Data-driven turbulence modeling applied to separated flows

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Do we have the right community?

"A mathematician, a physicist, and an engineer walk into a pub..." – A not-so-funny guy

Our background in a few (recent) examples:

Yegavian et al. (2015) PIV super-resolution with adjoint-based data assimilation.

Beneddine et al. (2016) Stability analysis of turbulent meanflows.

Franceschini et al. (on-going) Dynamic-based turbulence modeling.

Triple decomposition: $\boldsymbol{u} = \underset{\text{mean}}{\bar{\boldsymbol{u}}} + \underset{\text{coherent}}{\tilde{\boldsymbol{u}}} + \underset{\text{chaotic}}{\boldsymbol{u}'}$

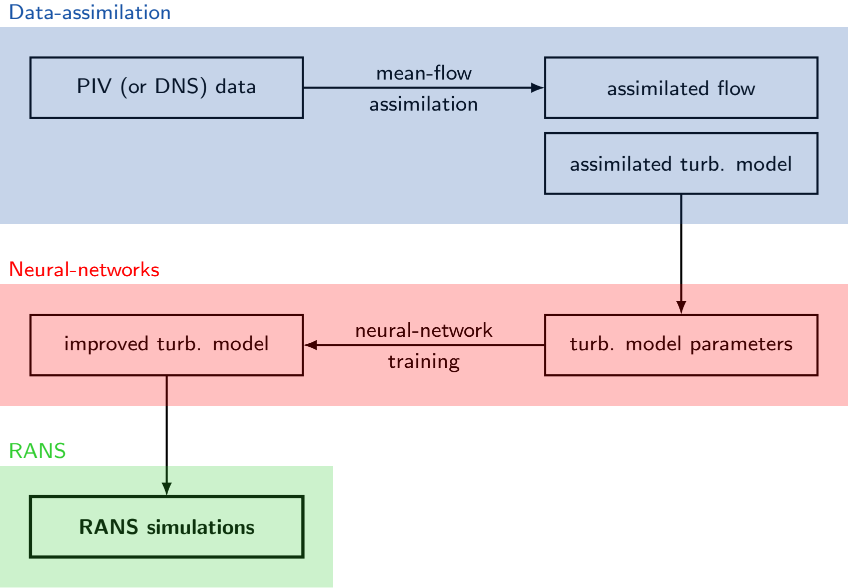
$$\bar{\boldsymbol{u}} \cdot \nabla \bar{\boldsymbol{u}} = -\nabla p - \langle \tilde{\boldsymbol{u}} \cdot \nabla \tilde{\boldsymbol{u}} \rangle + \nabla \cdot \left((\nu + \nu_t) \left(\nabla \bar{\boldsymbol{u}} + \nabla \bar{\boldsymbol{u}}^T \right) \right)$$

$$\nabla \cdot \bar{\boldsymbol{u}} = 0$$

Coherent part from linear optimal frequency response by the meanflow:

$$\langle \tilde{\boldsymbol{u}} \cdot \nabla \tilde{\boldsymbol{u}} \rangle = \int_0^\infty |a(\omega)|^2 \left(\boldsymbol{u}^*_{opt}(\omega) \cdot \nabla \boldsymbol{u}_{opt}(\omega) + \boldsymbol{u}_{opt}(\omega) \cdot \nabla \boldsymbol{u}^*_{opt}(\omega) \right) d\omega$$

Not today. Next time?



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A correction of Spalart-Allmaras model

Incompressible Reynolds-Averaged Navier-Stokes (RANS) with Spalart-Allmaras (SA) model:

$$\boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla p + \nabla \cdot \Big((\nu + \nu_t(\tilde{\nu})) \left(\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T \right) \Big)$$

$$\nabla \cdot \boldsymbol{u} = 0$$

$$\boldsymbol{u} \cdot \nabla \tilde{\nu} = \beta(\boldsymbol{x}) \underbrace{c_{b1}\, \tilde{S}(\boldsymbol{u}, \tilde{\nu})\, \tilde{\nu}}_{P(\boldsymbol{u},\tilde{\nu}) \text{ production}} + \underbrace{\frac{1}{\sigma} \nabla \cdot ((\nu + \tilde{\nu})\, \nabla \tilde{\nu})}_{D(\tilde{\nu}) \text{ dissipation}} - \underbrace{c_{w1}\, f_w(\boldsymbol{u}, \tilde{\nu}) \left(\frac{\tilde{\nu}}{d} \right)^2}_{W(\boldsymbol{u},\tilde{\nu}) \text{ destruction}}$$

Duraisamy and Singh (2016): use $\beta(\boldsymbol{x})$ to correct the production term $P(\boldsymbol{u}, \tilde{\nu})$.

