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# Resolution Requirements for DG-LES


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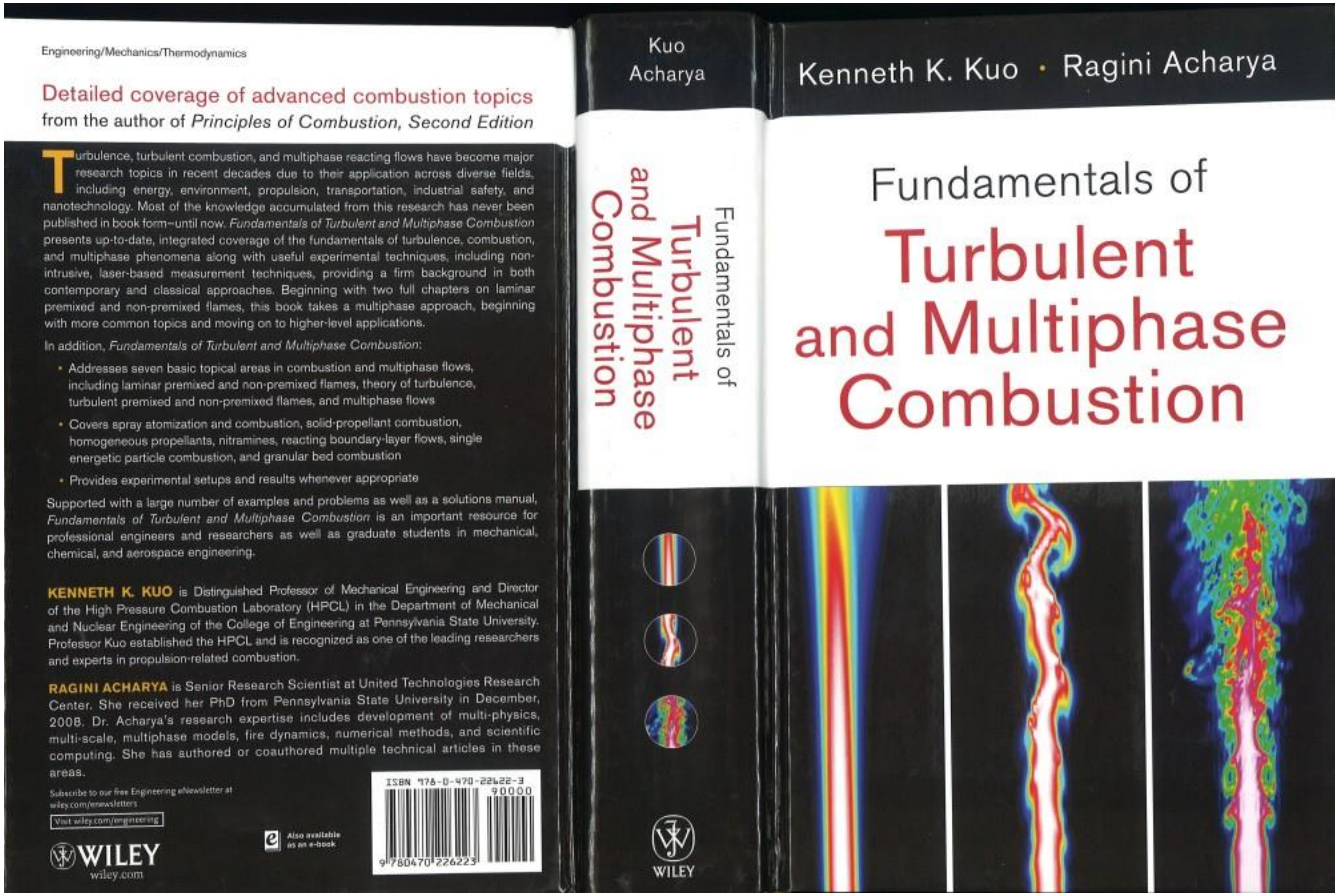
# Current Methodologies

