Appropriate differential Reynolds stress modeling for turbomachinery flows

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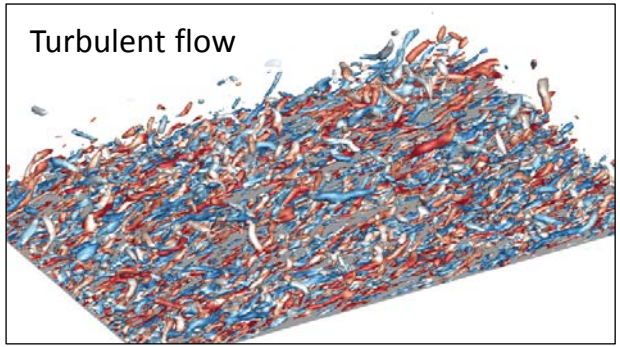
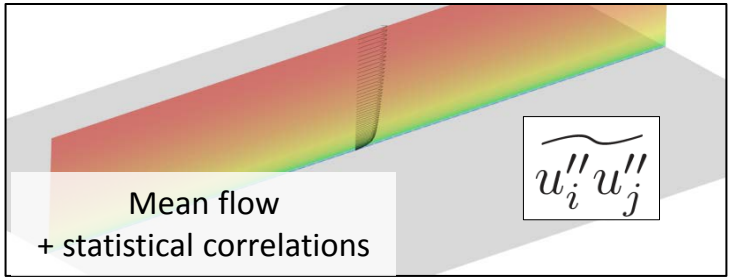
Overview



Statistical approach

$$u_i = \widetilde{u}_i + u_i''$$



Modelling

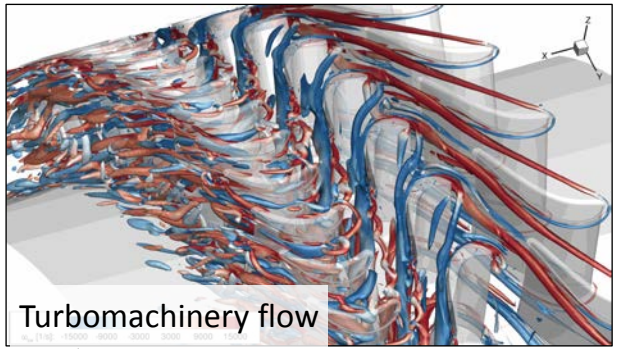
$$\frac{D}{Dt}(\overline{\rho}\widetilde{u}_i) = -\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\overline{\sigma}_{ij} - \overline{\rho}\widetilde{u_i'' u_j''}\right]$$

$$\frac{D}{Dt}\left(\overline{\rho}\widetilde{u_i'' u_j''}\right) = \dots$$

Differential
Reynolds stress model

Validation
Robustness

Application and
assessment



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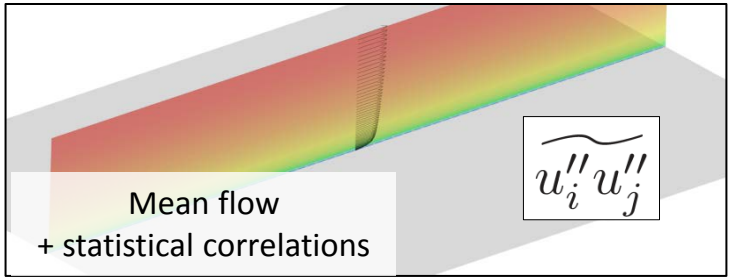
$$u_i = \widetilde{u}_i + u_i''$$



Modelling

$$\frac{D}{Dt}(\overline{\rho}\widetilde{u}_i) = -\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\overline{\sigma}_{ij} - \overline{\rho}\widetilde{u_i'' u_j''}\right]$$

$$\frac{D}{Dt}\left(\overline{\rho}\widetilde{u_i'' u_j''}\right) = \dots$$

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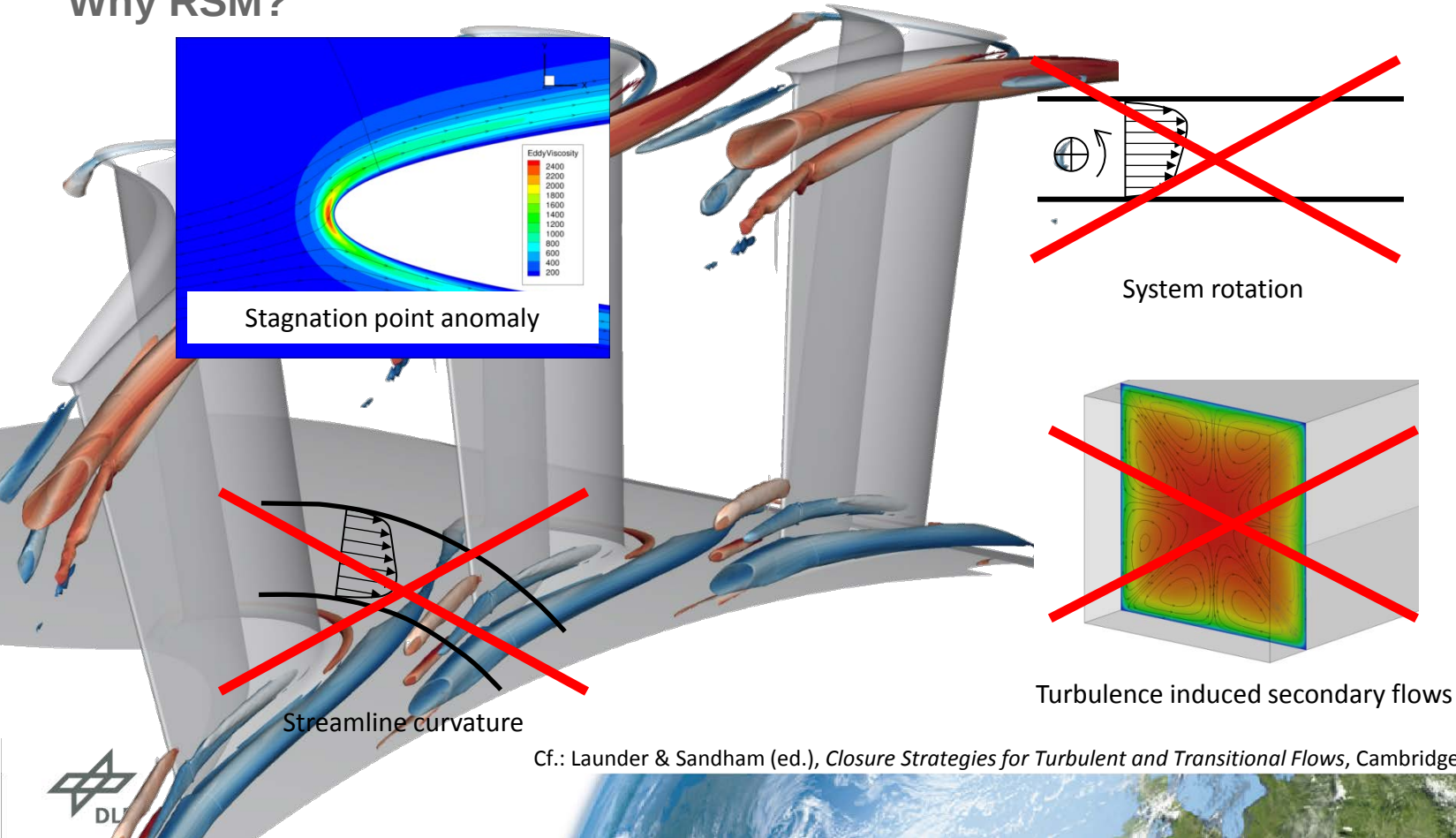
Why RSM?