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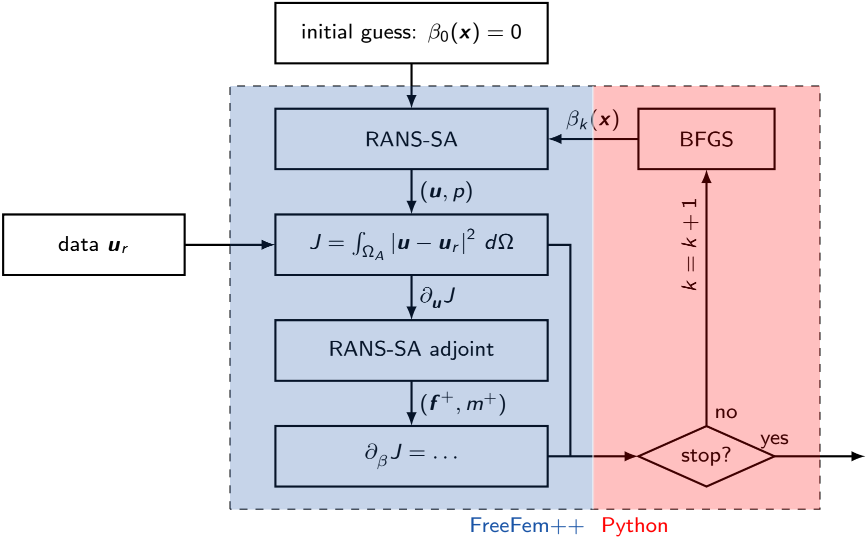
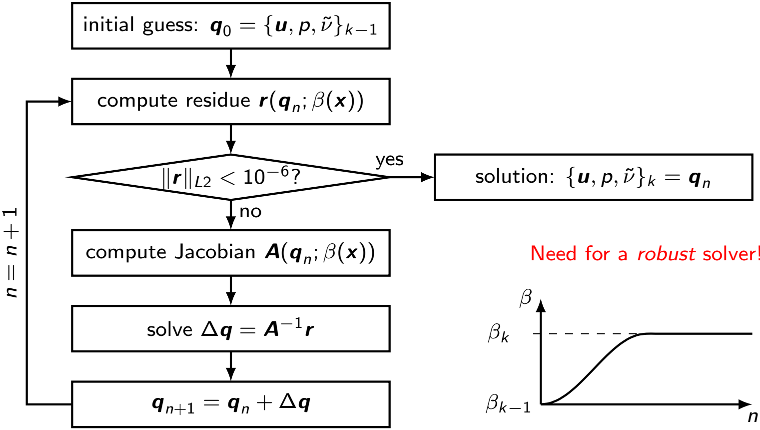
Adjust the production mask $\beta(\boldsymbol{x})$ to match the data $\boldsymbol{u}_r$, i.e. minimize the cost function J.



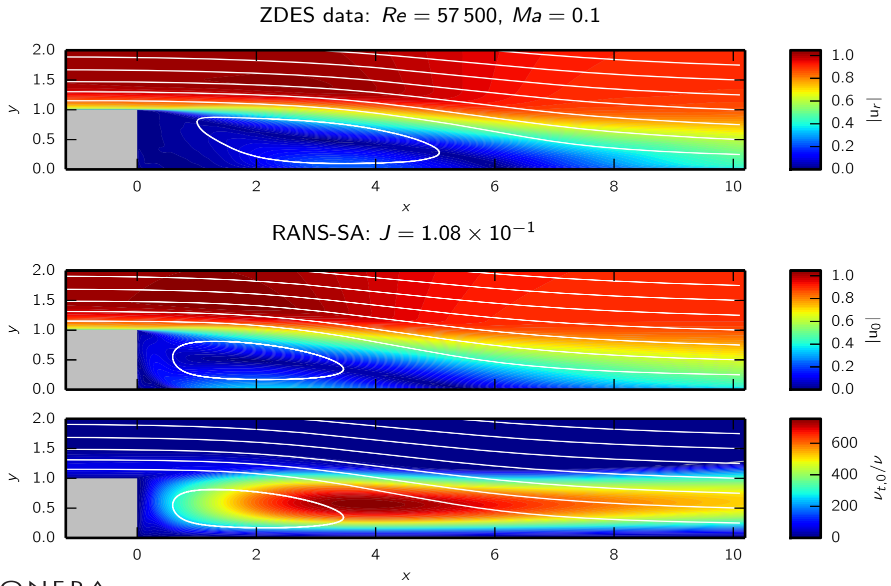
RANS solver

Newton's method for steady RANS equation with Spalart-Allmaras model (FreeFem++).



Flow assimilation (I)

Data: back-facing step simulations by Beneddine et al. (2016)

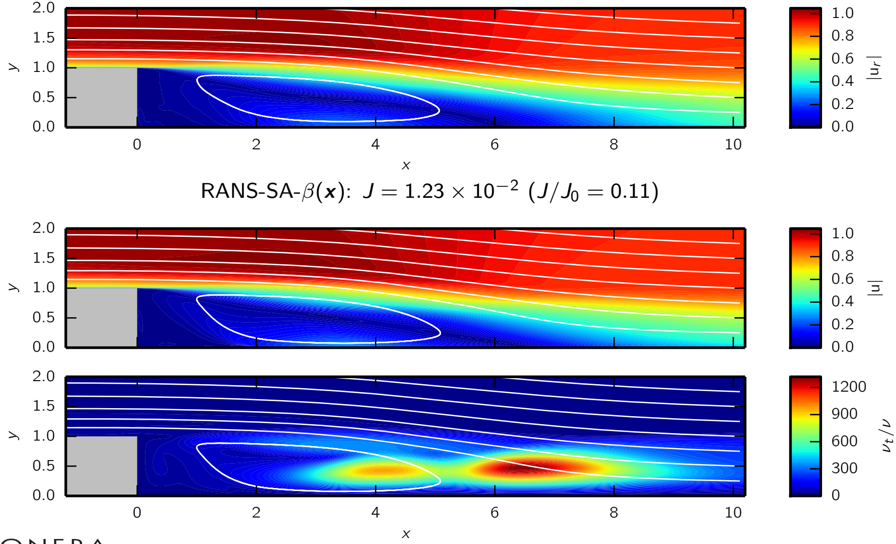


Flow assimilation (I)

Data: back-facing step simulations by Beneddine et al. (2016)