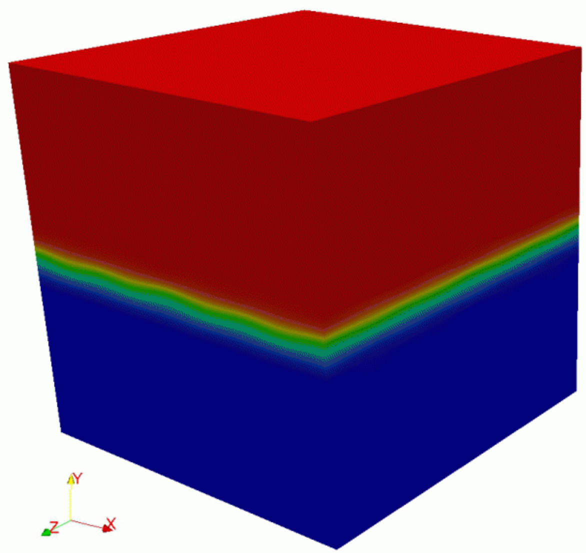



RANS LES DNS

# Resolution Study

Investigate the effect of p, Δ and h

- LES and DNS of a 3D temporally developing mixing layer



# Filtered Transport Equations

- Filtering

$$\langle Q(\mathbf{x},t)\rangle_\ell = \int_{-\infty}^{+\infty} Q(\mathbf{x}',t)G(\mathbf{x}',\mathbf{x})d\mathbf{x}' \qquad \langle Q(\mathbf{x},t)\rangle_L = \langle \rho Q\rangle_\ell / \langle \rho\rangle_\ell$$

$$\frac{\partial \langle\rho\rangle_\ell}{\partial t} + \frac{\partial \langle\rho\rangle_\ell \langle u_j\rangle_L}{\partial x_j} = 0$$

$$\frac{\partial \langle\rho\rangle_\ell \langle u_i\rangle_L}{\partial t} + \frac{\partial \langle\rho\rangle_\ell \langle u_j\rangle_L \langle u_i\rangle_L}{\partial x_j} = -\frac{\partial \langle p\rangle_\ell}{\partial x_i} + \frac{\partial \langle\tau_{ij}\rangle_\ell}{\partial x_j} - \frac{\partial \Sigma_{ij}}{\partial x_j}$$

$$\frac{\partial \langle\rho\rangle_\ell \langle\phi_\alpha\rangle_L}{\partial t} + \frac{\partial \langle\rho\rangle_\ell \langle u_j\rangle_L \langle\phi_\alpha\rangle_L}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\left\langle \gamma \frac{\partial \phi_\alpha}{\partial x_j}\right\rangle_\ell\right) - \frac{\partial M_j^\alpha}{\partial x_j}$$

<u>SGS stress</u> $$\Sigma_{ij} = \langle\rho\rangle_\ell \left(\langle u_i u_j\rangle_L - \langle u_i\rangle_L \langle u_j\rangle_L\right)$$

<u>SGS scalar flux</u> $$M_j^\alpha = \langle\rho\rangle_\ell \left(\langle u_j \phi_\alpha\rangle_L - \langle u_j\rangle_L \langle \phi_\alpha\rangle_L\right)$$

# Scalar-FDF SGS Closure

- Modeled (Fokker-Plank)