Cf.: Launder & Sandham (ed.), *Closure Strategies for Turbulent and Transitional Flows*, Cambridge University Press, 2002

Reynolds stress models in TRACE

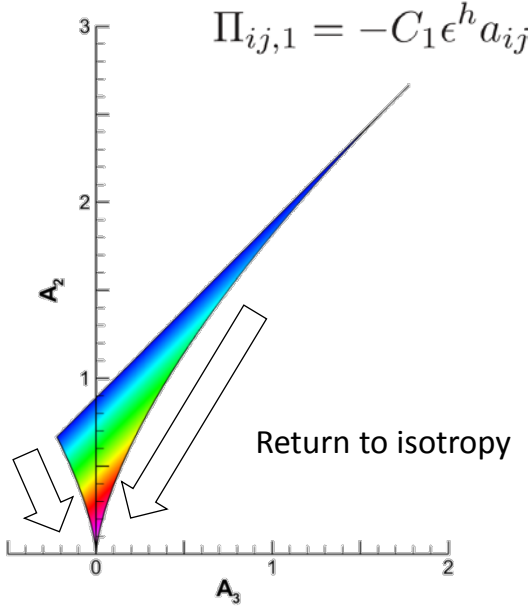
Model	Author	Comment
SSG/LRR-ω	Eisfeld	Switch between boundary layer and free shear layer
JH-ε^h	Jakirlic & Hanjalic	Near-wall modelling
JH-ω^h (Maduta)	Maduta & Jakirlic	
JH-ω^h	present work	

Pressure-strain models

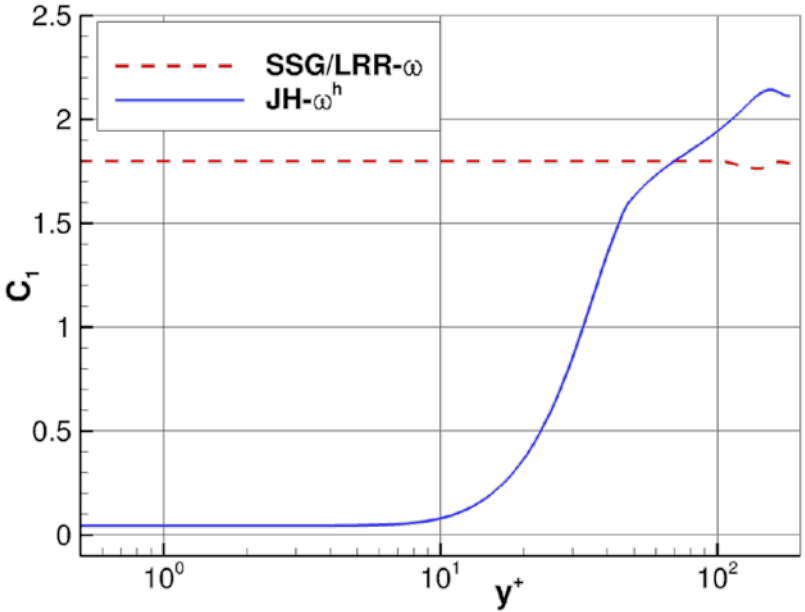
$$\overline{\rho}\Pi_{ij} = \overline{\rho}\left(\Pi_{ij,1} + \Pi_{ij,2}\right) = \overline{p'\left(\frac{\partial u_i''}{\partial x_j} + \frac{\partial u_j''}{\partial x_i}\right)}$$

Example: Slow part

$$\Pi_{ij,1} = -C_1 \epsilon^h a_{ij}$$



Coefficient function in boundary layer



Dissipation rate

$$\frac{D\left(\overline{\rho}\omega^h\right)}{Dt} = \frac{\partial}{\partial x_i}\left[\left(\frac{1}{2}\mu + \sigma_\omega \mu_T\right)\frac{\partial \omega^h}{\partial x_i}\right] + \alpha\frac{\overline{\rho}\omega^h}{k}P_k - \beta\overline{\rho}\left(\omega^h\right)^2 + CD_{k\omega} + \frac{1}{C_\mu k}P_{\epsilon 3}$$

Diffusion Production Destruction Cross diffusion Gradient production

Model
JH-ω^h (Maduta)
JH-ω^h
SSG/LRR-ω

Dissipation rate

$$\frac{D\left(\overline{\rho}\omega^h\right)}{Dt} = \frac{\partial}{\partial x_i}\left[\left(\frac{1}{2}\mu + \sigma_\omega \mu_T\right)\frac{\partial \omega^h}{\partial x_i}\right] + \alpha\frac{\overline{\rho}\omega^h}{k}P_k - \beta\overline{\rho}\left(\omega^h\right)^2 + CD_{k\omega} + \frac{1}{C_\mu k}P_{\epsilon 3}$$

Diffusion Production Destruction Cross diffusion Gradient production

Model	$CD_{k\omega}$
JH-ω^h (Maduta)	$\frac{2}{k}\left(\frac{1}{2}C_{\mathrm{cr}}\mu + \sigma_d\mu_T\right)\frac{\partial \omega^h}{\partial x_i}\frac{\partial k}{\partial x_i}$
JH-ω^h	$\sigma_d\frac{2\overline{\rho}}{\omega^h}\max\left[\frac{\partial \omega^h}{\partial x_i}\frac{\partial k}{\partial x_i}, 0\right]$
SSG/LRR-ω	$\sigma_d\frac{\overline{\rho}}{\omega}\frac{\partial \omega}{\partial x_j}\frac{\partial k}{\partial x_j}$

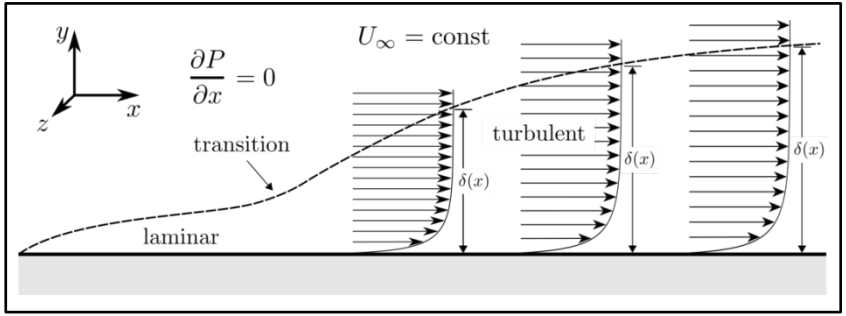
Dissipation rate

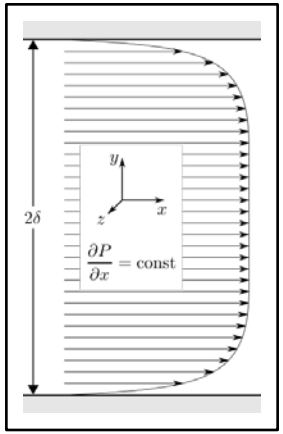
$$\frac{D\left(\overline{\rho}\omega^h\right)}{Dt} = \frac{\partial}{\partial x_i}\left[\left(\frac{1}{2}\mu + \sigma_\omega \mu_T\right)\frac{\partial \omega^h}{\partial x_i}\right] + \alpha\frac{\overline{\rho}\omega^h}{k}P_k - \beta\overline{\rho}\left(\omega^h\right)^2 + CD_{k\omega} + \frac{1}{C_\mu k}P_{\epsilon 3}$$

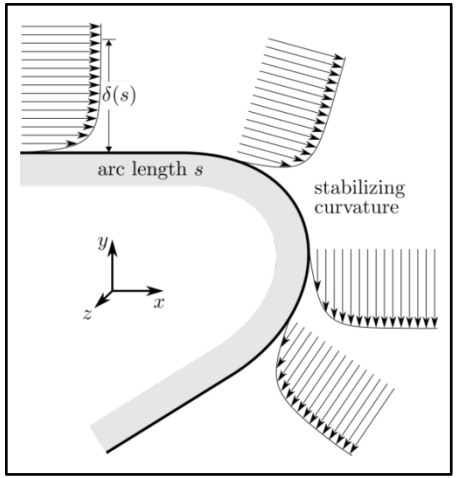
Diffusion Production Destruction Cross diffusion Gradient production

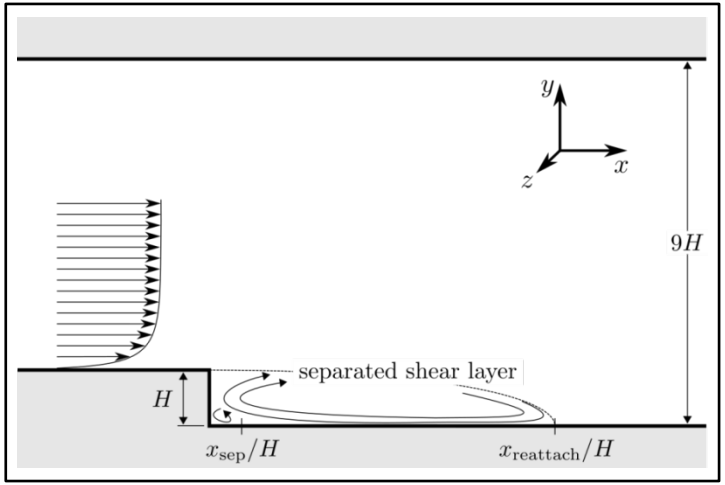
Model	$CD_{k\omega}$	Formulation $P_{\varepsilon 3}$
JH-ω^h (Maduta)	$\frac{2}{k}\left(\frac{1}{2}C_{\mathrm{cr}}\mu + \sigma_d\mu_T\right)\frac{\partial \omega^h}{\partial x_i}\frac{\partial k}{\partial x_i}$	Simplified
JH-ω^h	$\sigma_d\frac{2\overline{\rho}}{\omega^h}\max\left[\frac{\partial \omega^h}{\partial x_i}\frac{\partial k}{\partial x_i}, 0\right]$	Original
SSG/LRR-ω	$\sigma_d\frac{\overline{\rho}}{\omega}\frac{\partial \omega}{\partial x_j}\frac{\partial k}{\partial x_j}$	-