ZDES data: $Re = 57\,500$, $Ma = 0.1$

RANS-SA-$\beta(\boldsymbol{x})$: $J = 1.23 \times 10^{-2}$ $(J/J_0 = 0.11)$



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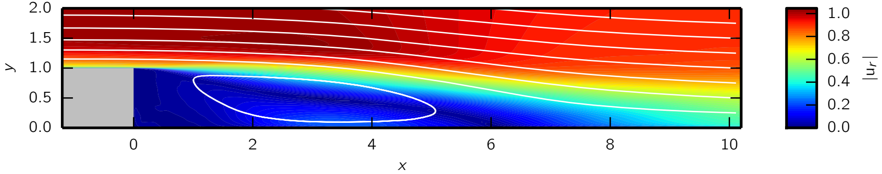
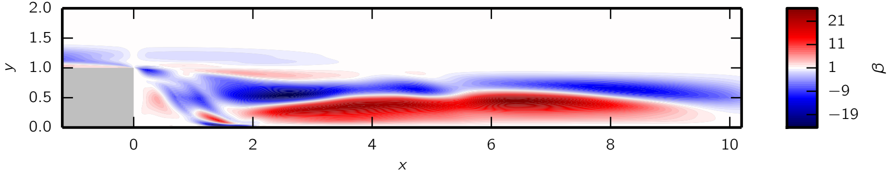


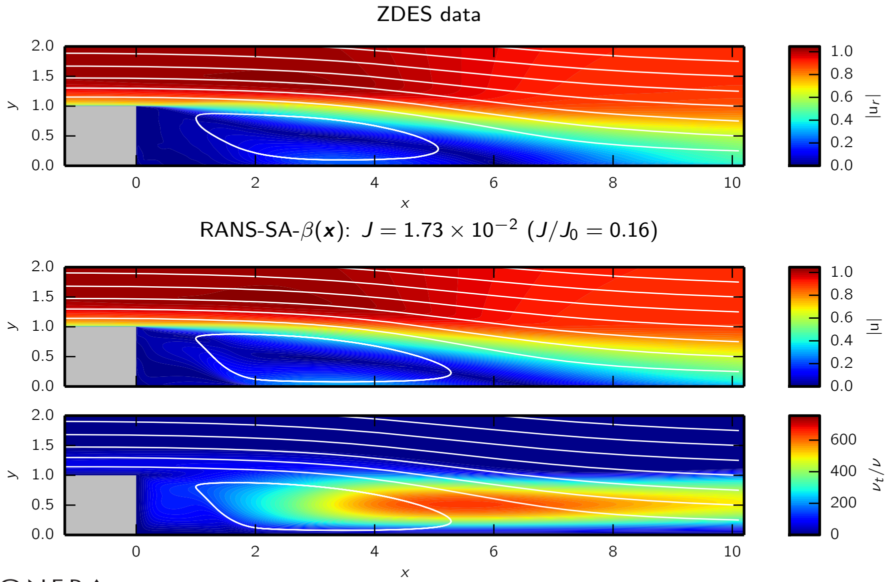
RANS-SA-$\beta(\boldsymbol{x})$: production mask β



$|\beta| \approx 20$ ▶ not a *small* correction of Spalart-Allmaras anymore!

Flow assimilation (II)

Modified cost-function to penalise extreme corrections of SA: $\tilde{J} = J + \gamma^2 \int_\Omega (\beta - 1)^2 \, d\Omega, \;\; \gamma^2 = 10^{-2}$



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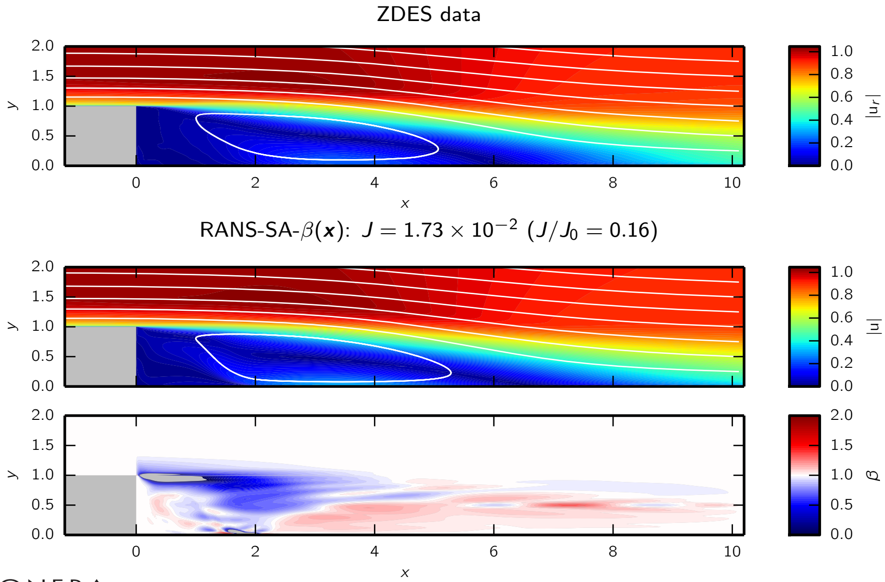