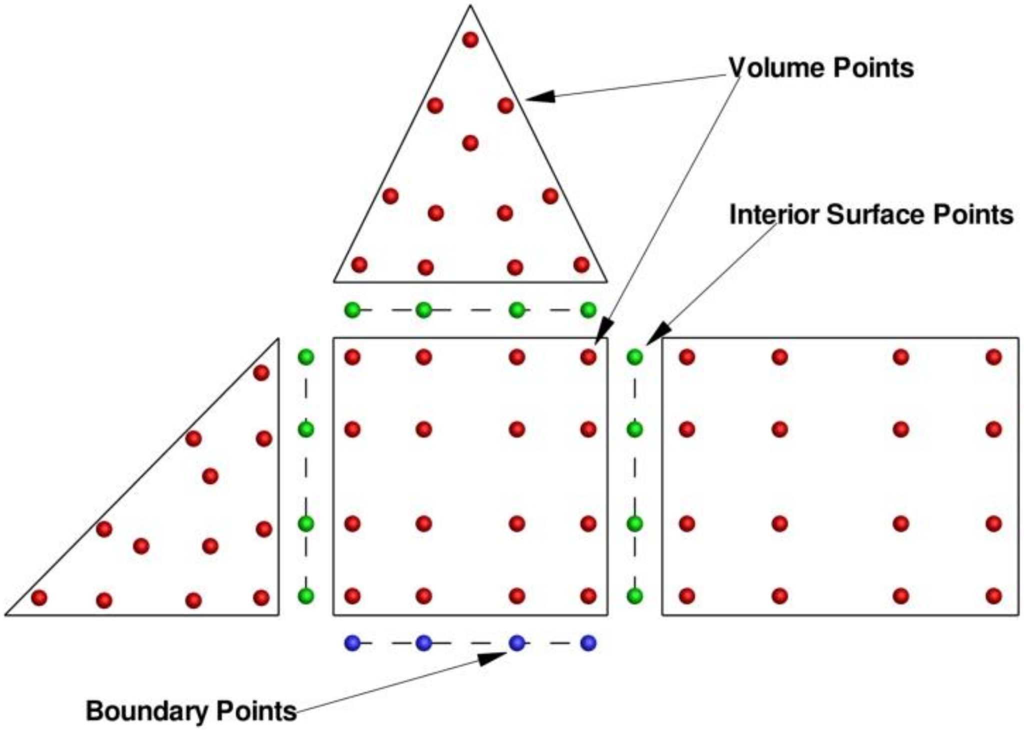
$$\frac{\partial F}{\partial t} + \frac{\partial \left( \langle u_j \rangle_L F \right)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ (\gamma + \gamma_t) \frac{\partial \left( F / \langle \rho \rangle_l \right)}{\partial x_j} \right] + \frac{\partial}{\partial \psi_k} \left[ \Omega_m F \left( \psi_k - \langle \phi_k \rangle_L \right) \right]$$

- SGS energy

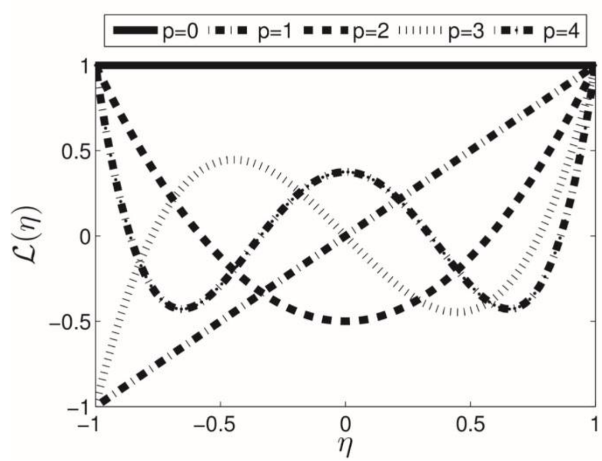
$$\frac{\partial (\langle \rho \rangle_\ell \tau_{\alpha\alpha})}{\partial t} + \frac{\partial [\langle \rho \rangle_\ell \langle u_j \rangle_L \tau_{\alpha\alpha}]}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ (\gamma + \gamma_t) \frac{\partial \tau_{\alpha\alpha}}{\partial x_j} \right] + 2(\gamma + \gamma_t) \left[ \frac{\partial (\langle \phi_\alpha \rangle_L)}{\partial x_j} \frac{\partial (\langle \phi_\alpha \rangle_L)}{\partial x_j} \right] - 2\Omega_m \langle \rho \rangle_\ell \tau_{\alpha\alpha}$$

# DG Methodology

- Discontinuous elements in space.
- Using basis Functions to approximate solution.
- Finite element method using Riemann solver for fluxes.