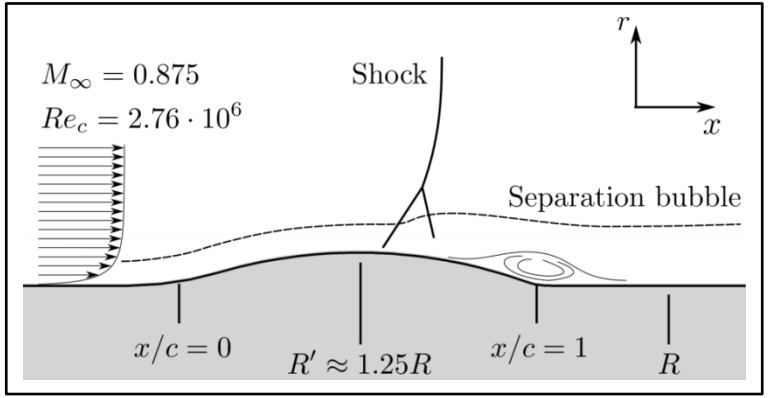
Validation


y
z
x
$\frac{\partial P}{\partial x} = 0$
$U_\infty = \text{const}$
transition
turbulent
laminar
$\delta(x)$
$\delta(x)$
$\delta(x)$


2δ
y
z
x
$\frac{\partial P}{\partial x} = \text{const}$


$\delta(s)$
arc length s
stabilizing curvature
y
z
x


y
z
x
$9H$
separated shear layer
H
x_{sep}/H
x_{reattach}/H


$M_\infty = 0.875$
$Re_c = 2.76 \cdot 10^6$
Shock
r
x
Separation bubble
$x/c = 0$
$R' \approx 1.25R$
$x/c = 1$
R

Overview

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Differential
Reynolds stress model

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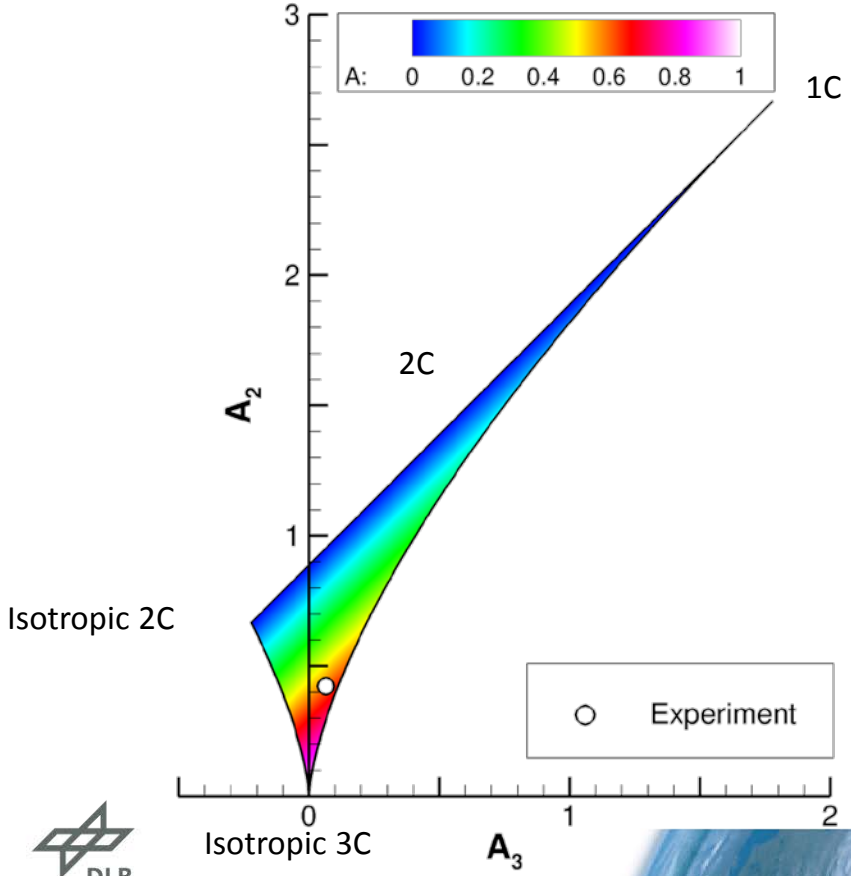
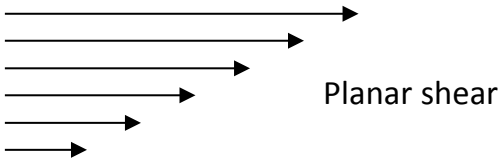
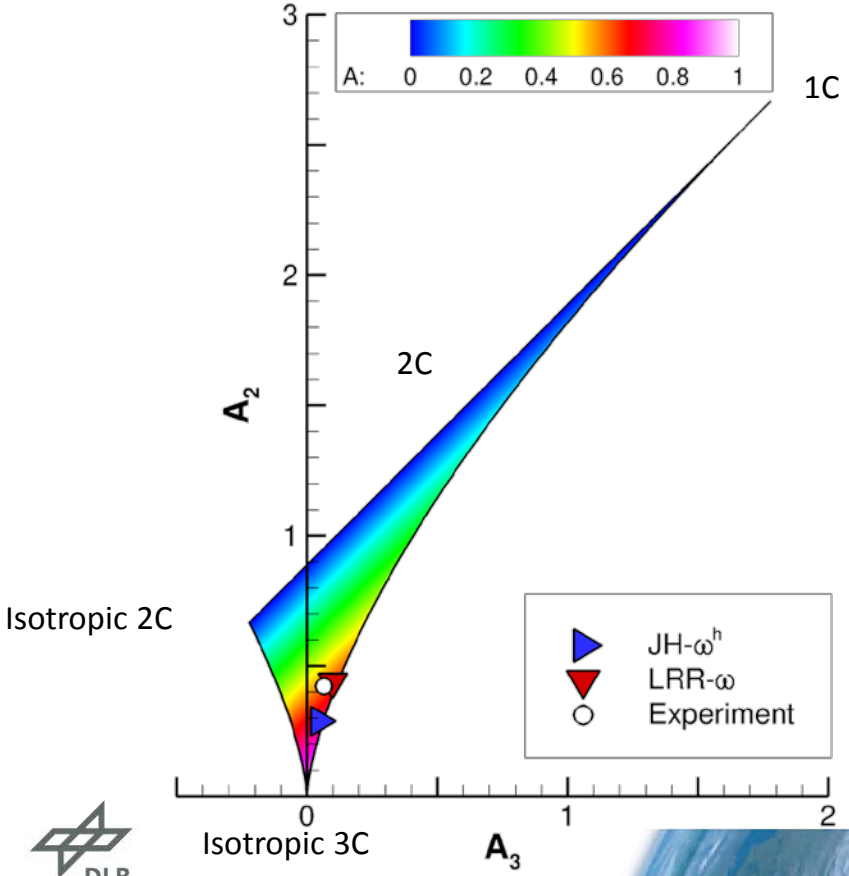
Laminarisation problem