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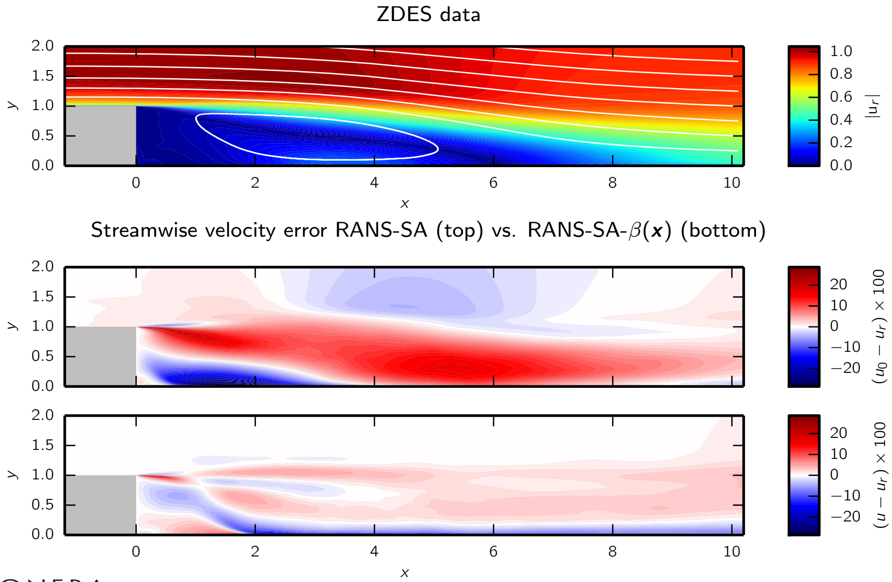
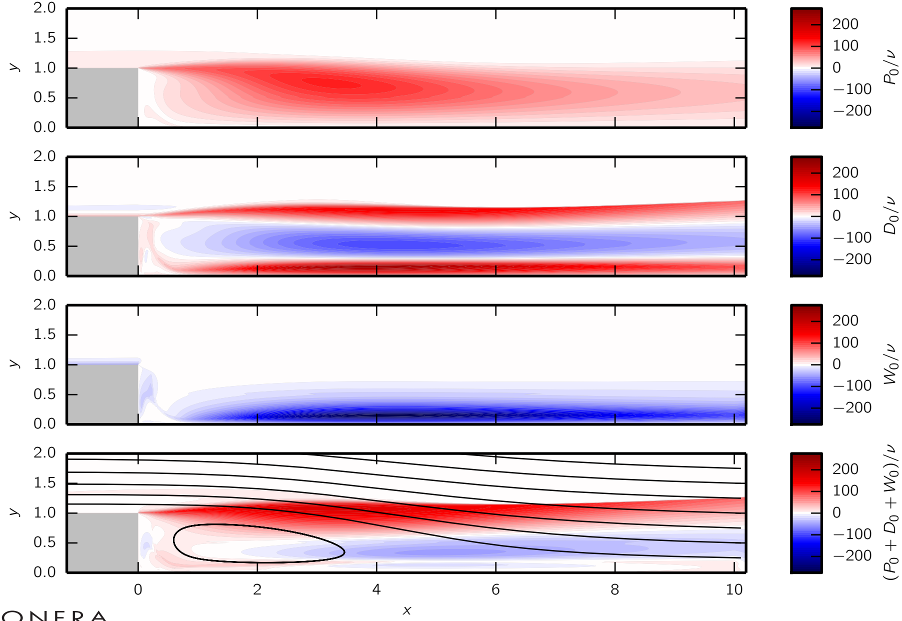


RANS-SA: production, dissipation and destruction



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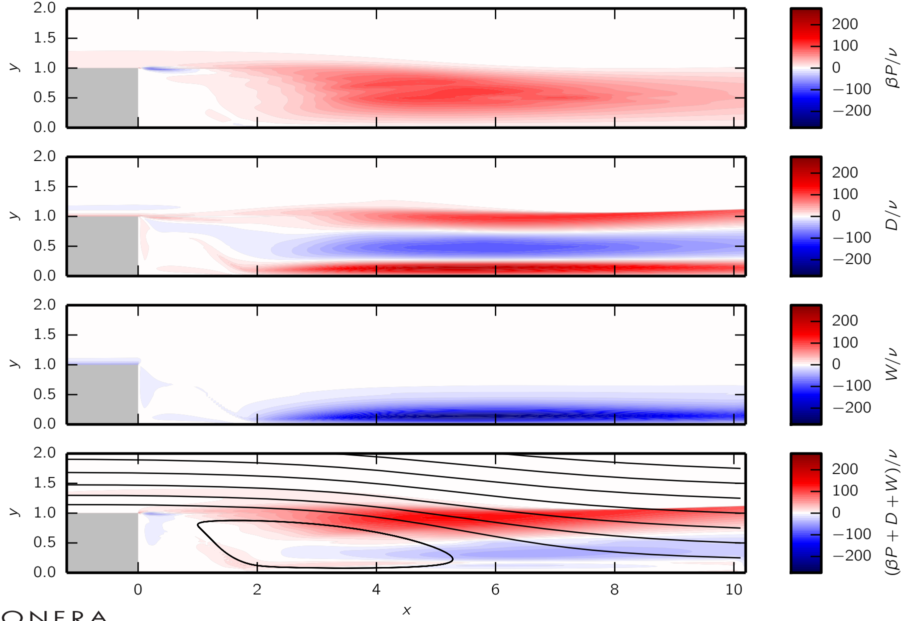
RANS-SA-$\beta(x)$: production, dissipation and destruction



A more “balanced” role for β?

Equilibrium between the three source terms in the boundary-layer, Spalart and Allmaras (1994):

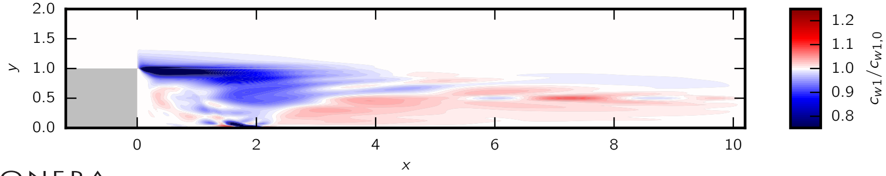
$$c_{w1} = \frac{c_{b1}}{\kappa^2} + \frac{1 + c_{b2}}{\sigma}$$