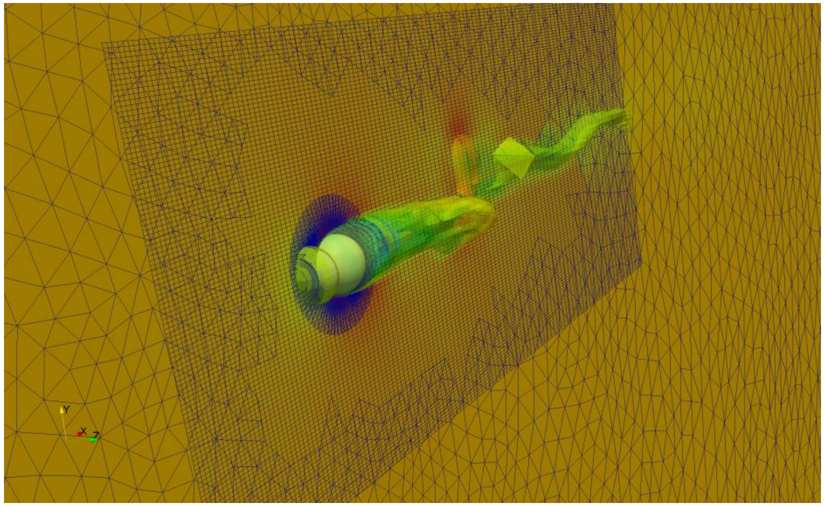


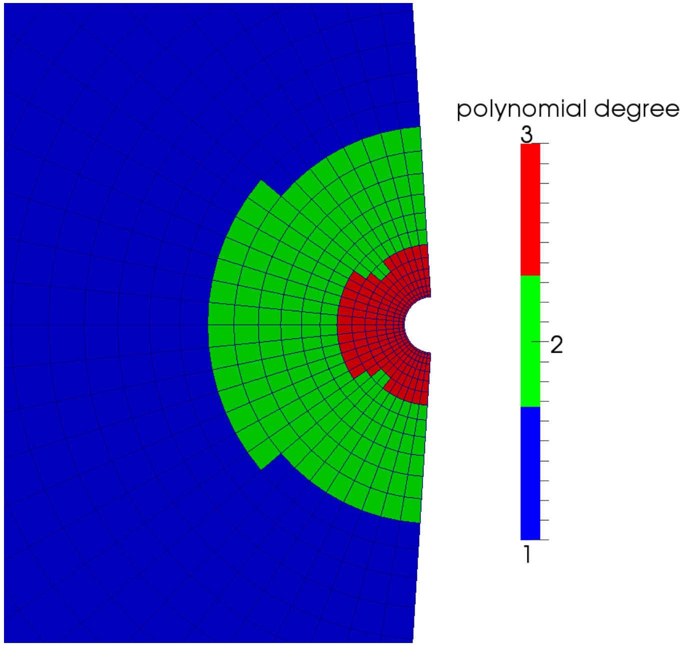

$$U_{app} = \sum_{j=1}^{s} a_j \mathcal{L}_{j,app}$$



# DG Solver Capabilities

- Hybrid mixed element unstructured meshes (tetrahedra, prisms, pyramids, and hexahedra)
- p-enrichment and h-refinement
- Curved mesh

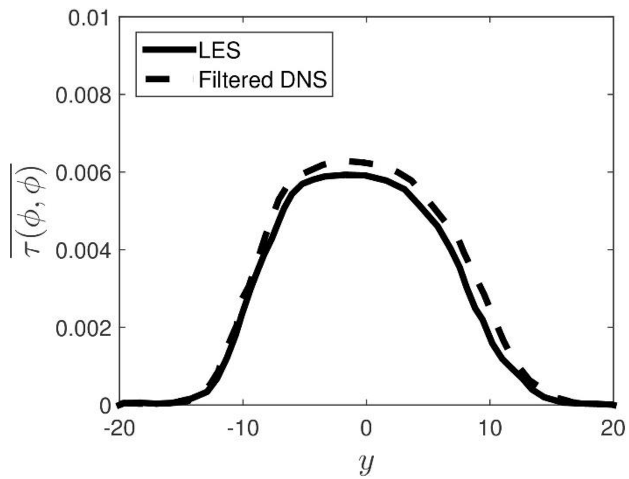




# Numerical Procedure

- 1 DNS case
  - $h=1/256$
  - $p=5$
- 27 LES cases
  - $h=1/128$, $1/64$ and $1/32$
  - $\Delta=1/64$, $1/32$ and $1/16$
  - $p=3$, $4$ and $5$
- Construct the $L_2$ norm error of subgrid scale energy ($\tau$), resolved energy ($R$) and total energy ($r$).