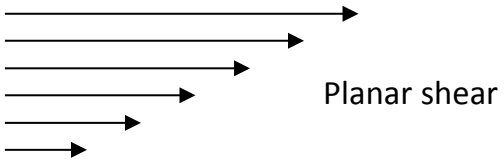




	LRR-ω	JH-ω^h

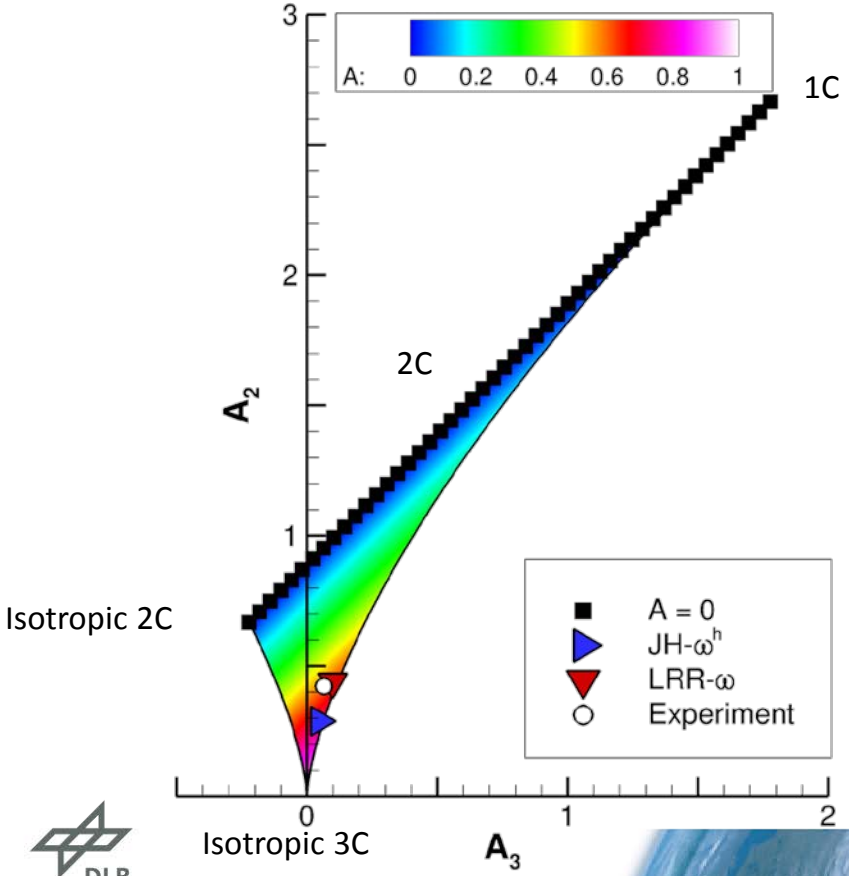
Laminarisation problem

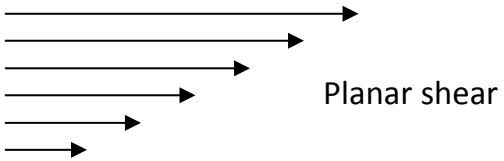




	LRR-ω	JH-ω^h

Laminarisation problem





	LRR-ω	JH-ω^h

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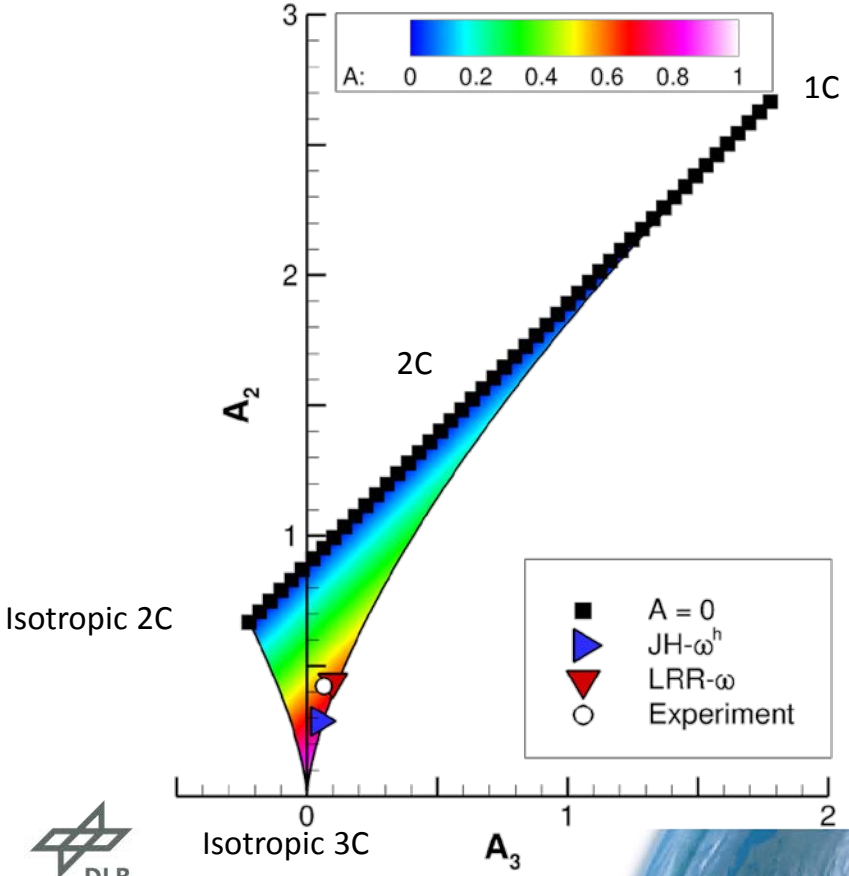
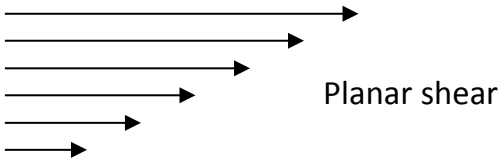
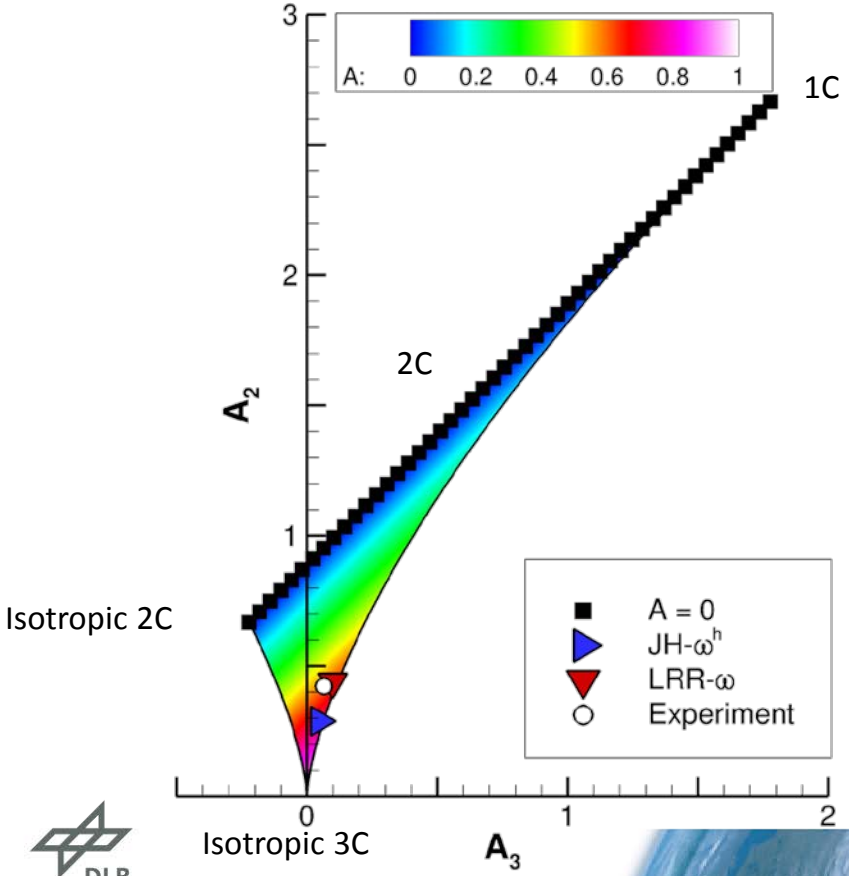