Equation for the turbulent viscosity $\tilde{\nu}$:

$$\boldsymbol{u} \cdot \nabla \tilde{\nu} = \underbrace{\beta(\boldsymbol{x})\, c_{b1}}_{c_{b1}\ \text{tuning}}\ \tilde{S}(\boldsymbol{u}, \tilde{\nu})\, \tilde{\nu} + \frac{1}{\sigma} \left[\nabla \cdot ((\nu + \tilde{\nu}\,) \nabla \tilde{\nu}\,) + c_{b2} |\nabla \tilde{\nu}|^2 \right] - c_{w1}\, f_w(\boldsymbol{u}, \tilde{\nu}) \left(\frac{\tilde{\nu}}{d} \right)^2$$

- Should we consider a *local* equilibrium in the model?

$$\hat{c}_{w1}(\boldsymbol{x}) = \beta(\boldsymbol{x}) \frac{c_{b1}}{\kappa^2} + \frac{1 + c_{b2}}{\sigma}$$



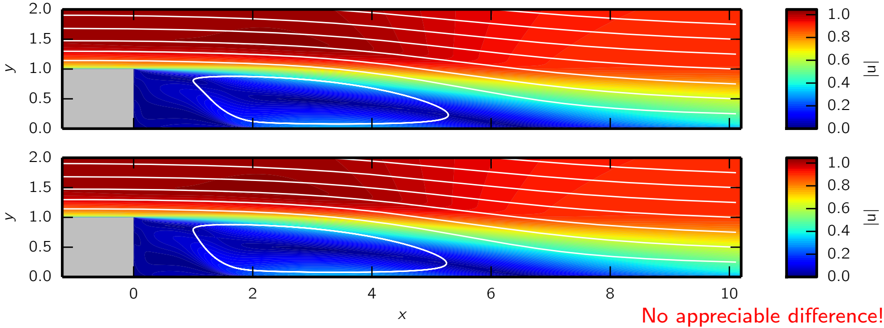
Flow assimilation (III)

Modified equation for the turbulent viscosity $\tilde{\nu}$:

$$\boldsymbol{u} \cdot \nabla \tilde{\nu} = \beta(\boldsymbol{x})(P(\boldsymbol{u}, \tilde{\nu}) + W_P(\boldsymbol{u}, \tilde{\nu})) + D(\tilde{\nu}) + W_D(\boldsymbol{u}, \tilde{\nu})$$

where $\hat{W}(\boldsymbol{u}, \tilde{\nu}, \beta) = -\hat{c}_{w1}(\boldsymbol{x})\, f_w \left(\frac{\tilde{\nu}}{d}\right)^2 = \beta(\boldsymbol{x}) \underbrace{\left(-\frac{c_{b1}}{\kappa^2} f_w \left(\frac{\tilde{\nu}}{d}\right)^2\right)}_{W_P(\boldsymbol{u},\tilde{\nu})} + \underbrace{\left(-\frac{1+c_{b2}}{\sigma} f_w \left(\frac{\tilde{\nu}}{d}\right)^2\right)}_{W_D(\boldsymbol{u},\tilde{\nu})}$

RANS-SA-$\beta(\boldsymbol{x})$ (top) vs. RANS-SA-$\beta(\boldsymbol{x})$-$\hat{c}_{w1}$ (bottom): assimilated flow



No appreciable difference!

Flow assimilation (III)

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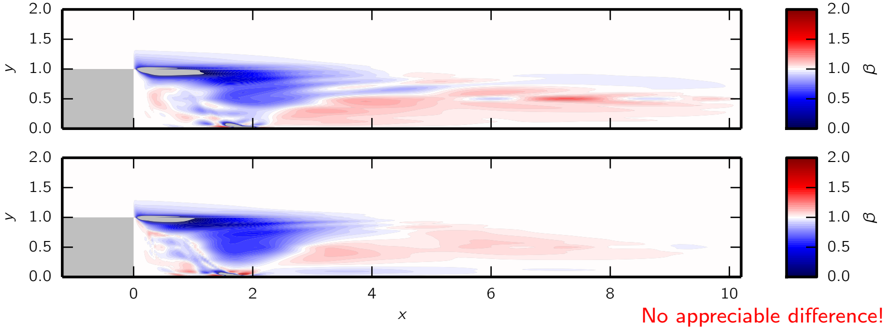
RANS-SA-$\beta(\boldsymbol{x})$ (top) vs. RANS-SA-$\beta(\boldsymbol{x})$-$\hat{c}_{w1}$ (bottom): production mask



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Flow assimilation (III)

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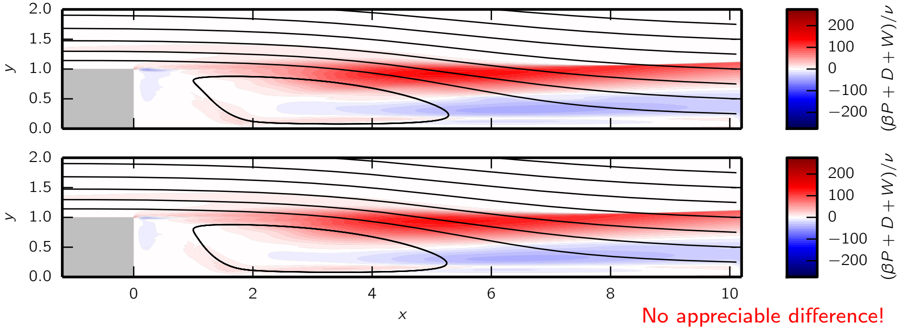
where

$$\hat{W}(\boldsymbol{u}, \tilde{\nu}, \beta) = -\hat{c}_{w1}(\boldsymbol{x})\, f_w \left(\frac{\tilde{\nu}}{d}\right)^2 = \beta(\boldsymbol{x}) \underbrace{\left(-\frac{c_{b1}}{\kappa^2} f_w \left(\frac{\tilde{\nu}}{d}\right)^2\right)}_{W_P(\boldsymbol{u},\tilde{\nu})} + \underbrace{\left(-\frac{1+c_{b2}}{\sigma} f_w \left(\frac{\tilde{\nu}}{d}\right)^2\right)}_{W_D(\boldsymbol{u},\tilde{\nu})}$$

RANS-SA-$\beta(\boldsymbol{x})$ (top) vs. RANS-SA-$\beta(\boldsymbol{x})$-$\hat{c}_{w1}$ (bottom): viscosity source term



No appreciable difference!