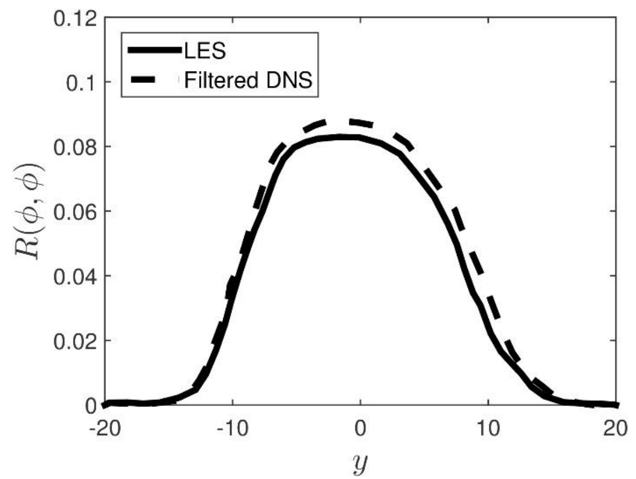
# Reynolds Stresses

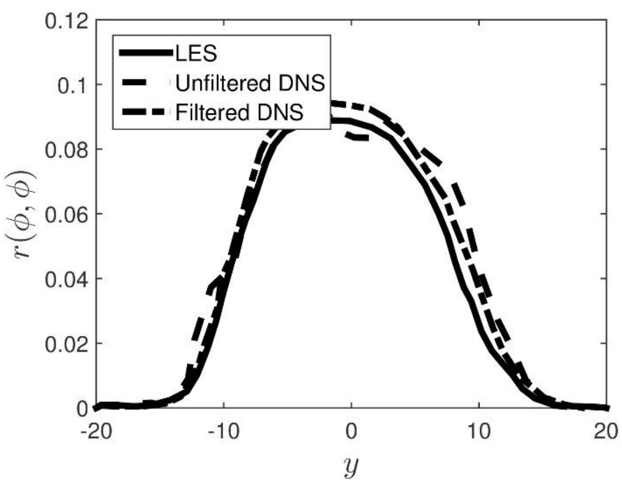


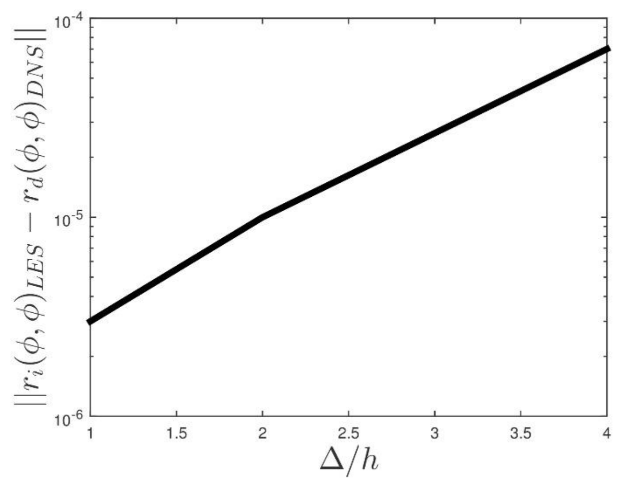

Δ=1/32

p=4

h=1/64

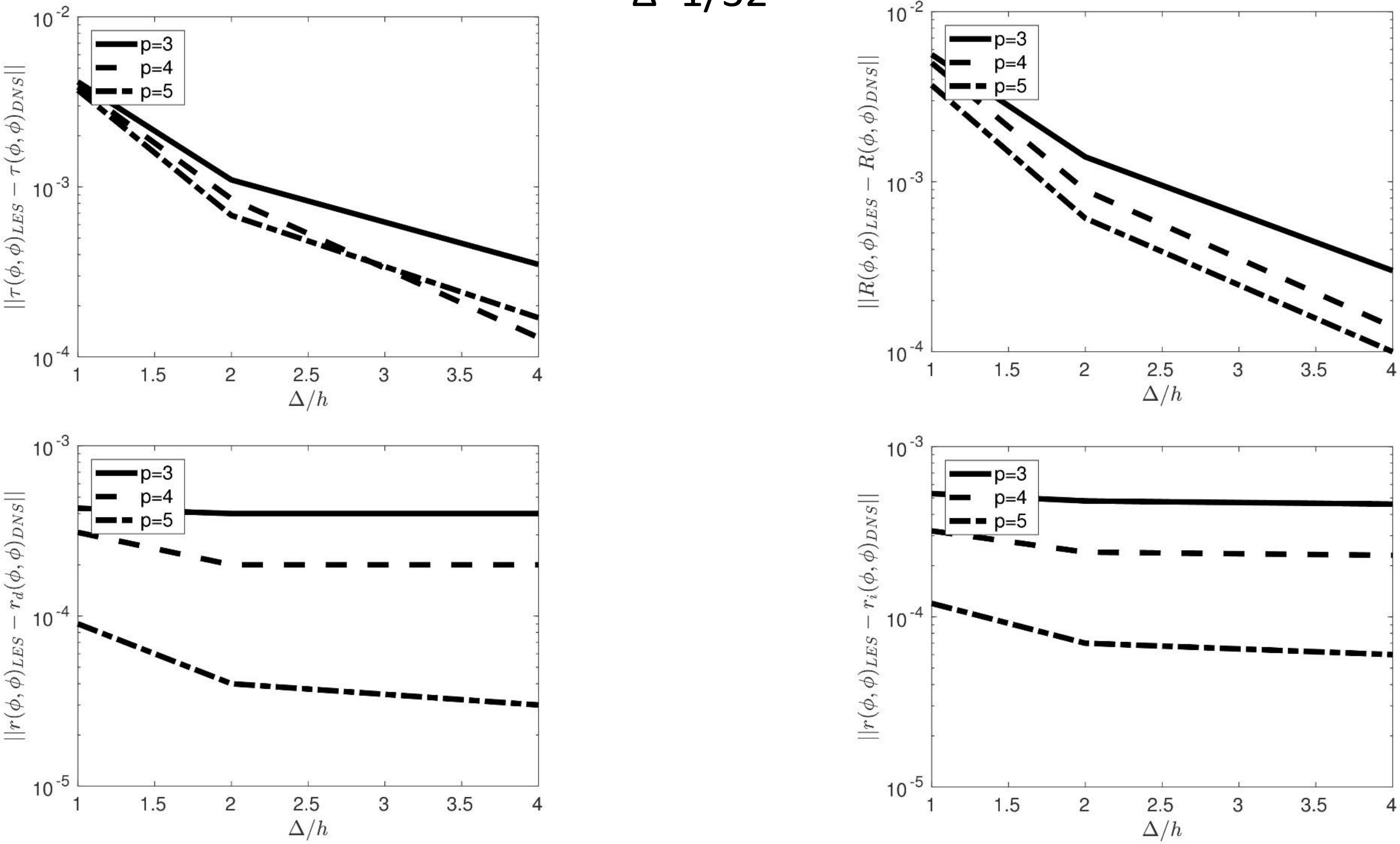