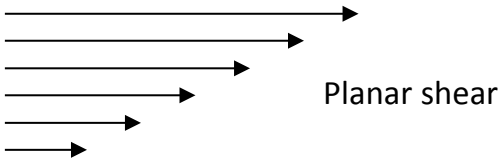


Stability analysis

	LRR-ω	JH-ω^h
A ≠ 0	Stable	Stable

Laminarisation problem





Stability analysis

	LRR-ω	JH-ω^h
A ≠ 0	Stable	Stable
A = 0	Unstable	All eigenvalues vanish

Numerical analysis of influencing factors

$$Re_T, \quad \frac{\omega}{S}, \quad \Pi_{ij,2}$$

Overview

Statistical approach

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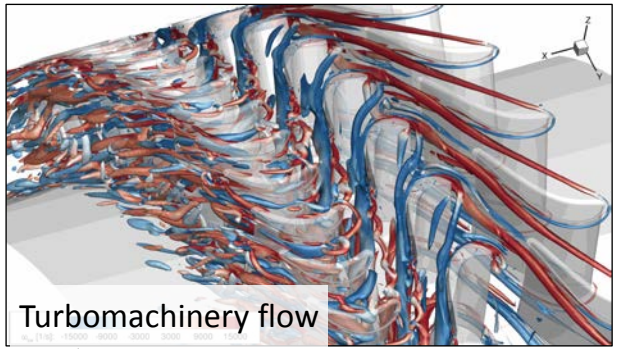
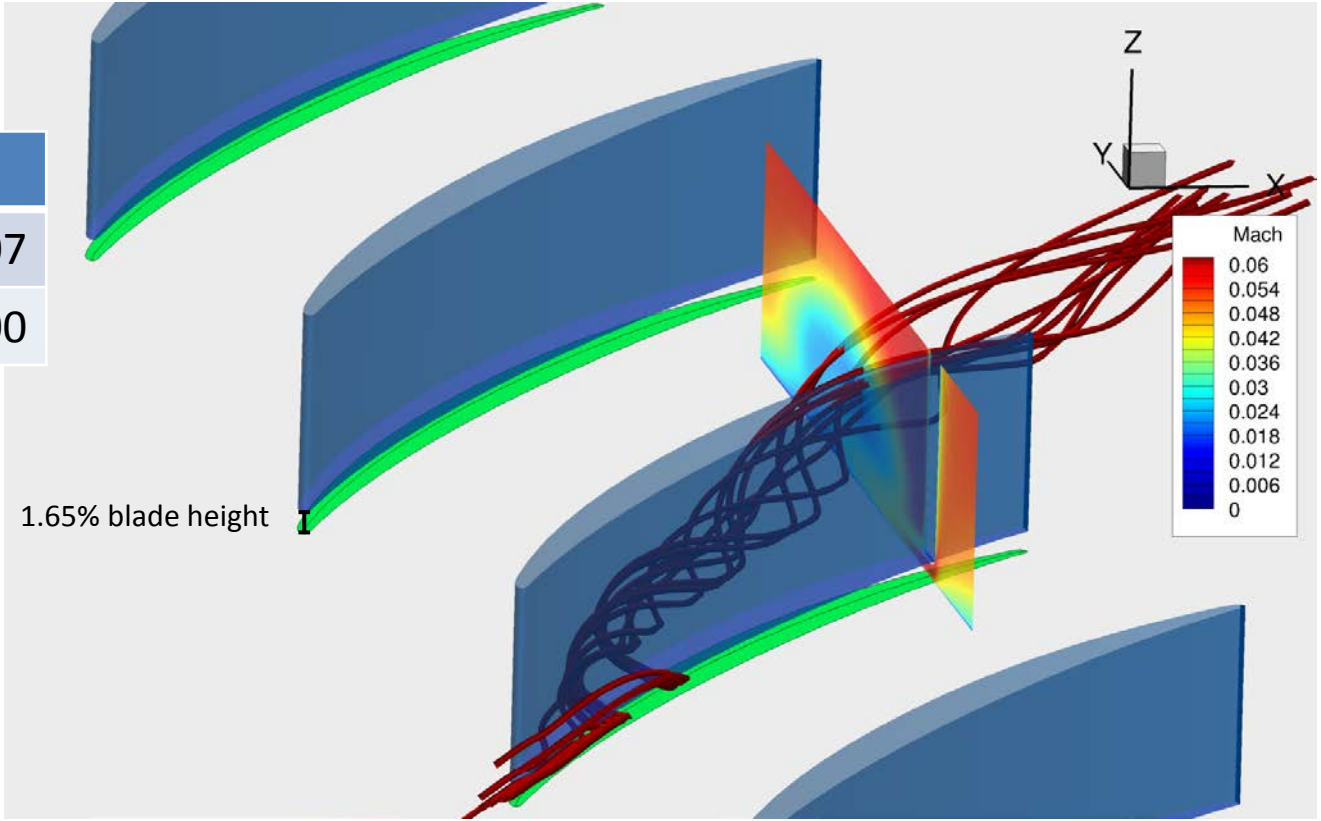
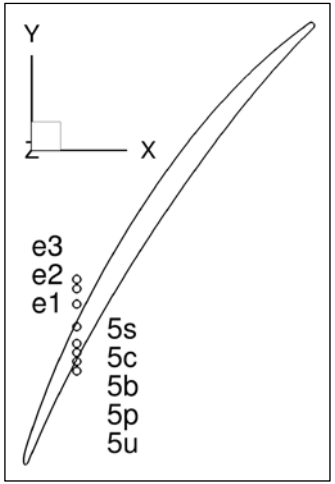