Neural network

Produce a general input/output (I/O) function that reproduces the SA correction.

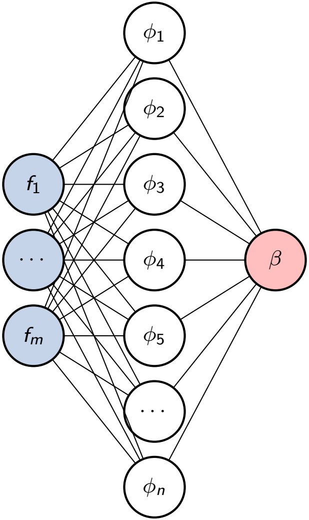
$$\boxed{\beta(\boldsymbol{x}) \rightarrow \beta(\boldsymbol{u}, \tilde{\nu}) \rightarrow \beta(\{f_j(\boldsymbol{u}, \tilde{\nu})\})}$$

- Not directly function of the state $(\boldsymbol{u}, \tilde{\nu})$ but on a set of observables $\{f_j(\boldsymbol{u}, \tilde{\nu})\}$
- Neural network (`scikit-learn` package): 1 layer of n=100 neurons.

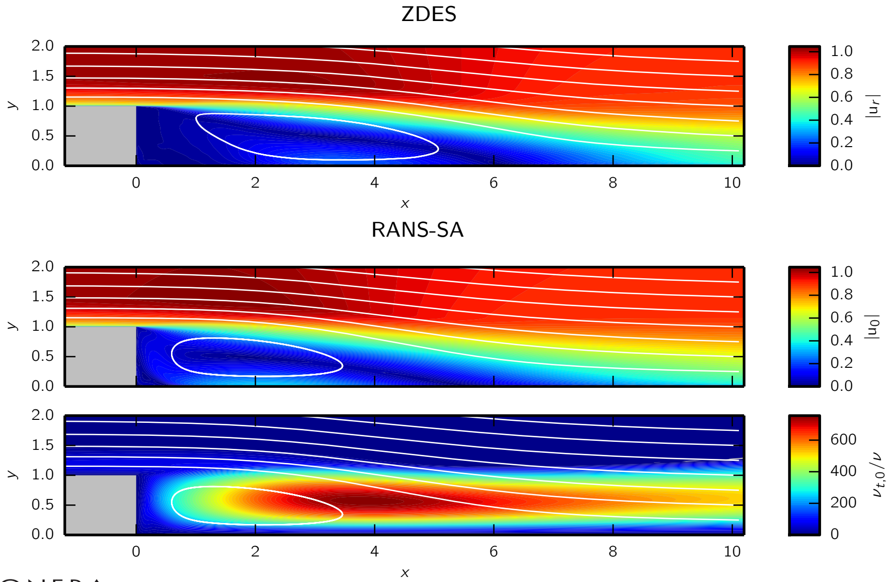
$$\phi_i = \max\left(0, b_i + \sum_{j=1}^{m} a_{ij}\, f_j(\boldsymbol{u}, \tilde{\nu})\right)$$

$$\beta = \beta_0 + \sum_{i=1}^{n} w_i \phi_i$$

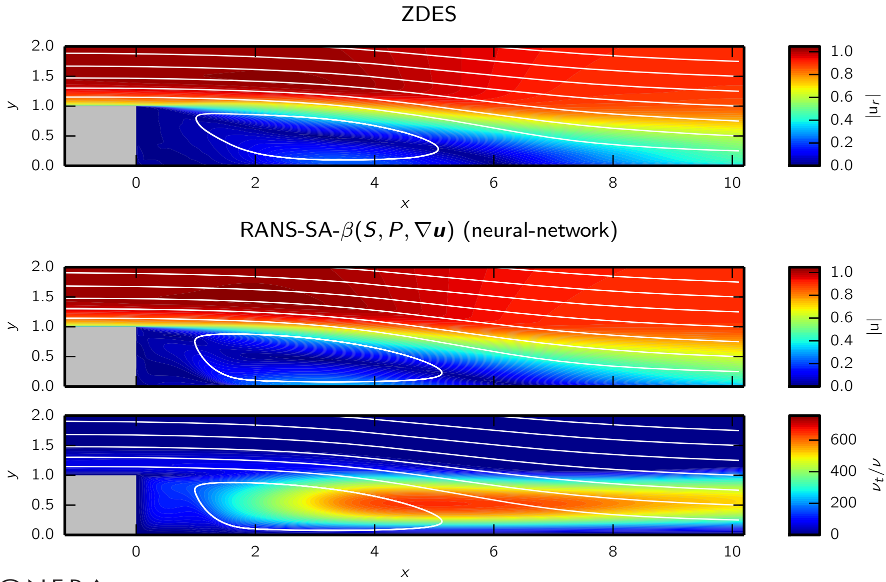
- Which observables $f_j(\boldsymbol{u}, \tilde{\nu})$?
  First attempt: $\boxed{\{f_j\} = \{\ln(S), P, \nabla \boldsymbol{u}\}}$



