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# **Resolution Requirements** for DG-LES

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### **Current Methodologies**

Engineering/Mechanics/Thermodynamics

#### Detailed coverage of advanced combustion topics from the author of *Principles of Combustion, Second Edition*

Turbulence, turbulent combustion, and multiphase reacting flows have become major research topics in recent decades due to their application across diverse fields, including energy, environment, propulsion, transportation, industrial safety, and nanotechnology. Most of the knowledge accumulated from this research has never been published in book form-until new. *Fundamentals of Turbulent and Multiphase Combustion*, and multiphase phenomena along with useful experimental techniques, including nontinutusive, laser-based measurement techniques, providing a firm background in both contemporary and classical approaches. Beginning with two full chapters on laminar premixed and non-premixed flames, this book takes a multiphase approach, beginning with more common topics and moving on to higher-level applications.

In addition, Fundamentals of Turbulent and Multiphase Combustion:

- Addresses seven basic topical areas in combustion and multiphase flows, including laminar premixed and non-premixed flames, theory of turbulence, turbulent premixed and non-premixed flames, and multiphase flows
- Covers spray atomization and combustion, solid-propellant combustion, homogeneous propellants, nitramines, reacting boundary-layer flows, single energetic particle combustion, and granular bed combustion
- · Provides experimental setups and results whenever appropriate

Supported with a large number of examples and problems as well as a solutions manual, Fundamentals of Turbulant and Multiphase Combustion is an important resource for professional engineers and researchers as well as graduate students in mechanical, chemical, and aerospace engineering.

KENNETH K. KUO is Distinguished Professor of Mechanical Engineering and Director of the High Pressure Combustion Laboratory (HPCL) in the Department of Mechanical and Nuclear Engineering of the College of Engineering at Pennsylvania State University. Professor Kuo established the HPCL and is recognized as one of the leading researchers and experts in propulsion-related combustion.

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#### Fundamentals of Turbulent and Multiphase Combustion



RANS LES DNS



Investigate the effect of p,  $\Delta$  and h

#### □ LES and DNS of a 3D temporally developing mixing layer



## **Filtered Transport Equations**

□ Filtering

$$\langle Q(\mathbf{x},t) \rangle_{\ell} = \int_{-\infty}^{+\infty} Q(\mathbf{x}',t) G(\mathbf{x}',\mathbf{x}) d\mathbf{x}' \qquad \langle Q(\mathbf{x},t) \rangle_{L} = \langle \rho Q \rangle_{\ell} / \langle \rho \rangle_{\ell}$$

$$\frac{\partial \langle \rho \rangle_{\ell}}{\partial t} + \frac{\partial \langle \rho \rangle_{\ell} \langle u_{j} \rangle_{L}}{\partial x_{j}} = 0$$

$$\frac{\partial \langle \rho \rangle_{\ell} \langle u_{i} \rangle_{L}}{\partial t} + \frac{\partial \langle \rho \rangle_{\ell} \langle u_{j} \rangle_{L} \langle u_{i} \rangle_{L}}{\partial x_{j}} = -\frac{\partial \langle p \rangle_{\ell}}{\partial x_{i}} + \frac{\partial \langle \tau_{ij} \rangle_{\ell}}{\partial x_{j}} - \frac{\partial \Sigma_{ij}}{\partial x_{j}}$$

$$\frac{\partial \langle \rho \rangle_{\ell} \langle \phi_{\alpha} \rangle_{L}}{\partial t} + \frac{\partial \langle \rho \rangle_{\ell} \langle u_{j} \rangle_{L} \langle \phi_{\alpha} \rangle_{L}}{\partial x_{j}} = \frac{\partial}{\partial x_{j}} \left( \left\langle \gamma \frac{\partial \phi_{\alpha}}{\partial x_{j}} \right\rangle_{\ell} \right) - \frac{\partial M_{j}^{\alpha}}{\partial x_{j}}$$

$$\frac{\text{SGS stress}}{\text{SGS scalar flux}} \qquad \Sigma_{ij} = \langle \rho \rangle_{\ell} \left( \langle u_i u_j \rangle_L - \langle u_i \rangle_L \langle u_j \rangle_L \right)$$

$$\frac{\text{SGS scalar flux}}{M_j^{\alpha}} = \langle \rho \rangle_{\ell} \left( \langle u_j \phi_{\alpha} \rangle_L - \langle u_j \rangle_L \langle \phi_{\alpha} \rangle_L \right)$$

#### **Scalar-FDF SGS Closure**

#### Modeled (Fokker-Plank)

$$\frac{\partial F}{\partial t} + \frac{\partial \left( \langle u_j \rangle_L F \right)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ (\gamma + \gamma_t) \frac{\partial \left( F / \langle \rho \rangle_l \right)}{\partial x_j} \right] + \frac{\partial}{\partial \psi_k} \left[ \Omega_m F \left( \psi_k - \langle \phi_k \rangle_L \right) \right]$$

#### □ SGS energy

$$\frac{\partial(\langle \rho \rangle_{\ell} \tau_{\alpha \alpha})}{\partial t} + \frac{\partial[\langle \rho \rangle_{\ell} \langle u_{j} \rangle_{L} \tau_{\alpha \alpha}]}{\partial x_{j}} = \frac{\partial}{\partial x_{j}} \left[ (\gamma + \gamma_{t}) \frac{\partial \tau_{\alpha \alpha}}{\partial x_{j}} \right] + 2(\gamma + \gamma_{t}) \left[ \frac{\partial(\langle \phi_{\alpha} \rangle_{L})}{\partial x_{j}} \frac{\partial(\langle \phi_{\alpha} \rangle_{L})}{\partial x_{j}} - 2\Omega_{m} \langle \rho \rangle_{\ell} \tau_{\alpha \alpha} \right]$$

# **DG Methodology**

- Discontinuous elements in space.
- Using basis Functions to approximate solution.
- Finite element method using Riemann solver for fluxes.



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# **DG Solver Capabilities**

- Hybrid mixed element unstructured meshes (tetrahedra, prisms, pyramids, and hexahedra)
- p-enrichment and h-refinement
- Curved mesh





# **Numerical Procedure**

- 1 DNS case
  - □ h=1/256
  - 🖵 p=5
- 27 LES cases
  - □ h=1/128, 1/64 and 1/32
  - □ Δ=1/64, 1/32 and 1/16
  - p=3, 4 and 5
- Construct the L<sub>2</sub> norm error of subgrid scale energy (τ), resolved energy (R) and total energy (r).

### **Reynolds Stresses**



# h-refinement



# p-enrichment



## **Δ-refinement**



### **Summary**

- DG-LES simulator
  - p-enrichment
    - □ As p goes higher, the error converges to zero for all Reynolds stresses.
  - h-refinement
    - Similar to the p-enrichment, the LES Reynolds stresses converges to the DNS results for finer resolution.
  - Δ-refinement
    - $\Box$  Where the h is fixed, the best solution is obtained at  $\Delta/h=1$ .
  - Generic filter
    - $\Box$  Convergence as  $\Delta$ /h decreases.

#### Thank you!