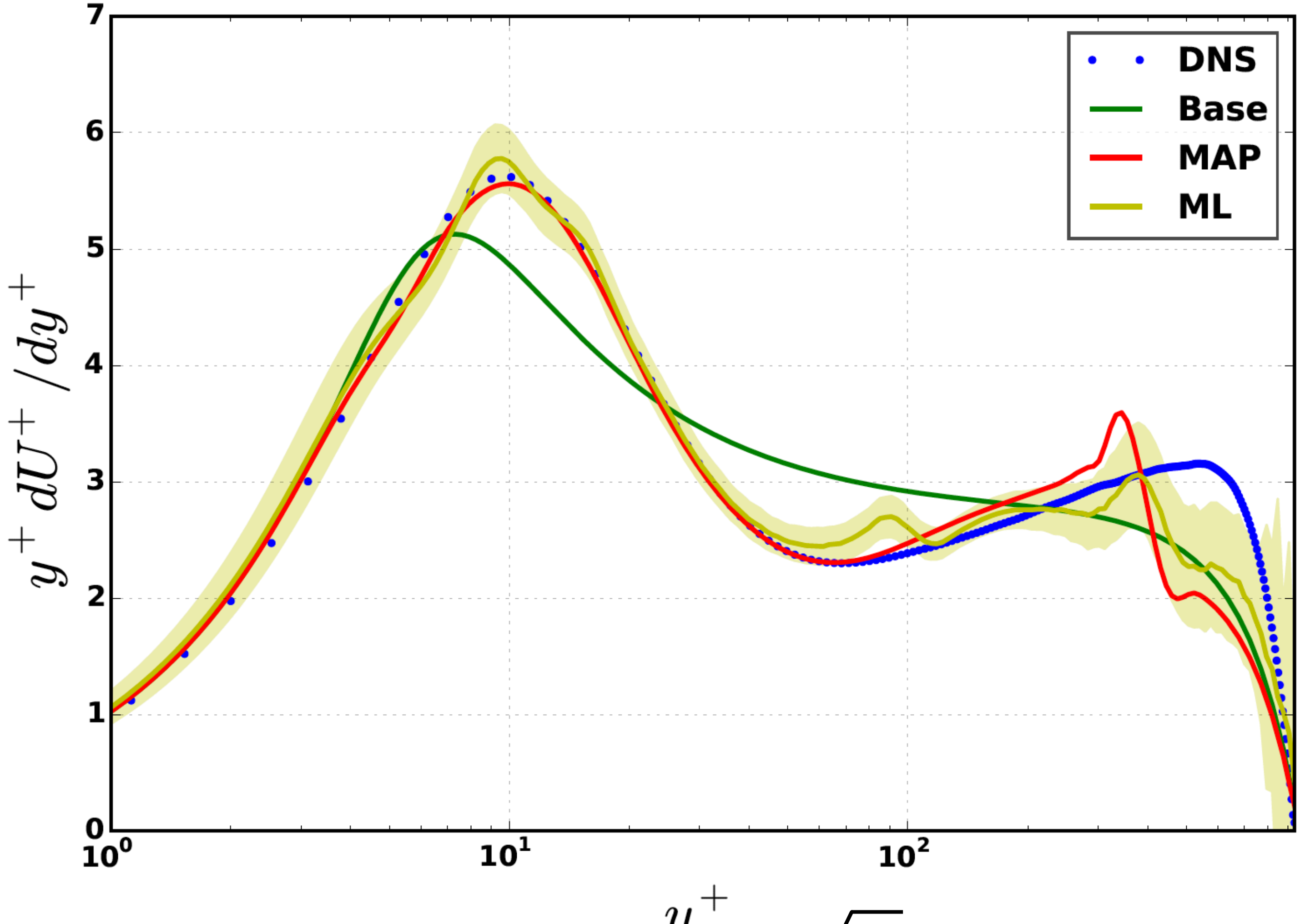




Barycentric map perturbations:
Iaccarino '12
Duraisamy '13
Xiao '15



Prediction with Machine-Learning Injection (Re_τ = 950)



$$\eta = \{Sk/\epsilon, P/\epsilon, y\sqrt{k}/\nu\}$$