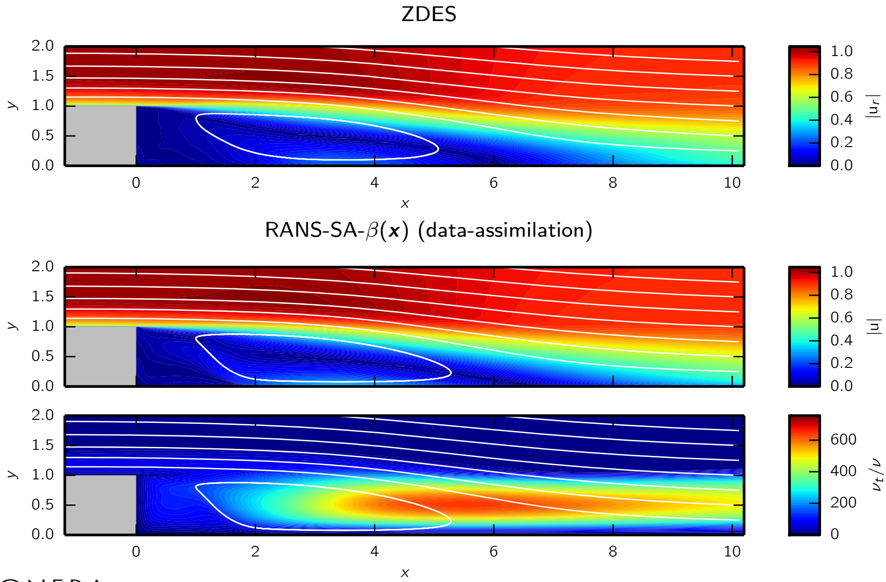
Validation



Validation


ZDES
2.0
1.5
1.0
0.5
0.0
y
0
2
4
6
8
10
x
1.0
0.8
0.6
0.4
0.2
0.0
$|u_r|$
RANS-SA-$\beta(S, P, \nabla \boldsymbol{u})$ (neural-network)
$|u|$
600
400
200
0
ν_t/ν

Validation



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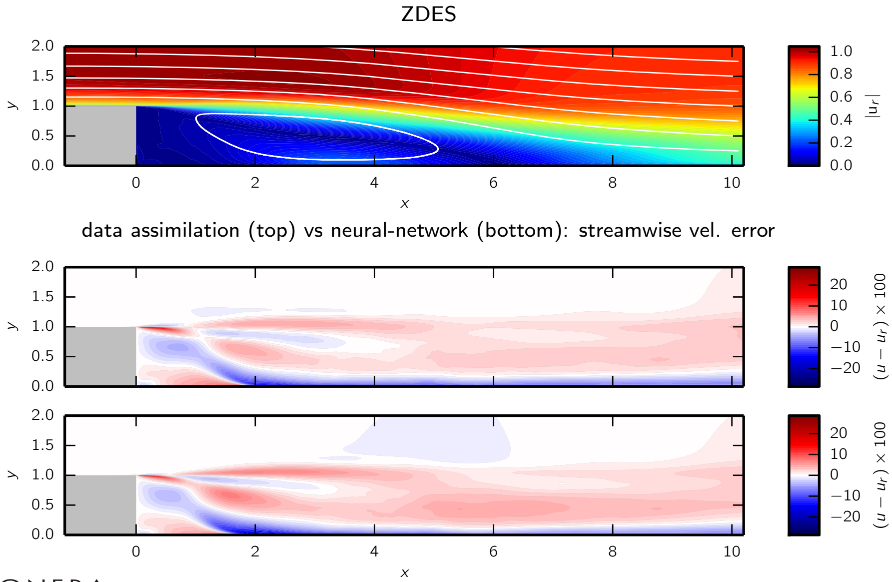
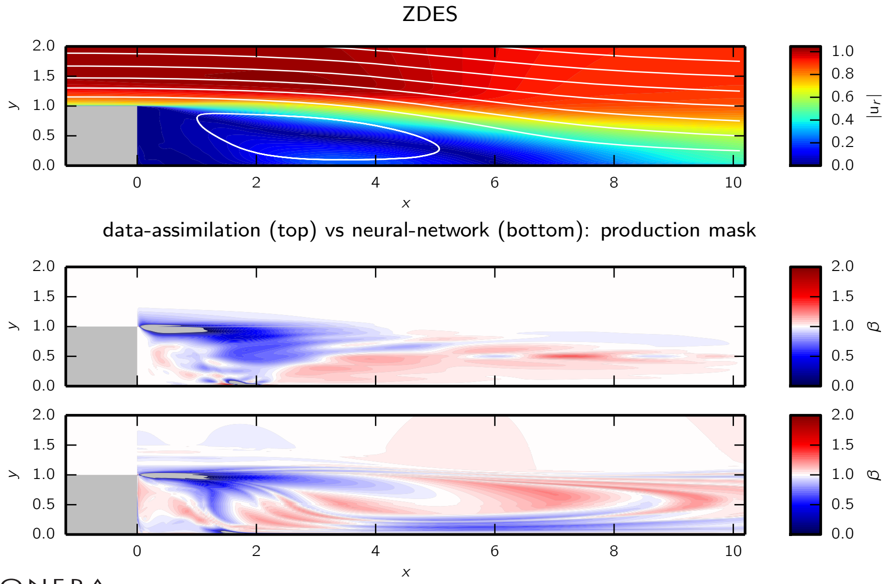
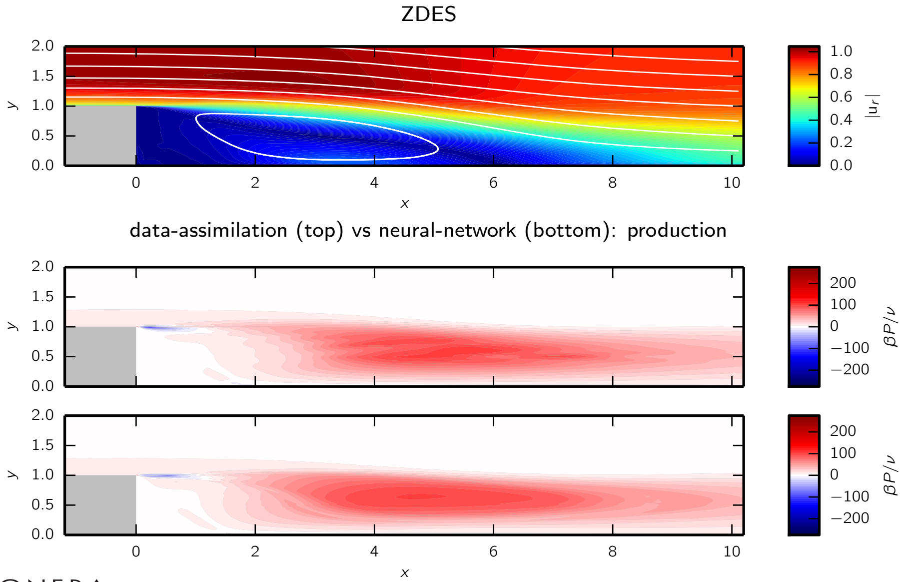
Validation