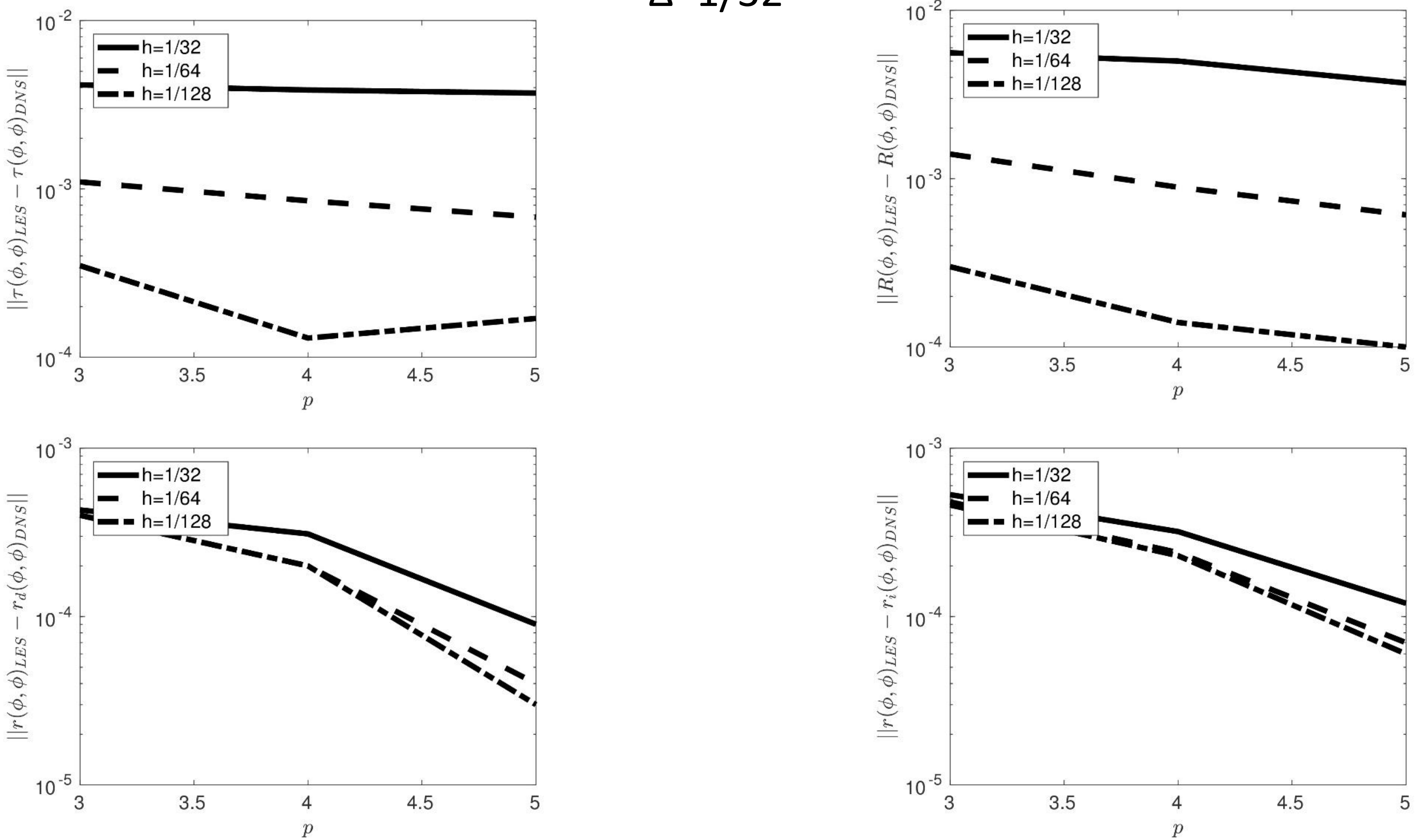
# h-refinement

Δ=1/32



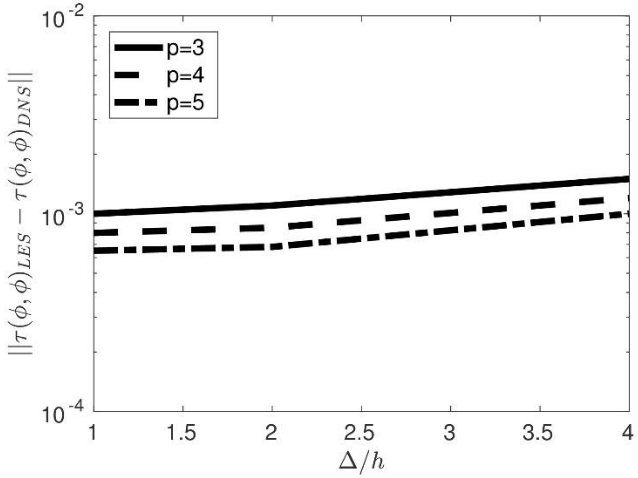
# p-enrichment

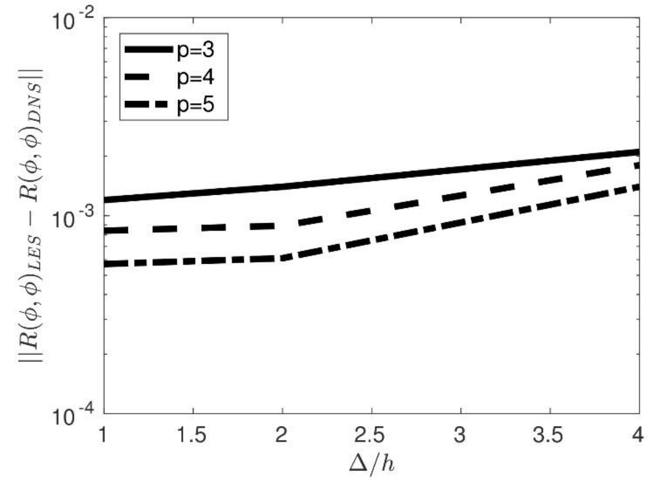
Δ=1/32

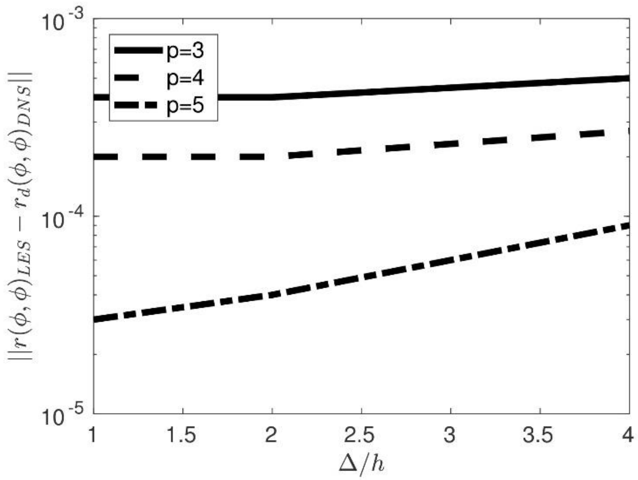


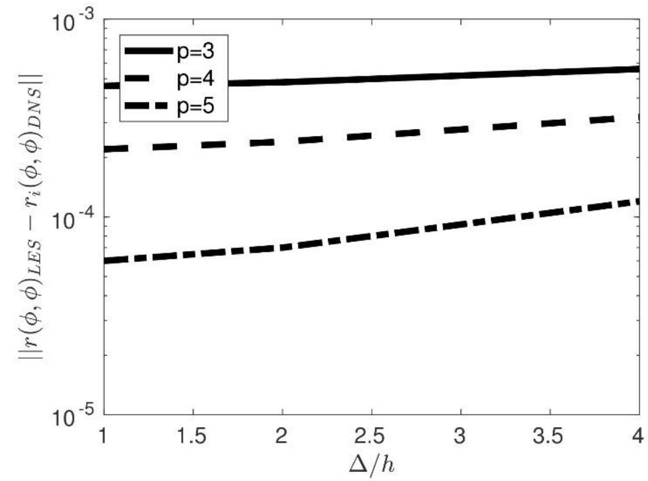
# Δ-refinement

h=1/64



# Summary

- DG-LES simulator
  - p-enrichment
    - As p goes higher, the error converges to zero for all Reynolds stresses.
  - h-refinement
    - Similar to the p-enrichment, the LES Reynolds stresses converges to the DNS results for finer resolution.
  - Δ-refinement
    - Where the h is fixed, the best solution is obtained at $\Delta/h=1$.
  - Generic filter
    - Convergence as $\Delta/h$ decreases.

Thank you!