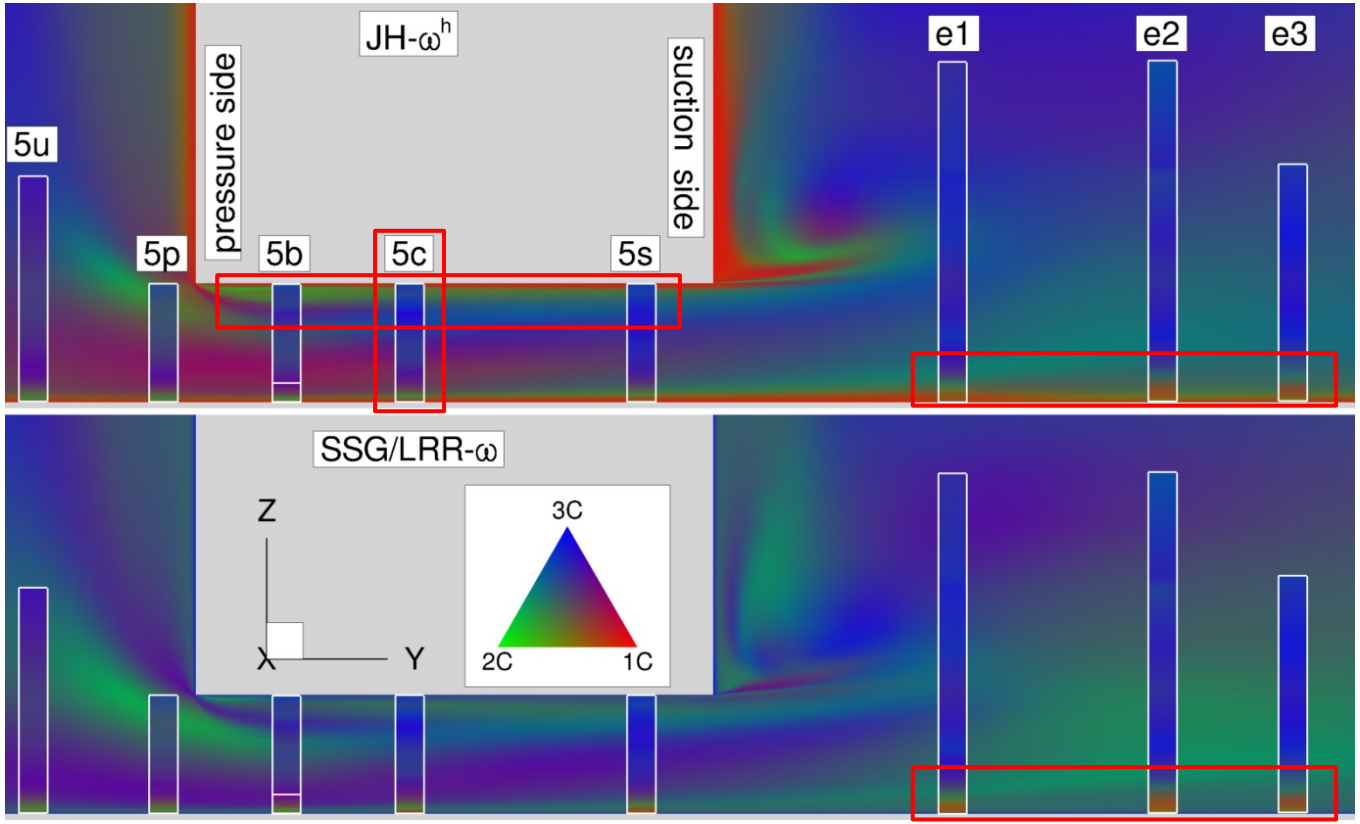
Virginia Tech Compressor Cascade

Parameter	
M	0.07
Re	400,000

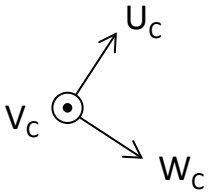


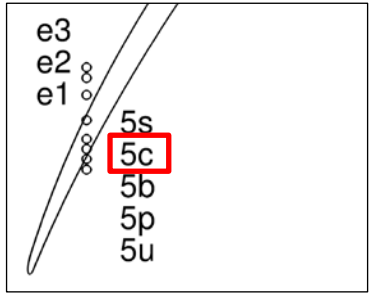
Tip gap flow: Reynolds stress tensor

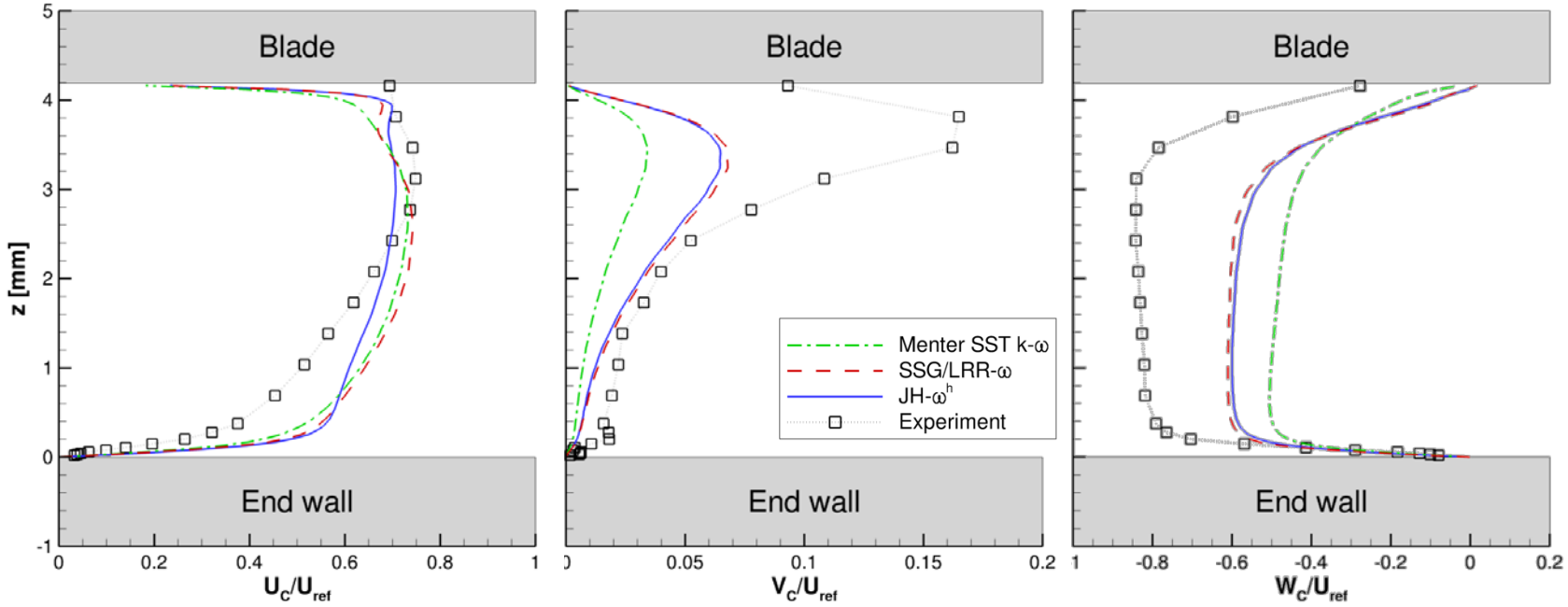