ZDES
2.0
1.5
1.0
0.5
0.0
y
0
2
4
6
8
10
x
1.0
0.8
0.6
0.4
0.2
0.0
$|u_r|$
data assimilation (top) vs neural-network (bottom): streamwise vel. error
20
10
0
−10
−20
$(u - u_r) \times 100$

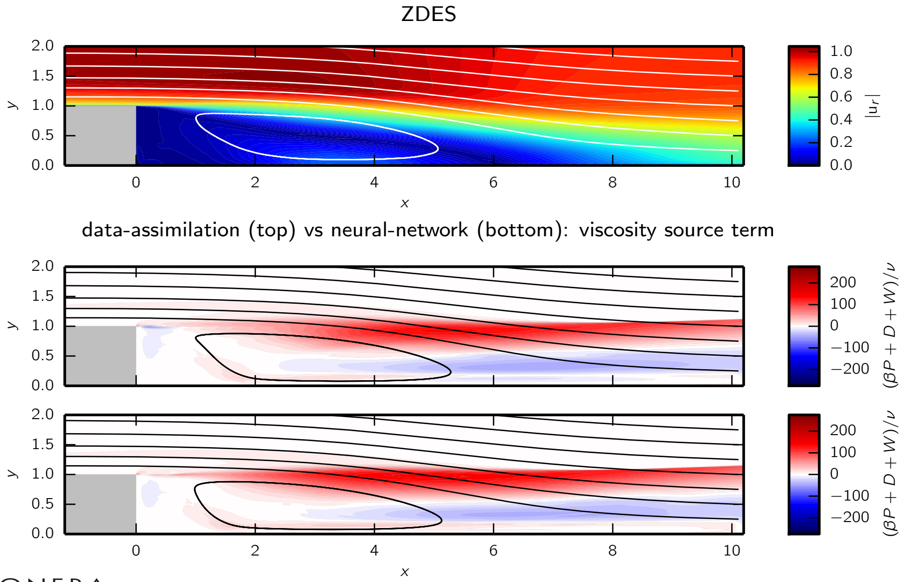
Validation


ZDES
2.0
1.5
1.0
0.5
0.0
y
0
2
4
6
8
10
x
1.0
0.8
0.6
0.4
0.2
0.0
$|u_r|$
data-assimilation (top) vs neural-network (bottom): production mask
2.0
1.5
1.0
0.5
0.0
y
β
0
2
4
6
8
10
x

Validation


ZDES
data-assimilation (top) vs neural-network (bottom): production
y
x
$|u_r|$
$\beta P/\nu$
2.0
1.5
1.0
0.5
0.0
0
2
4
6
8
10
0.8
0.6
0.4
0.2
200
100
−100
−200

Validation



ZDES
2.0
1.5
1.0
0.5
0.0
y
0
2
4
6
8
10
x
1.0
0.8
0.6
0.4
0.2
0.0
$|u_r|$
data-assimilation (top) vs neural-network (bottom): viscosity source term
200
100
0
−100
−200
$(\beta P + D + W)/\nu$

Conclusions

- A correction of Spalart-Allmaras is fitted to ZDES data.
  - Adjoint-based assimilation algorithm.
  - No appriciable difference between balanced and unbalanced c_{w1}.
- A neural network is trained to generalize the identified correction...
- ...and tested on the *training* flow-case.
- The algorithm can be (is already) generalised for different cost functions:
  - pressure matching,
  - skin friction matching
  - ...

  and different measuments techniques, i.e. include a measurment operator in the procedure.

Next step(s):

- Explore different input variables for the neural-network model.
- Blind-test on a *different-but-similar* flow case.
- Expand the training database: CALL FOR incompressible CASES!

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Thank you for your attention.

Any questions?

Beneddine, S., Sipp, D., Arnault, A., Dandois, J., and Lesshafft, L. (2016). Conditions for validity of mean flow stability analysis. *Journal of Fluid Mechanics*, 798:485–504.

Duraisamy, K. and Singh, A. (2016). Informing turbulence closures with computational and experimental data. In *54th AIAA Aerospace Sciences Meeting*, San Diego, California, USA.

Spalart, P. and Allmaras, S. (1994). A one-equation turbulence model for aerodynamic flows. *La Recherche Arospatiale*, 1:pp. 5–21.

Yegavian, R., Leclaire, B., Champagnt, F., and Marquet, O. (2015). Performance assessment of piv super-resolution with adjoint-based data assimilation. In *11th International Symposium on PIV*, Santa Barbara, California, USA.