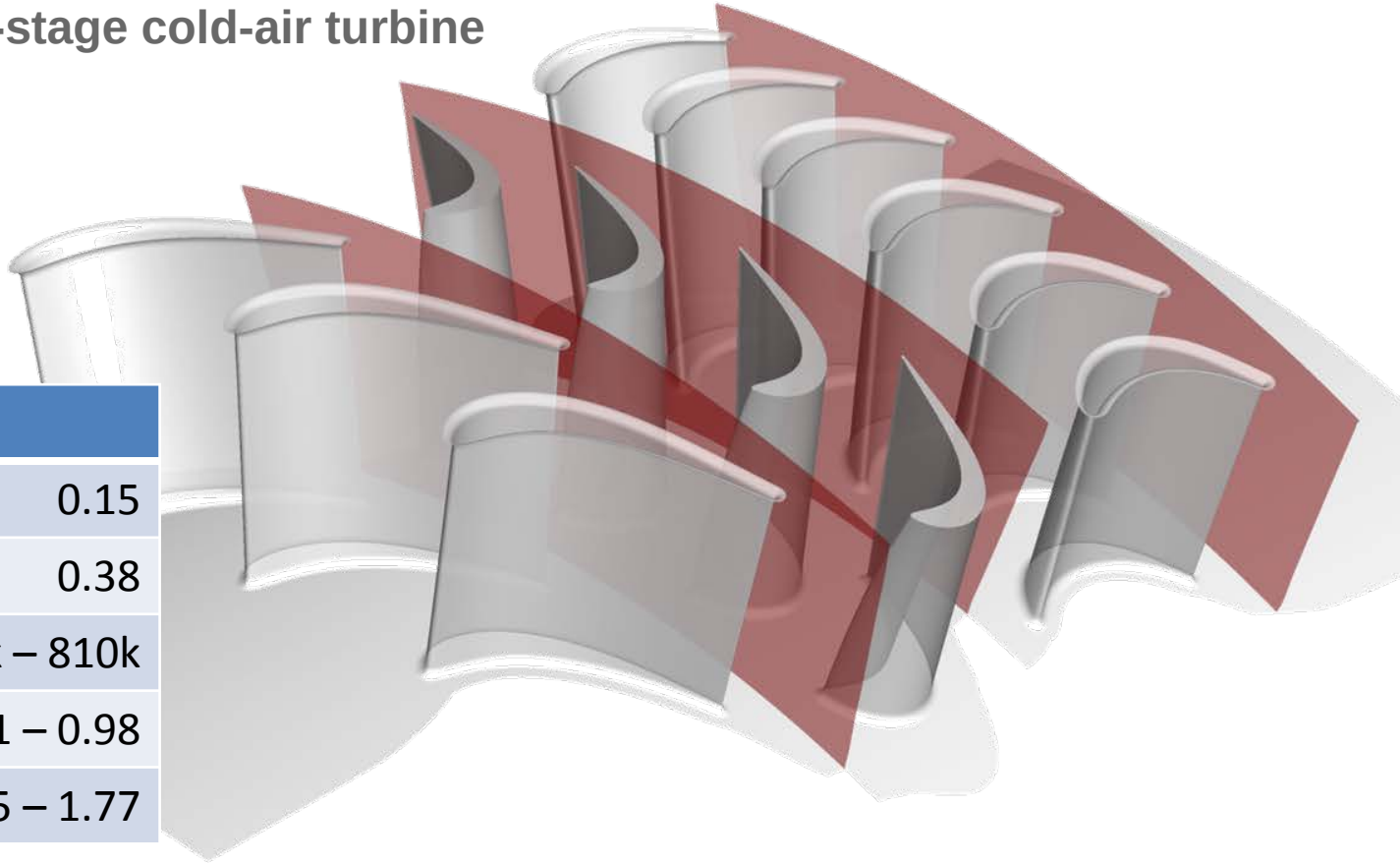
Tip gap flow: velocity vector





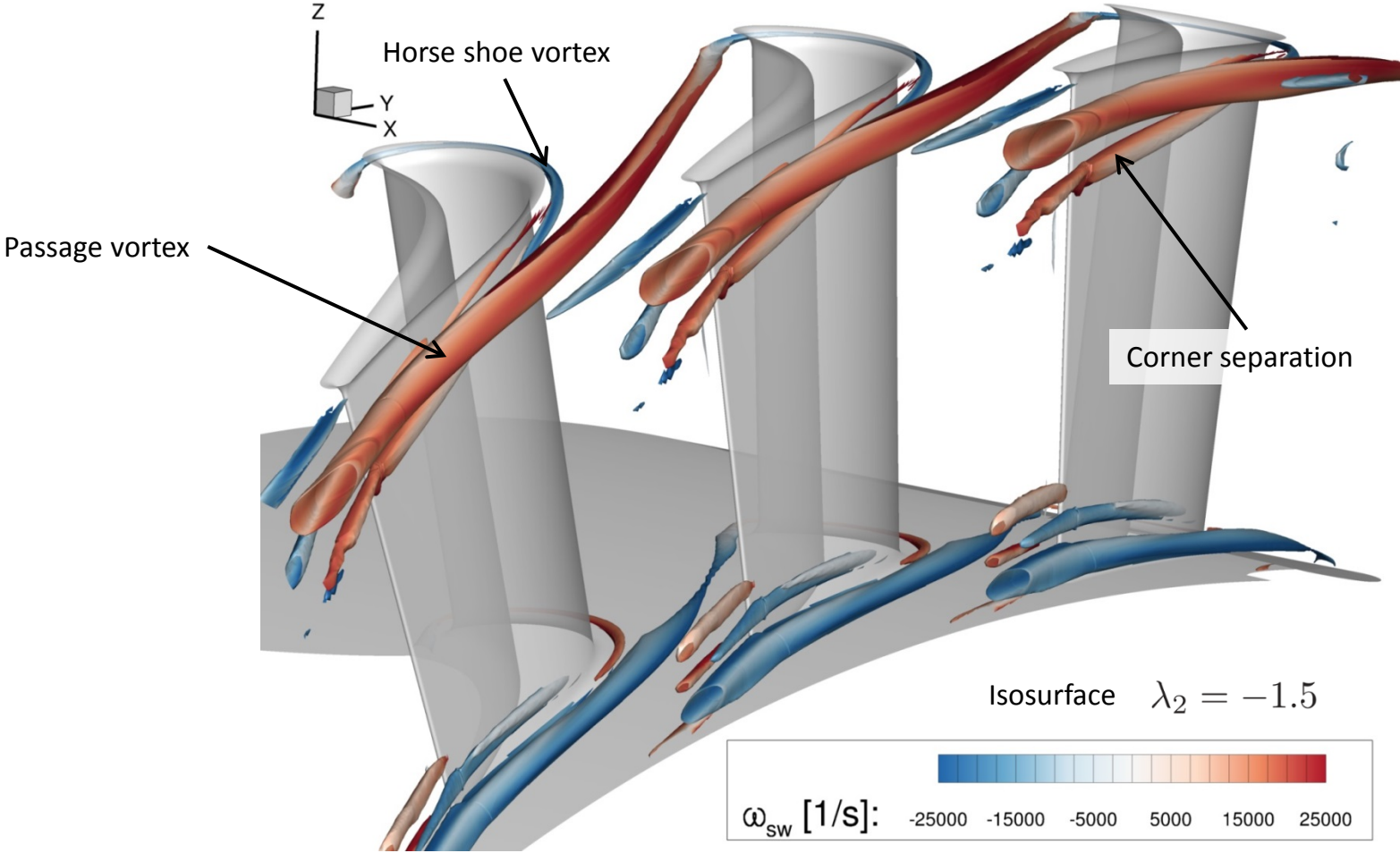


RWTH Aachen 1.5-stage cold-air turbine

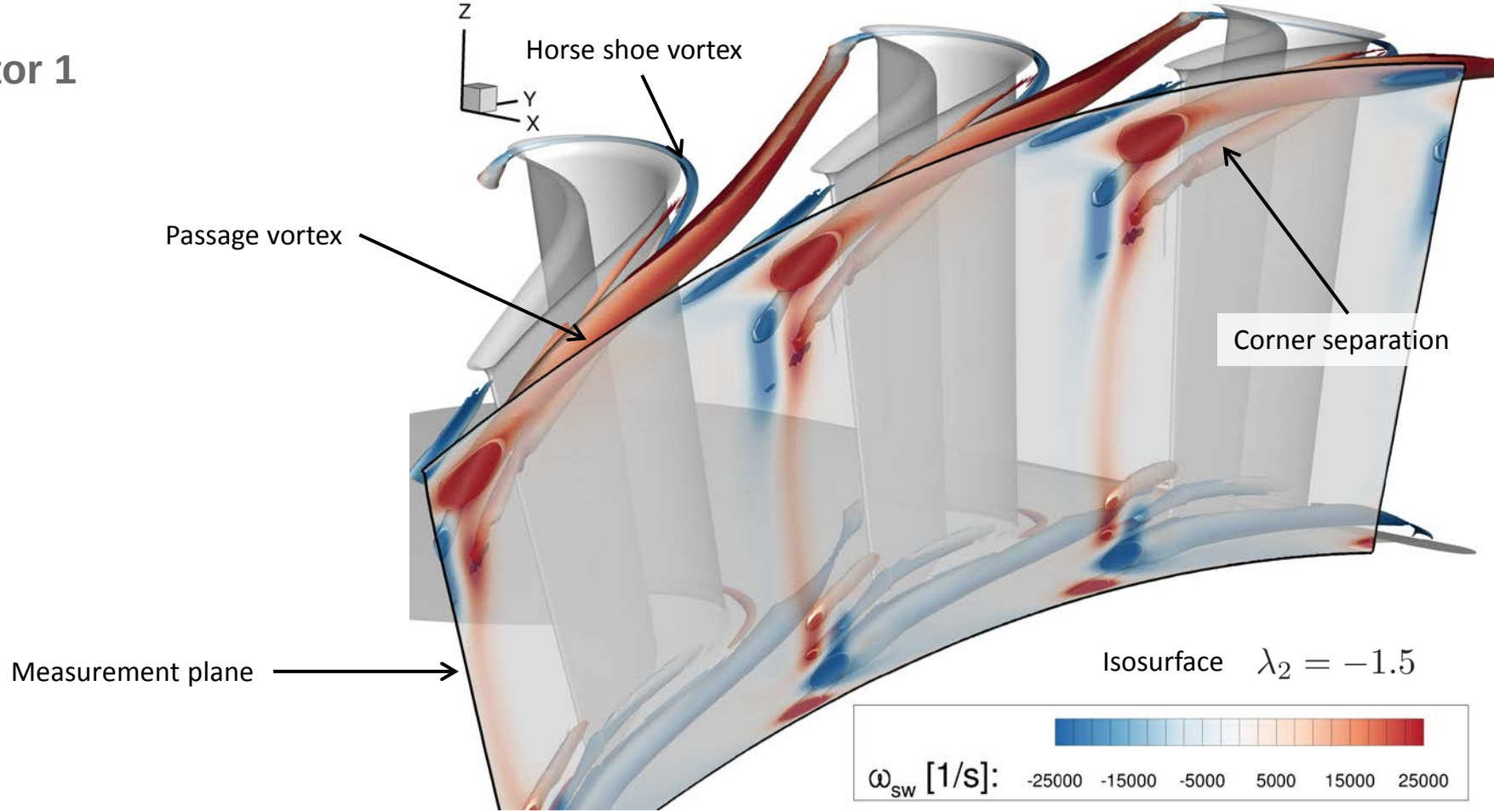


Parameter	
M_{in}	0.15
M_{out}	0.38
Re_{2th}	330k – 810k
t/c	0.71 – 0.98
h/l_{ax}	1.45 – 1.77

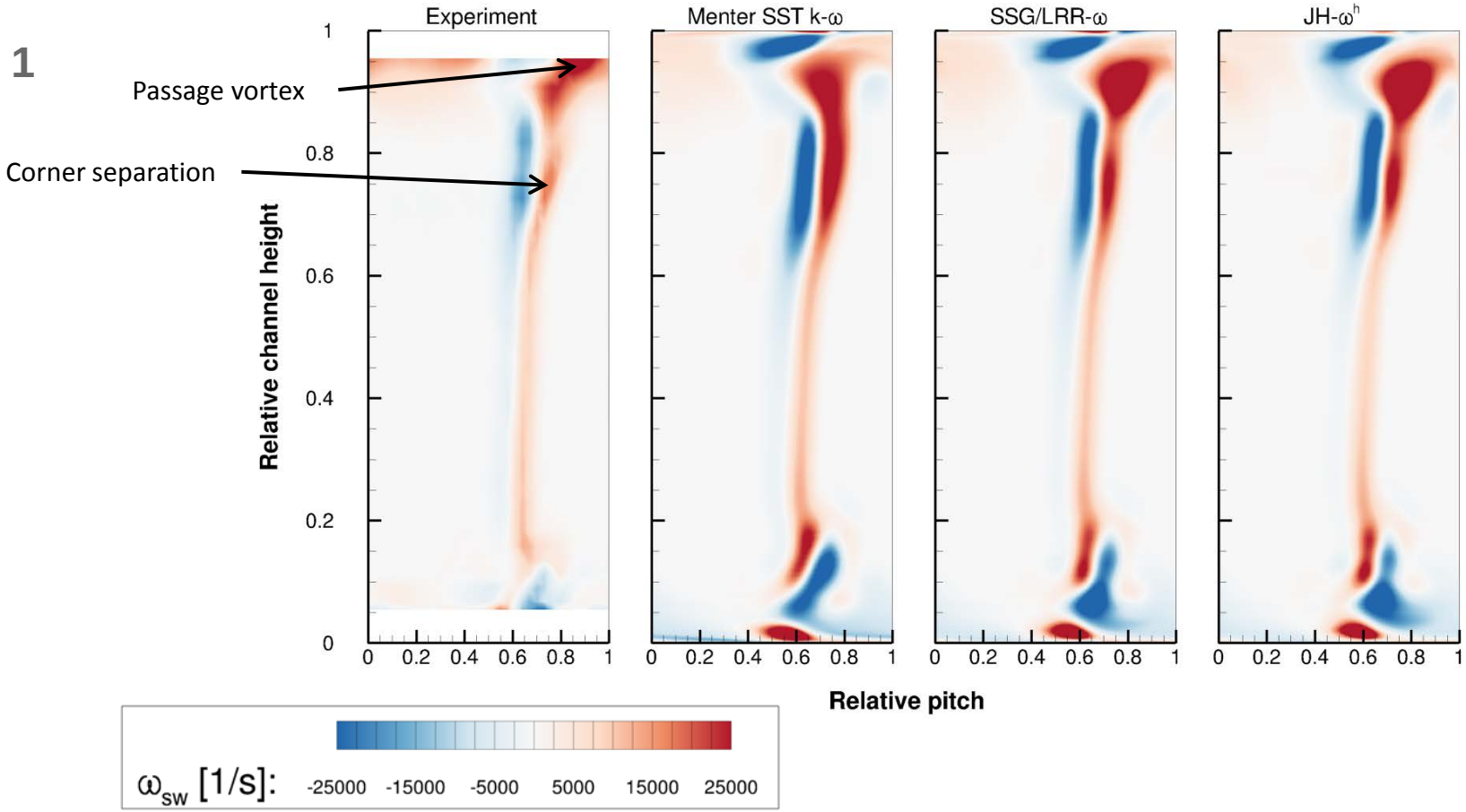
Stator 1


Z
Y
X
Horse shoe vortex
Passage vortex
Corner separation
Isosurface $\lambda_2 = -1.5$
ω_{sw} [1/s]:
-25000
-15000
-5000
5000
15000
25000

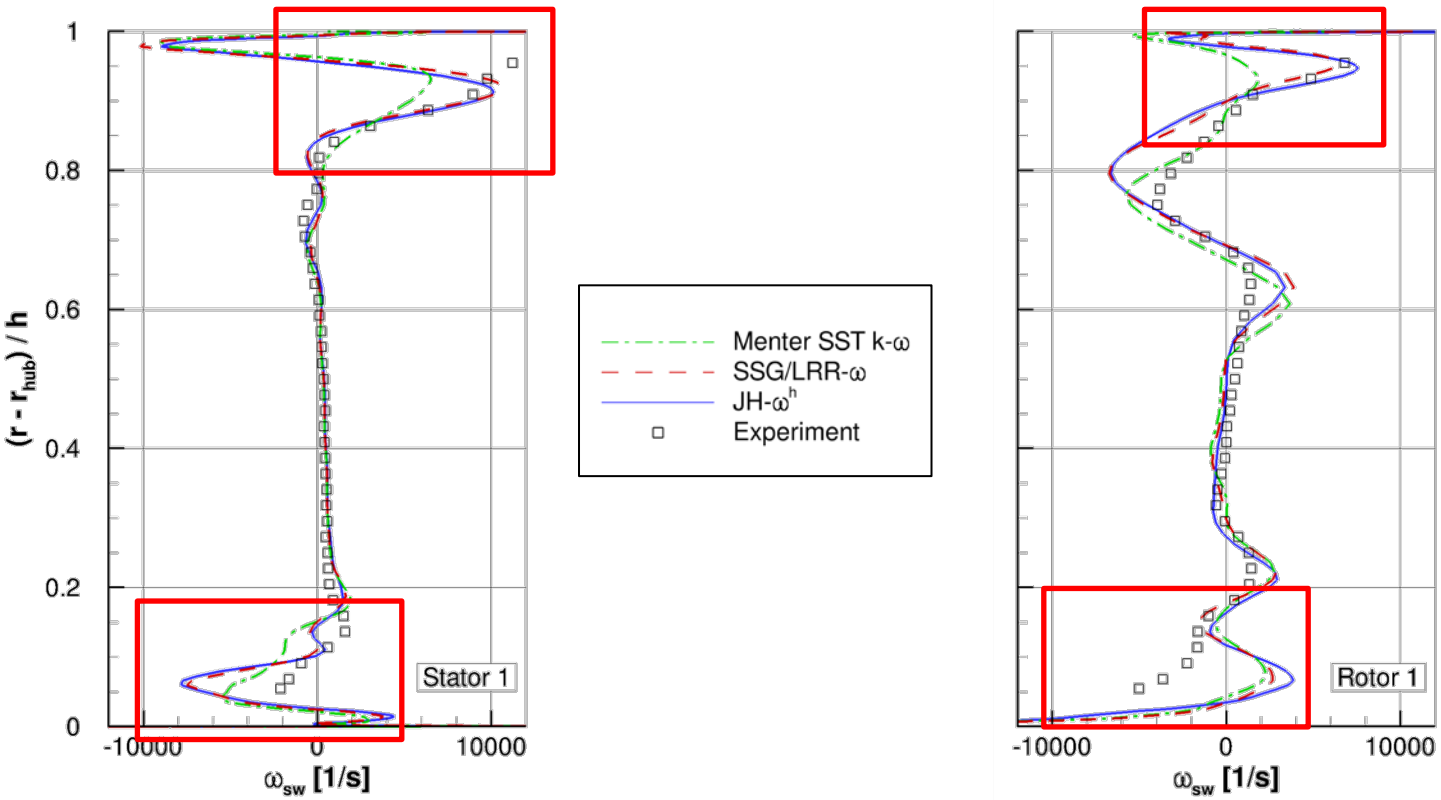
Stator 1


Z
Y
X
Horse shoe vortex
Passage vortex
Corner separation
Measurement plane
Isosurface $\lambda_2 = -1.5$
ω_{sw} [1/s]:
-25000
-15000
-5000
5000
15000
25000

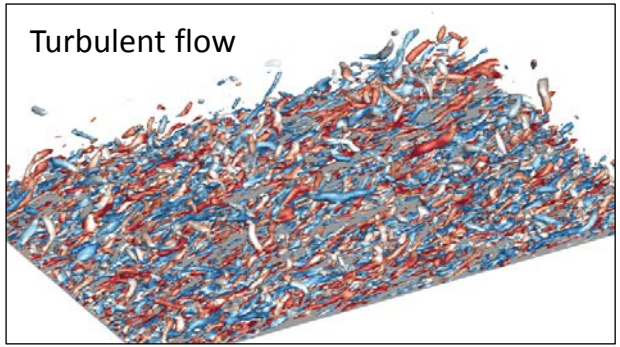
Stator 1


Experiment
Menter SST k-ω
SSG/LRR-ω
JH-ω^h
Passage vortex
Corner separation
Relative channel height
Relative pitch
0
0.2
0.4
0.6
0.8
1
ω_sw [1/s]:
-25000
-15000
-5000
5000
15000
25000

Circumferential averages



Summary



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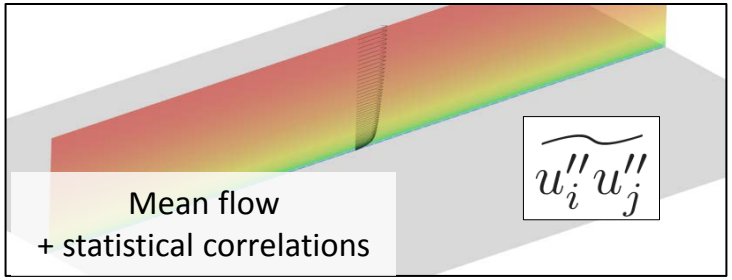

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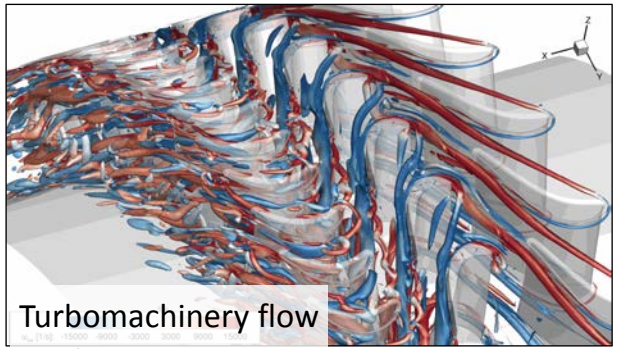
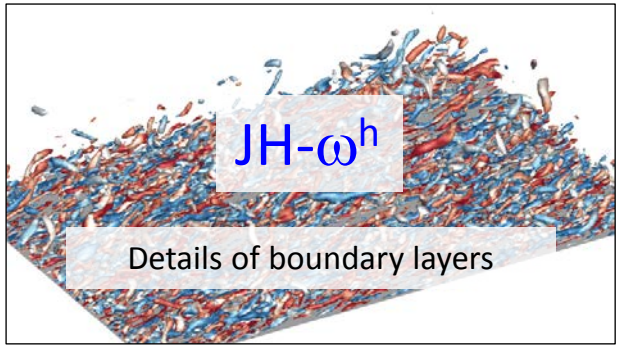


Conclusion



Outlook